Speed-Accuracy Trade-off in Collective Decision Making

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Collective Decision Making in two alternative choice tasks

Birds deciding whether to migrate or not

Leader election in a two party system

Social information assimilation + decision-making = Socio-Cognitive Networks
Collective Decision Making in Socio-Cognitive Networks

Drift Diffusion Model and the Free Response Paradigm

- Models human decision making in two alternative choice tasks
- Evidence evolution in a two alternative choice task is modeled by
  \[ dx(t) = \beta dt + dW(t), \quad x(t) = x_0 \]

- Decision process at time \( \tau \) is
  \[
  \begin{cases} 
  x(\tau) > \eta, & \text{choose alternative 1,} \\
  x(\tau) < -\eta, & \text{choose alternative 2,} \\
  \text{else,} & \text{collect more evidence.}
  \end{cases}
  \]

Social Interaction and the DeGroot Model

- \( p \): vector of opinions in a network
- \( A \): row stochastic matrix
- models consensus seeking in a social network by
  \[ p(t + 1) = Ap(t). \]

- same as the celebrated consensus dynamics in multi-agent systems
- Continuous time consensus seeking in a social network modeled by
  \[ \dot{p}(t) = -Lp(t), \quad p(0) = p_0 \]
  \( L \) = Laplacian Matrix
Asymptotic Optimality of the Coupled DDM

- $n$ decision-makers collect noisy signals and interact with each other
- the evidence aggregation process well modeled by
  \[
  dx(t) = -Lx(t)dt + \beta 1_n dt + \sigma dW(t), \quad x(0) = 0_n. \tag{1}
  \]

Quantities of interest:
- Expected decision times
- Error rates (probability of wrong decision)

Evidence vector: $x(t) = x_{cen}(t)1_n + \epsilon(t)$

- $\epsilon_k(t) \to \mathcal{N}(0, 1/\mu_k)$,
- $\frac{1}{\mu_k} = \sum_{p=2}^{n} \frac{1}{2\lambda_p} u_k^{(p)}$
- $\mu_k$ is a certainty index determined purely by the interaction graph

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Quantities of interest:
- Expected decision times
- Error rates (probability of wrong decision)

Standard approach:
- solve first passage time associated with the FP equation for (1)
- an elliptic PDE with $n$ variables

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Asymptotic optimality

\[
\frac{x_k(t) - \beta t}{\sqrt{t}} = \frac{x_{cen}(t) - \beta t}{\sqrt{t}} + \frac{\epsilon_k(t)}{\sqrt{t}} \implies x_k(t) = x_{cen}(t) + o(1)
\]
Decoupled Approximation to the Coupled DDM

- Decoupled approximation to $\varepsilon(t)$
  
  \[ d\varepsilon(t) = -L\varepsilon(t)dt + \left( I_n - \frac{1}{n} 1_n 1_n^\top \right) dW_n(t), \varepsilon(0) = 0 \]

- $\varepsilon_k(t)$ is a continuous Gaussian process and converges to $\mathcal{N}(0, 1/\mu_k)$

- Approximate $\varepsilon_k(t)$ by the O-U process
  
  \[ d\varepsilon_k(t) = -\frac{\varepsilon_k(t)}{2} dW(t), \varepsilon_k(0) = 0 \]

Efficiency of approximation

\[
\lim_{t \to +\infty} \text{corr}(\varepsilon_k(t), \varepsilon_k(t)) = \mu_k \sum_{p=1}^{n} \frac{1}{2 \text{eig}_p(L + \text{diag}(\mu/2))} (\bar{u}_p)^2 - \frac{2}{n}
\]

- Approximate evidence at node $k$: $x_{\text{cen}}(t) + \varepsilon_k(t)$
- Decision time and Error Rate: need to solve $n$ elliptic PDEs with two variables opposed to a PDE with $n$ variables earlier
Numerical Illustration: Decoupled Approximation

Further Approximations

- bound the contribution by the O-U process \( \varepsilon_k(t) \)
- for sufficiently large \( K \), with high probability

\[
\max_{s \in [0, t]} |\varepsilon_k(t)| \leq \frac{K}{\sqrt{\mu_k}}
\]

- effective threshold for the centralized DDM belongs to the set

\[
(\eta - K/\sqrt{\mu_k}, \eta + K/\sqrt{\mu_k})
\]

The reduced DDM approximates the coupled DDM well.

Further Approximations

Bounds on Decision Time and Error Rates

- coupled DDM at each node well approximated by centralized DDM with a modified threshold

\[
\frac{\eta_k - K/\sqrt{\mu_k}}{\beta} \tanh \left( \beta n \left( \eta_k - \frac{K}{\sqrt{\mu_k}} \right) \right) \leq ET_k \leq \frac{\eta_k + K}{\beta} \tanh \left( \beta n \left( \eta_k + \frac{K}{\sqrt{\mu_k}} \right) \right)
\]

\[
\frac{1}{1 + \exp \left( 2\beta n (\eta_k + \frac{K}{\sqrt{\mu_k}}) \right)} \leq ER_k \leq \frac{1}{1 + \exp \left( 2\beta n (\eta_k - \frac{K}{\sqrt{\mu_k}}) \right)}
\]
The centralized DDM with corrected thresholds approximates the coupled DDM well.

Expected decision time

Log odds of error rates

Conclusions and Future Directions

Conclusions:
1. towards rigorous modeling and analysis of socio-cognitive networks
2. coupled DDM as model for social decision-making in 2-AC tasks
3. a computationally tractable decoupled approximation to coupled DDM
4. further approximation by the threshold corrected centralized DDM
5. ideas extend to multi-alternative choