Analysis and Compensation of Oscillations Induced by Control Valve Stiction

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Abstract—A time-domain approach (TDA) is formulated to analyze oscillations that are induced by control valve stiction in feedback control loops. Analytical relationships are established between the proportional-integral controller parameters, and the oscillation amplitude and period of the process output. Based on the relationships and the robustness to model uncertainties, a new compensation method by tuning controller parameters is proposed to reduce the oscillation amplitude to a desired value. Compared with the describing function approach, the TDA achieves a significant improvement in accuracy in calculating the oscillation amplitude and period. By contrast to the counterparts in the literature, the proposed compensation method is quantitative and avoids tuning the controller parameters in a trial-and-error manner to compensate oscillations. Experimental examples illustrate the effectiveness of the obtained results.

Index Terms—Control valve, describing function, oscillation, stiction compensation.

I. INTRODUCTION

Industrial surveys [1]–[3] reported that about 20–30% of control loops in process industries suffer from poor performance due to control valve nonlinearities. Among these nonlinearities, the control valve stiction usually leads to oscillations in closed-loop systems, which has received increasing attention recently [4]. The first step to deal with control valve stiction is to detect the existence of stiction and quantify its severity. Once the stiction has been detected and quantified, a typical method for eliminating or mitigating its negative effects is to perform valve maintenance or replacement; however, this is feasible only during the plant shutdowns, which are usually scheduled every six months to three years [5]. Consequently, it is desirable to continue the operation of sticky control valves and to apply some compensation methods to reduce or remove the oscillations caused by control valve stiction.

Existing compensation methods can be classified mainly into four categories, namely, the knocker method [6]–[8], the constant reinforcement method [9], the two-movement method [5], [10], [11], and the controller tuning method [12], [13]. In particular, the controller tuning method does not change the control loop configuration, and it is relatively easy to implement in practice. However, the existing controller tuning methods typically have one severe drawback: they are based on the describing function approach (DFA) [14], [15] for the oscillation analysis, which involves a rough approximation of control valve stiction and has large errors in calculating the oscillation amplitude and period. As a result, only qualitative tuning suggestions can be provided on increasing or decreasing the controller parameters [12], [13] and, thus, the controller parameters are determined in a trial-and-error manner. Aside from the literature on the compensation of control valve stiction, there also exists relevant work based on the DFA on the limit cycle analysis and controller design for systems with hysteresis-type nonlinearities [16]–[20]. However, the aforementioned limitation of the DFA remains.

This paper has two main contributions.

1) A time-domain approach (TDA) is formulated to establish analytical relationships between the proportional-integral (PI) controller parameters and the oscillation amplitude and period of the process output. The proposed TDA is compared with the widely-accepted DFA: numerical and laboratorial studies show that the TDA greatly outperforms the DFA in terms of accuracy in calculating the oscillation amplitude and period.

2) Based on these relationships, together with the robustness to model uncertainties as another specification, a new oscillation compensation method is proposed to obtain the PI controller parameters for reducing the oscillation amplitude to a desired value. The proposed compensation method is quantitative, while the existing counterparts are qualitative. Thus, the controller parameters are no longer tuned in a trial-and-error manner for oscillation compensation.

The rest of this paper is organized as follows. Section II describes the research problem. The proposed TDA is presented in Section III, while the DFA is given in Appendix. The proposed oscillation compensation method is provided in Section IV. The obtained results are supported through numerical and laboratorial examples in Section V. Concluding remarks are given in Section VI.

II. PROBLEM FORMULATION

Consider a feedback control loop in Fig. 1, where a continuous-time process \( G(s) \) is actuated by a sticky control valve \( f \). Here \( r(t), e(t), u(t), m(t), y(t), w(t), \) and \( y_m(t) \)
denote the reference, control error, controller output, valve position, process output, process noise, and measured process output, respectively. The parameter $c$ is a real-valued constant to take care of the static offset of $y_m(t)$ so that $y(t)$ is a deviation signal from zero. The process $G(s)$ is described by a first-order plus dead time (FOPDT) model,

$$G(s) = \frac{K_p}{T_p s + 1} e^{-\theta s}. \quad (1)$$

A PI controller $C(s)$ is used

$$C(s) = K_c + \frac{K_i}{s}. \quad (2)$$

There are several data-driven stiction models in the literature [4], among which He’s stiction model [21] is adopted here to describe the behavior of $f$ in oscillations owing to its effectiveness in modeling valve stiction [11]. The signature plot of He’s stiction model (plotting $m(t)$ versus $u(t)$) for an oscillatory $u(t)$ in $[u_{\text{min}}, u_{\text{max}}]$ is shown in Fig. 2. The flowchart of He’s stiction model is presented in Fig. 3, where the discrete-time signal $u[k]$ is connected with its continuous-time counterpart $u(t)$ as $u[k] = u(kh)$ for $k = 0, 1, 2, \cdots$ with the sampling period $h$.

The two parameters $f_s$ and $f_d$ in Fig. 3 stand for the static and kinetic friction bands, respectively. The variable $u_r$ is the residual force acting on the valve which has not resulted in a valve movement, and the variable $u_{\text{cum}}$ is a current cumulative force acting on the valve.

The following assumptions are made throughout the paper.

**A1.** The process and sticky control valve can be described by $G(s)$ in (1) and He’s stiction model in Fig. 3, respectively. The signals $u(t), m(t)$ and $y_m(t)$ are measurable. The parameters of $G(s)$, namely, $K_p, T_p$ and $\theta$, the parameters $f_s$ and $f_d$ of He’s stiction model, and the static offset $c$ are known a priori, and the sign of the process gain $K_p$ is positive.

**A2.** The reference $r(t)$ stays at a known constant value $r_0$. Under some nonzero initial condition, the valve stiction leads to oscillations in the control loop, in which the valve position $m(t)$ moves back and forth at two known positions $m_1$ and $m_2$.

Assumption A1 is not restrictive. First, mathematical models of some processes, developed from related physical principles, are indeed the FOPDT models, e.g., the liquid storage process and the continuous stirred-tank reactor [22]. Second, the open-loop step response of most self-regulating processes is an S-shaped curve and can be well described by the FOPDT model [23]. Owing to the above reasons, the FOPDT model has been extensively adopted in the field of process control, and many model-based proportional-integral-derivative (PID) controller design methods have been developed based on the FOPDT model as the process dynamics [24], [25]. He’s stiction model is widely used to describe sticky control valves [21], [11]. With the measurements of $u(t), m(t)$ and $y_m(t)$, all the unknown system parameters $(K_p, T_p, \theta, c, f_s,$ and $f_d)$ can readily be estimated by identification methods [26]. The phenomenon stated in Assumption A2, namely, oscillations appear under a constant reference, and the valve moves between two positions, has been observed from many industrial control loops [4]. This assumption is further analyzed at the end of Section IV and also supported by Example 2 in Section V. Additionally, since the measurement of $m(t)$ is available, Assumption A2 is easy to confirm.

### III. Time-Domain Analysis of Oscillations

This section formulates a TDA to analyze oscillations induced by control valve stiction under Assumptions A1 and A2.

#### A. Main Idea

If the control valve has no stiction, the valve position $m(t)$, under a constant reference $r_0$, can reach its steady value $m_{ss}$ defined as
The FOPDT model (1) experiences step responses in oscillations, so with this objective, the main idea of the TDA is as follows. The process output \( y(t) \) (top, solid), reference \( r(t) \) (top, dash), controller output \( u(t) \) (bottom, solid), valve position \( m(t) \) (bottom, dash) and desired valve position \( m_{ss} \) (bottom, dashdot).}

\[
m_{ss} \triangleq (r_0 - c)/K_p.
\]

However \( m(t) \) cannot arrive at \( m_{ss} \) when the stiction is present; instead, \( m(t) \) jumps around \( m_{ss} \) in a periodic manner, leading to oscillations in the loop. Since the static offset \( c \) always appears together with \( r_0 \), we denote \( r_{ss} \triangleq r_0 - c \) as the nominal reference for the control loop with zero offset, and use \( y(t) \) instead of \( y_n(t) \) in calculating the oscillation amplitude.

When the oscillations appear, the signals evolve as illustrated in Fig. 4. During one time period \([t_A, t_E]\), the valve jumps up at the time instant \( t_A \) from the lower position \( m_2 \) to the upper position \( m_1 \), and holds this value until \( t_C \) when it moves back to \( m_2 \). At \( t_B \), \( m(t) \) experiences an upward jump again. Define two time instants \( t_B \triangleq t_A + \theta \) and \( t_D \triangleq t_C + \theta \), and two time intervals \( T_1 \triangleq t_C - t_A \) and \( T_2 \triangleq t_E - t_C \), where \( \theta \) is the time delay of the FOPDT model (1). The oscillations imply that \( T_1 > \theta \) and \( T_2 > \theta \). Note that as shown in Fig. 4, the valve position \( m(t) \) can be asymmetric with respect to its desired value \( m_{ss} \). When the control valve moves, e.g., at the time instants \( t_A \) and \( t_C \), the relationship between \( u(t) \) and \( m(t) \) is

\[
m(t) = u(t) \pm f_d
\]

where the summation (subtraction) sign is applied if \( u(t) \) is decreasing (increasing).

The key objective of oscillation analysis is to compute \( T_1 \) and \( T_2 \), which will be shown later to characterize the oscillation. With this objective, the main idea of the TDA is as follows. The FOPDT model (1) experiences step responses in oscillations, so that the time-domain expressions of \( y(t) \) and \( u(t) \) can be obtained. With the character of the valve motion in (3), the values of \( y(t) \) and \( u(t) \) at some particular time instants \( t_A, t_B, t_C, t_D \), and \( t_E \) must satisfy certain conditions to formulate the oscillations. These conditions lead to the analytical relationships between the controller parameters and the oscillation amplitude and period of the process output.

![Fig. 4. Illustration of oscillatory signals in the feedback control loop: the process output \( y(t) \) (top, solid), reference \( r(t) \) (top, dash), controller output \( u(t) \) (bottom, solid), valve position \( m(t) \) (bottom, dash) and desired valve position \( m_{ss} \) (bottom, dashdot).](image)

![Fig. 5. (a) FOPDT model \( G(s) \), (b) a connection of a time delay operator \( e^{-\theta s} \) and a first-order model \( G_0(s) = K_p/(T_p s + 1) \) with an intermittent signal \( m_0(t) \).](image)

### B. Analysis of Process Output \( y(t) \)

The variation of the process output \( y(t) \) during one oscillation period is analyzed in this section according to the step response analysis of an FOPDT model [22], [27]. We decouple \( G(s) \) in (1) as a time delay operator \( e^{-\theta s} \) followed by a first-order model \( G_0(s) = K_p/(T_p s + 1) \), as shown in Fig. 5. Because \( T_1 > \theta \) and \( m(t) = m_2 \) for \( t \in [t_A - T_2, t_A] \), the subsystem \( G_0(s) \) of \( G(s) \) takes the constant value \( m_2 \) as the input during the time interval \([t_A, t_B]\). Then, the process output \( y(t) \), which is the same as the output of \( G_0(s) \), is a step response of a first-order model \( G_0(s) \) with the initial state \( y(t_A) \) and the input \( m_2 \) in the time interval \([t_A, t_B]\) [27], i.e.,

\[
y(t) = K_p m_2 + [y(t_A) - K_p m_2] e^{-(t-t_A)/T_p}, \ t \in [t_A, t_B].
\]

Thus, \( y(t) \) at \( t_B \) is

\[
y(t_B) = K_p m_2 + [y(t_A) - K_p m_2] e^{-\theta/T_p}.
\]

Because the input to \( G_0(s) \) is switched from the minimum value \( m_2 \) to the maximum value \( m_1 \) at \( t_B \), \( y(t) \) is minimized at this moment, namely, \( y_{\min} = y(t_B) \). Similarly, \( y_{\max} = y(t_D) \).

Rewrite (5) to let \( y(t_B) \) be the independent variable, i.e.,

\[
y(t_A) = K_p m_2 + [y(t_B) - K_p m_2] e^{\theta/T_p}.
\]

Similarly, the subsystem \( G_0(s) \) encounters a constant input \( m_1 \) in \([t_B, t_C]\), then

\[
y(t_C) = K_p m_1 + [y(t_B) - K_p m_1] e^{-\theta/T_p}.
\]

Moreover, the values of \( y(t) \) at the time instants \( t_D \) and \( t_E \) are, respectively,

\[
y(t_D) = K_p m_1 + [y(t_C) - K_p m_1] e^{-\theta/T_p}
\]

\[
y(t_E) = K_p m_2 + [y(t_D) - K_p m_2] e^{-\theta/T_p}.
\]

The existence of an oscillation requires \( y(t_A) = y(t_E) \). This condition, together with (6) and (9), results in

\[
[y(t_B) - K_p m_2] e^{\theta/T_p} = [y(t_D) - K_p m_2] e^{-\theta/T_p}.
\]

Substituting (7) and (8) into (10) to eliminate \( y(t_D) \), we obtain

\[
y(t_B) = K_p \frac{m_2 + J e^{-\theta/T_p} - m_1 e^{-(T_1 + T_2)/T_p}}{1 - e^{-(T_1 + T_2)/T_p}}.
\]
where $J \triangleq m_1 - m_2 = f_s - f_d$ is the jump amplitude when the valve moves. Recall that the maximum of $y(t)$ is reached at $t_D$. The oscillation amplitude of $y(t)$ can be defined as

$$Y(T_1, T_2) \triangleq y_{\text{max}} - y_{\text{min}} = y(t_D) - y(t_B)$$

$$= K_p m_1 + [y(t_B) - K_p m_1] e^{-T_1/T_p} - y(t_B)$$

$$= [K_p m_1 - y(t_B)] \left(1 - e^{-T_1/T_p}\right).$$

With $y(t_B)$ in (11), we can rewrite $Y(T_1, T_2)$ as

$$Y(T_1, T_2) = K_p J \left(1 - e^{-T_1/T_p}\right)(1 - e^{-T_2/T_p}). \tag{12}$$

It is noted that the oscillation amplitude $Y(T_1, T_2)$ in (12) is determined fully by $K_p$, $T_p$, $J$, $T_1$, and $T_2$.

C. Analysis of Controller Output $u(t)$

Since $Y(T_1, T_2)$ in (12) involves two unknown period parameters $T_1$ and $T_2$, we formulate two equalities on $T_1$ and $T_2$ by analyzing the characteristics of the controller output $u(t)$ in this section. By using the relationship (3), $u(t)$ and $m(t)$ at the time instants $t_A$ and $t_C$ are connected as

$$u(t_A) = m_1 + f_d$$

$$u(t_C) = m_2 - f_d. \tag{13}$$

Because the contribution of the integrator in the PI controller (2) at an arbitrary time $t_X$ is $K_i \int_{t_A}^{t_X} e(\tau) d\tau = u(t_X) - K_i e(t_X)$, and the control error is $e(t) = r_{ss} - y(t)$, the controller output $u(t)$ for $t \in [t_A, t_B]$ is [24]

$$u(t) = K_c e(t) + u(t_A) - K_i e(t_A) + K_i \int_{t_A}^{t} e(\tau) d\tau$$

$$= u(t_A) + K_c [y(t_A) - y(t)] + K_i r_{ss}(t - t_A) - K_i \int_{t_A}^{t} y(\tau) d\tau. \tag{15}$$

Substituting (4) into (15) and calculating the integral term, one obtains

$$u(t) = u(t_A) + (K_c - K_i T_p) [y(t_A) - y(t)]$$

$$+ K_i (r_{ss} - K_p m_2)(t - t_A).$$

Hence, $u(t)$ at the time instant $t_B$ is

$$u(t_B) = u(t_A) + (K_c - K_i T_p) [y(t_A) - y(t_B)]$$

$$+ K_i (r_{ss} - K_p m_2) \theta. \tag{16}$$

Similarly, $u(t)$’s at the time instants $t_C$, $t_D$, and $t_E$ are, respectively,

$$u(t_C) = u(t_B) + (K_c - K_i T_p) [y(t_B) - y(t_C)]$$

$$+ K_i (r_{ss} - K_p m_1)(T_1 - \theta)$$

$$+ K_i (r_{ss} - K_p m_1) \theta \tag{17}$$

$$u(t_D) = u(t_C) + (K_c - K_i T_p) [y(t_C) - y(t_D)]$$

$$+ K_i (r_{ss} - K_p m_1) \theta$$

$$u(t_E) = u(t_D) + (K_c - K_i T_p) [y(t_D) - y(t_E)]$$

$$+ K_i (r_{ss} - K_p m_2)(T_2 - \theta). \tag{19}$$

To formulate the oscillations, $u(t_A) = u(t_E)$ and $y(t_A) = y(t_E)$. With the definition $m_{ss} = r_{ss}/K_p$, the summation of both sides of (16)–(19) yields the first equality on $T_1$ and $T_2$ as

$$H_1(T_1, T_2) \triangleq m_1 T_1 + m_2 T_2 - m_{ss} (T_1 + T_2) = 0 \tag{20}$$

which indicates that the desired valve position $m_{ss}$ is the time average of $m_1$ and $m_2$.

The presence of the oscillations also implies that the value $u(t_C)$ in (14) is equal to the right-hand side of (17), i.e.,

$$u(t_C) = m_2 - f_d$$

$$= u(t_A) + (K_c - K_i T_p) [y(t_A) - y(t_C)] + K_i r_{ss} T_1$$

$$- K_i K_p [m_2 \theta + m_1 (T_1 - \theta)] \tag{21}$$

where $u(t_B)$ in (17) is represented by (16). Replacing $y(t_A)$ and $y(t_C)$ in (21) with (6) and (7), respectively, and using (11), (13) and $J = m_1 - m_2$ ultimately give the second equality on $T_1$ and $T_2$ as

$$H_2(T_1, T_2) \triangleq f_s + f_d - K_i K_p (m_1 T_1 - \theta J) + K_i r_{ss} T_1$$

$$+ (K_c - K_i T_p) K_p \left[\frac{e^{\frac{T_1}{T_p}}}{1 - e^{-\frac{T_1}{T_p}}} \left(e^{\frac{T_1}{T_p}} \right) - 1\right] = 0. \tag{22}$$

D. Solving for the Oscillation Period Parameters

Given He’s stiction model parameters, $f_s$, $f_d$, $m_1$, and $m_2$, the process model parameters, $K_p$, $T_p$, $\theta$, the controller parameters, $K_c$ and $K_i$, the reference $r_0$, and the static offset $c$, (20) and (22) yield two equalities for $T_1$ and $T_2$. Since these are nonlinear equations, the Newton–Raphson method [28] is utilized to solve them. Define $\Phi \triangleq [T_1, T_2]^T$ and $H(\Phi) \triangleq [H_1(\Phi), H_2(\Phi)]^T$. Then the problem can be formulated as

$$H(\Phi) = [0, 0]^T. \tag{23}$$

A solution can be obtained using the iterative formula

$$\Phi^{i+1} = \Phi^i + J_a^{-1}(\Phi) \cdot H(\Phi) \tag{23}$$

where $J_a(\Phi)$ is Jacobian matrix of $H(\Phi)$, namely,

$$J_a(\Phi) \triangleq \begin{bmatrix} \frac{\partial H_1}{\partial T_1} & \frac{\partial H_1}{\partial T_2} \\ \frac{\partial H_2}{\partial T_1} & \frac{\partial H_2}{\partial T_2} \end{bmatrix}$$

which can be easily calculated from (20) and (22). Note that the solution $\Phi^*$ completely characterizes the behavior of the oscillations, because the oscillation amplitude is related with the period parameters $T_1$ and $T_2$ as (12).

IV. OSCILLATION COMPENSATION METHOD

The objective of the proposed oscillation compensation method is to reduce the oscillation amplitude of $y(t)$, namely,
controller, the maximum value of $T_p$ by adjusting the PI controller parameters. Besides the two equalities (20) and (22), there are other inequalities that have to be satisfied. This section presents these inequalities and the detailed steps of the oscillation compensation methods.

Recall that Assumption A2 states that the valve moves back and forth at the two positions $m_1$ and $m_2$ in the oscillations. This requires $u(t) \in \{m_2 - J, m_1 + J\}$, $t \in [t_A, t_E]$. Note that $y(t)$ is minimized and maximized at the time instants $t_B$ and $t_D$, respectively. Due to the existence of the integrator in the PI controller, the maximum value of $u(t)$ does not appear at $t_B$; instead, it is reached in the time interval $[t_B, t_D]$. The controller output $u(t)$ in the time interval $[t_B, t_D]$ can be expressed as

$$u(t) = u(t_B) + (K_c - K_Tp)[y(t_B) - y(t)] + K_i(r_{ss} - K_p m_1)(t - t_B).$$

Thus, the derivative of $u(t)$ with respect to $t$ is

$$\dot{u}(t) = -(K_c - K_Tp)\dot{y}(t) + K_i(r_{ss} - K_p m_1). \quad (24)$$

Note that in the time interval $[t_B, t_D]$, $y(t) = K_p m_1 + [y(t_B) - K_p m_1]e^{-(t - t_B)/T_p}$.

Substituting the derivative of $y(t)$ into (24) and letting $\dot{u}(t_1) = 0$ yield

$$t_1 = t_B + T_p \ln \frac{(K_c - K_Tp)[y(t_B) - K_p m_1]}{K_i T_p(K_p m_1 - r_{ss})} \in [t_B, t_D] \quad (25)$$

where $t_1$ corresponds to the time instant when $u(t)$ reaches its maximum value. Correspondingly, $y(t_1)$ at $t_1$ becomes

$$y(t_1) = \frac{K_c K_p m_1 - K_Tp r_{ss}}{K_c - K_Tp}.$$ 

Thus, the upper threshold of $u(t)$ is limited to

$$u(t_1) = u(t_B) + (K_c - K_Tp)\dot{y}(t_1) - K_p m_1 + K_Tp r_{ss} + K_i T_p(r_{ss} - K_p m_1) \ln \frac{(K_c - K_Tp)[y(t_B) - K_p m_1]}{K_i T_p(K_p m_1 - r_{ss})}$$

$$< m_1 + J. \quad (26)$$

Note that in (25), it is clear that $y(t_B) - K_p m_1 < 0$, $K_p m_1 - r_{ss} > 0$; hence, the definition of the natural logarithm requires

$$K_c - K_Tp < 0 \quad (27)$$

which is valid in general since the integral time constant $T_i = K_c/K_i$ is usually expected to be less than the process time constant $T_p$ [24]. Analogously, the lower threshold of $u(t)$ is obtained as

$$u(t_2) = u(t_D) + (K_c - K_Tp)\dot{y}(t_2) - K_p m_2 + K_Tp r_{ss} + K_i T_p(r_{ss} - K_p m_2) \ln \frac{(K_c - K_Tp)[y(t_D) - K_p m_2]}{K_i T_p(K_p m_2 - r_{ss})}$$

$$> m_2 - J \quad (28)$$

where the time instant $t_2$ is

$$t_2 = t_D + T_p \ln \frac{(K_c - K_Tp)[y(t_D) - K_p m_2]}{K_i T_p(K_p m_2 - r_{ss})} \in [t_D, t_E + \theta].$$

Equation (22) gives one equality constraint for the controller parameters $(K_c, K_i)$. There is one degree of freedom in tuning the controller parameters. Several important constraints such as rejection of load disturbance, attenuation of measurement noise, and robustness to model uncertainties can give another specification to obtain unique controller parameters. Here, the robustness to model uncertainties is considered as another constraint to uniquely determine the controller parameters. The robustness of the feedback control loop without valve nonlinearity is usually measured by the maximum value of the sensitivity function of the control loop [24], i.e.,

$$M_s \triangleq \max_{\omega} \left| \frac{1}{1 + G(j\omega)C(j\omega)} \right|.$$ 

Owing to the acceptable range of $M_s \in [1.2, 1.6]$, we select the controller parameters $(K_c, K_i)$ being the closest to the robustness level $M_s = 1.4$ and satisfying the inequality constraints (26), (27), and (28) as the optimal choice.

In summary, the oscillation compensation method finds the controller parameters $K_c$ and $K_i$, such that $Y(T_1, T_2)$ is equal to a desired oscillation amplitude $Y_d$, and the following constraints are satisfied,

Equalities: (12), (20), (22),

Inequalities: (26), (27), (28),

$$K_c > 0, K_i > 0, M_s \approx 1.4.$$ 

The oscillation compensation method requires the estimation of the process model parameters $K_p, T_p$, and $\theta$, the static offset $c$, He’s stiction model parameters $f_s$ and $f_d$, and the two valve positions $m_1$ and $m_2$ from the discrete-time measurements $\{u[k], m[k], y_m[k]\}_{k=0,1}$. A few approaches exist in the literature for the identification of He’s stiction and process model. In this context, the process model parameters $K_p, T_p$, and $\theta$, together with the static offset $c$, are identified by a discrete-time autoregression with extra input (ARX) model identification and a model-order reduction [26], [29], [30]. It is well known that a high-order ARX model, defined as

$$y_m[k] = -\sum_{i=1}^{n_a} a_i y_m[k - i] + \sum_{j=1}^{n_b} b_j m[k - j - n_k] + c_d + n[k]$$ 

can approximate any linear time-invariant systems arbitrarily well even if the measurement noise is heavily colored [30]. With given structure parameters $(n_a, n_b, n_k)$, the parameter vector $\gamma \triangleq [a_1, \ldots, a_{n_a}, b_1, \ldots, b_{n_b}, c_d]^T$ of the ARX model can be estimated by the least-square method as

$$\hat{\gamma}(n_a, n_b, n_k) = (\Phi^T \Phi)^{-1} \Phi^T Y_m$$ 

where $\Phi \triangleq [\phi^T(1), \ldots, \phi^T(N)]^T$ with $\phi(k) \triangleq [-y_m[k - 1], \ldots, -y_m[k - n_k], m[k - 1 - n_k], \ldots, m[k - n_k - n_k], 1]^T$ and $Y_m \triangleq [y_m[1], \ldots, y_m[N]]^T$. Then, the FOPTD model $G(s)$ is obtained by applying the prediction-error method [26] to the step response of the estimated ARX model. Similarly, the stiction parameters $f_s$ and $f_d$ of He’s stiction model are...
estimated as
\[
(\hat{f}_s, \hat{f}_d) = \arg\min_{f_s, f_d} \frac{1}{N} \sum_{k=1}^{N} [m[k] - f(u[k]; f_s, f_d)]^2.
\] (29)

After collecting the measurements \( \{u[k], m[k], y_m[k]\}_{k=1}^{N} \) at some time instant \( t_N = Nh \), the steps of the proposed oscillation compensation method to achieve \( Y(T_1, T_2) = Y_d \) are listed as follows.

1) Identify the process model parameters \( K_p, T_p, \) and \( \theta \), the static offset \( c \), He’s stiction model parameters \( f_s \) and \( f_d \), and the two valve positions \( m_1 \) and \( m_2 \) from the measurements \( \{u[k], m[k], y_m[k]\}_{k=1}^{N} \) in an offline manner by the above-mentioned identification approaches.

2) Design the oscillation compensation controller to achieve \( Y(T_1, T_2) = Y_d \).
   a) Check whether Assumption A2 is valid from the measurement of the valve position \( m[k] \). If \( m[k] \) does move between two positions, then continue to the next step; otherwise, stop.
   b) Calculate the expected period parameters \( T_1 \) and \( T_2 \) by applying the Newton–Raphson method to (12) and (20), in order to achieve the desired amplitude \( Y_d \).
   c) Formulate the curve (22) that the controller parameters \( K_c \) and \( K_i \) satisfy, and find the feasible region of \( (K_c, K_i) \) on the curve confined by the inequalities (26), (27), and (28).
   d) Select the pair in the feasible region of \( (K_c, K_i) \) with the robustness level \( M_s \) closest to \( 1.4 \) as the final choice.

3) Change the controller parameters \( (K_c, K_i) \) manually, and put the new controller in operation afterward.

A natural question is about the minimum oscillation amplitude \( Y_{dm} \) that the designed controller achieves. The problem is formulated as
\[
Y_{dm} = \min_{K_c, K_i} Y_d \\
\text{s.t.} \ (12), \ (20), \ (22), \ (26), \ (27), \ (28), \\
K_c > 0, \ K_i > 0.
\] (30)

The value of \( Y_{dm} \) can be obtained when \( Y_d \) is gradually decreased until the case appears that a feasible region for the controller parameters \( (K_c, K_i) \) cannot be found.

Another concern is whether the phenomenon that the valve moves between two positions is common. This problem is difficult to directly solve; however, an indirect solution exists as follows. The feasible regions of the controller parameters \( (K_c, K_i) \) are calculated as the above Step 2C) for different values of oscillation amplitude \( Y_d \). If the feasible regions are quite broad, then it is rational that the valve movements between two positions are not restrictive. This solution is demonstrated in Example 2 of Section V.

V. EXPERIMENTAL EXAMPLES

Experimental examples are provided to illustrate the effectiveness of the proposed TDA and the oscillation compensation method.

The experimental configuration is schematically depicted in Fig. 6, with the snap of the experimental device shown in Fig. 7. In the experiments, the controlled plant \( G(s) \) in Fig. 1 is a water tank system (specifically, the bottom-middle water tank in Fig. 7), whose cross-sectional area is about \( 320 \text{ cm}^2 \). The water level of the water tank system in Fig. 6 is manipulated by one inlet valve and one outlet valve. The inlet valve is an electric control valve with a smart valve positioner, which can measure the valve position. The control valve becomes sticky after tightening the valve stem packing screw. The outlet valve is a manual valve, and its opening position is fixed during the experiments. The level of the water tank system is measured by a sensor. The samples of the water level and valve position are transmitted to a DCS platform of Siemens PCS7 with the
Fig. 8. All the measured signals in Example 1: the measured process output \( y_m[k] \) (top, solid), reference \( r[k] \) (top, dash), controller output \( u[k] \) (bottom, solid), and valve position \( m[k] \) (bottom, dash).

sampling period \( h = 0.5 \) s. The discrete-time counterpart of the PI controller \( C(s) \) in Fig. 1 is implemented at the DCS platform. The PI controller \( C(s) \) drives the electric control valve to change the water level. Thus, the feedback control loop in Fig. 1 is formed, where \( y_m(t), m(t), u(t) \), and \( r(t) \) stand for the water level of the tank system, the valve position, the controller output, and the setpoint, respectively.

**Example 1:** This example compares the TDA in Section III with the DFA in Appendix for oscillation analysis. The reference \( r(t) \) is kept at a constant value \( r_0 = 35 \). The PI controller in use is

\[
C(s) = 0.5 \left( 1 + \frac{1}{50s} \right).
\]

The discrete-time signals \( u[k], m[k], \) and \( y_m[k] \) shown in Fig. 8 are oscillatory due to the presence of control valve stiction. Based on the oscillatory data segment \( \{u[k], m[k], y_m[k]\}_{k=1}^{6000} \), the process model and He’s stiction model are identified separately. First, the process model of \( G(s) \) is identified by the identification approach given in Section IV,

\[
\hat{G}(s) = \frac{3.1765}{168.1329s + 1} e^{-3.1985s}, \hat{c} = -126.9125.
\]

The simulated process output \( \hat{y}_m[k] \) from the FOPDT model is given in Fig. 9. The fitness between \( y_m[k] \) and \( \hat{y}_m[k] \) is 93.4154%, where the fitness value is defined as

\[
F_G = \left( 1 - \frac{\|\hat{y}_m[k] - y_m[k]\|}{\|y_m[k] - E\{y_m[k]\}\|} \right) \times 100%
\]

with \( E\{\cdot\} \) standing for the mean value of the operand and \( \|\cdot\| \) the Euclidean norm. The closeness between \( y_m[k] \) and \( \hat{y}_m[k] \) indicates that \( \hat{G}(s) \) is able to well capture the process dynamics. Second, the stiction parameters \( f_s \) and \( f_d \) of He’s stiction model are estimated via (29), and the two noise-free valve positions \( m_1 \) and \( m_2 \) are also obtained from the simulated output \( \hat{m}[k] \) of He’s stiction model. The results are

\[
f_s = 9.8409, f_d = 4.4591, \hat{m}_1 = 52.8342, \hat{m}_2 = 47.4524.
\]

The fitness between the simulated valve position \( \hat{m}[k] \) and measured valve position \( m[k] \), defined as

\[
F_H = \left( 1 - \frac{\|\hat{m}[k] - m[k]\|}{\|m[k] - E\{m[k]\}\|} \right) \times 100%
\]

is 87.4762%. The simulated valve position \( \hat{m}[k] \) in Fig. 10 proves that the quality of He’s stiction model is good.

The actual oscillation period and amplitude parameters are determined from the oscillatory signals in Fig. 8 as \( T_1 = 501.5 \) s,
These results prove the effectiveness of the proposed TDA. The errors in the parameters calculated from the TDA and DFA is presented forth at two positions. The feasible region for \((K_c, T_i)\) is plotted in Fig. 12, together with the corresponding oscillation amplitude \(Y\). The feasibility region is quite broad, especially for the well-accepted controller design with \(K_c \in [0.2, 1.5]\) [24]. Hence, the assumption that \(m(t)\) moves between two positions is not a concern for the controller design.

Second, the minimum value of the oscillation amplitude \(Y_{dm}\) in (30) is calculated as discussed in Section IV, \(Y_{dm} = 4.4720\), with the corresponding oscillation periods \(T_{1m} = 130.4568\) and \(T_{2m} = 69.0250\) s.

Third, the compensation objective is to reduce the oscillation amplitude by half through tuning the controller parameters, that is, \(Y_2 = Y/2 = 6.5620\). Based on the identification results shown in Example 1, the expected period parameters \(T_1\) and \(T_2\) are obtained from solving (12) and (20) by the Newton–Raphson method, namely, \(T_1 = 197.6899\) and \(T_2 = 104.5981\) s. Then, the desired controller parameters \((K_c, K_i)\) are determined by (22), subject to the constraints (26), (27), and (28). The final feasible region for \((K_c, K_i)\) is shown in Fig. 13. The robustness level \(M_s\) for the feasible region of \((K_c, K_i)\) is shown in Fig. 14. It reveals that the optimal parameter pair is \(K_c = 1.7210\) and \(K_i = 0.0738\) (denoted as “*” in Fig. 13), which gives \(M_s = 1.3363\) closest to 1.4000, is appropriate for the robustness consideration.

**Example 2:** This example verifies the performance of the proposed oscillation compensation method. First, we address the concern whether the valve position \(m(t)\) moves back and forth at two positions. The feasible region for \((K_c, T_i)\) is plotted in Fig. 12, together with the corresponding oscillation amplitude \(Y\). The feasible region is quite broad, especially for the well-accepted controller design with \(K_c \in [0.2, 1.5]\) [24]. Hence, the assumption that \(m(t)\) moves between two positions is not a concern for the controller design.

As a comparison, the oscillation amplitude and period are also calculated by the DFA from (31) in Appendix. The enlargements of local trajectories of the negative reciprocal describing function of He’s stiction model \(-1/Y_N(A)\) and the open-loop frequency response \(G(j\omega)C(j\omega)\) are plotted in Fig. 11. The crossing point between two trajectories indicates there exists an oscillation, and the corresponding parameters are \(\hat{A} = 6.8567\) and \(\hat{\omega} = 0.0092\). Then, the oscillation period parameters are estimated to be \(\hat{T}_1 = 369.0083\) and \(\hat{T}_2 = 311.8006\) s, and the oscillation amplitude \(\hat{Y} = 13.0407\). The comparison between the estimated parameters from the TDA and DFA is presented in Table I. The errors in the parameters calculated from the TDA are significantly less than their counterparts from the DFA, especially for the period parameters \(T_1\) and \(T_2\). This comparison proves that the proposed TDA outperform greatly the DFA in analyzing the oscillations.

In Table I, the comparisons between the TDA and DFA based on laboratorial experiment (LE) in Example 1, where the percentage error is defined as (Estimated − Actual)/Actual x 100%.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(T_1)</th>
<th>(T_2)</th>
<th>(Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LE</td>
<td>501.5</td>
<td>270.0</td>
<td>13.1239</td>
</tr>
<tr>
<td>TDA</td>
<td>512.2937</td>
<td>271.9557</td>
<td>13.1603</td>
</tr>
<tr>
<td>TDA error (%)</td>
<td>2.1513</td>
<td>0.3910</td>
<td>0.2758</td>
</tr>
<tr>
<td>DFA</td>
<td>369.0083</td>
<td>311.8006</td>
<td>13.0407</td>
</tr>
<tr>
<td>DFA error (%)</td>
<td>-26.4191</td>
<td>15.4817</td>
<td>-0.6340</td>
</tr>
</tbody>
</table>

- Fig. 10. Identification of He’s stiction model in Example 1: the controller output \(u[k]\), valve position \(m[k]\) and estimated valve position \(\hat{m}[k]\) from He’s stiction model.
- Fig. 11. Trajectories of negative reciprocal describing function \(-1/Y_N(A)\) for He’s stiction model and the frequency response \(G(j\omega)C(j\omega)\) for the open-loop system including the FOPDT process and PI controller for Example 1.

In Table I, the comparisons between the TDA and DFA based on laboratorial experiment (LE) in Example 1, where the percentage error is defined as (Estimated − Actual)/Actual x 100%.
Next, in the laboratorial experiment, the compensation is implemented at the time instant $t = 4000$ s after the calculation of new controller parameters. The signals after compensation are shown in Fig. 8, from which we can read that the amplitude and the period after the compensation become 6.6430 and 314.5 s, respectively. This is close to the expected amplitude and period reduction from $Y = 13.1239$ to $Y_d = 6.5620$ and from $T_1 + T_2 = 771.5$ to $T_1 + T_2 = 302.2880$ s, respectively. Note also that the actual valve behaves more complexly than jumping between the only two valve positions after the compensation, especially at the transient state of the oscillation. Even so, the new controller is still effective, which also illustrates the robustness of the proposed compensation method.

Finally, let us compare the proposed compensation method with the method proposed by Mohammad and Huang [13]. In the present circumstance, the latter only suggests qualitatively to reduce the integral effect in the controller $C(s)$. As a result, the determination of $K_c$ and $K_i$ has to be done in a trial-and-error manner. By contrast, the proposed method directly gives the desired controller parameters.

VI. CONCLUSION

In this paper, the TDA was formulated to analyze oscillations in feedback control loops with sticky control valve described by He’s stiction model. The analytical relationships were established for the PI controller parameters and the oscillation amplitude and period parameters. Based on the relationships as well as other constraints including the one on the robustness to model uncertainties, the oscillation compensation method was proposed to design the PI controller parameters to reduce the oscillation amplitude to a desired value. Compared with the DFA, the proposed TDA is much more accurate in calculating the oscillation parameters. Compared with the qualitative-type controller tuning methods in the literature, the proposed oscillation compensation method is quantitative, and directly gives the desired controller parameters. Therefore, the proposed method can significantly improve the efficiency in compensating oscillations caused by control valve stiction. Laboratory examples in Section V validate the performance of the obtained results. The proposed TDA and the compensation method were developed based on He’s stiction model. As implied by the development procedure of the TDA, the generalization to other types of stiction models is feasible, even though the mathematical complexity may be higher. The generalization may be left as one future work.

APPENDIX

DESCRIPTING FUNCTION APPROACH

The oscillations are analyzed by the DFA [15] here. Without loss of generality, the constant reference $r_0$ and the static offset $c$
are assumed to be zero, because they only introduce a coordinate shift, and have no significant impact on the DFA. In this case, \( m_1 > 0 > m_2 \). The input \( u(t) \) is a sinusoidal signal, i.e.,

\[
u(t) = A \sin(\omega t)
\]

with \( A > \max(m_1, -m_2) \). Then, the valve output \( m(t) \) in one period, with \( \phi \equiv \omega t \), is

\[
m(\phi) = \begin{cases} 
m_2, & 0 < \phi \leq \phi_1, 
m_1, & \phi_1 < \phi \leq \phi_2, 
m_2, & \phi_2 < \phi \leq 2\pi
\end{cases}
\]

where \( \phi_1 \equiv \arcsin(m_1/A) \), and \( \phi_2 \equiv \pi - \arcsin(m_2/A) \). Furthermore, the fundamental Fourier coefficients \( a_1 \) and \( b_1 \) are

\[
a_1 = \frac{1}{\pi} \int_0^{2\pi} y(\phi) \cos(\phi) d\phi = \frac{1}{\pi} \int_0^{\phi_1} m_2 \cos(\phi) d\phi + \frac{1}{\pi} \int_{\phi_1}^{\phi_2} m_1 \cos(\phi) d\phi + \frac{1}{\pi} \int_{\phi_2}^{2\pi} m_2 \cos(\phi) d\phi
\]

\[
= -\frac{1}{\pi} (m_1 - m_2) [\sin(\phi_1) - \sin(\phi_2)]
\]

\[
b_1 = \frac{1}{\pi} \int_0^{2\pi} y(\phi) \sin(\phi) d\phi = \frac{1}{\pi} (m_1 - m_2) [\cos(\phi_1) - \cos(\phi_2)].
\]

Note that the constant term of Fourier series \( a_0 \) is nonzero because He’s stiction model is not odd symmetric with respect to the origin. However, for the development of the describing function for He’s stiction model, this direct current component is neglected hereafter. The describing function of He’s stiction model, with the valve output \( m(t) \) moving between two known positions \( m_1 \) and \( m_2 \), is

\[
Y_N(A, \omega) \equiv \frac{b_1 + ja_1}{A} = \frac{m_1 - m_2}{\pi A} [\cos(\phi_1) - \cos(\phi_2) - j \sin(\phi_1) + j \sin(\phi_2)] = \frac{J}{\pi A} (e^{-j\phi_1} - e^{j\phi_2}).
\]

The resulting describing function of He’s stiction model relies on the amplitude \( A \), but is independent of the frequency \( 2\pi/\omega \) of the input. Thus, \( Y_N(A, \omega) \) can be shortened to \( Y_N(A) \). According to the calculation in [13], the frequency response of the open-loop system including a FOPDT process (1) and PI controller (2) is

\[
G(j\omega)C(j\omega) = -\frac{1}{Y_N(A)}
\]

Hence, the crossing point between \( G(j\omega)C(j\omega) \) with \(-1/Y_N(A)\) indicates the existence of an oscillation [15]. To follow the same notations as the TDA, the corresponding oscillation amplitude and periods are denoted as \( Y \equiv Y_N(A, \omega) \), \( T_1 \equiv \phi_2 - \phi_1 \), and \( T_2 \equiv 2\pi/\omega \), respectively.

References

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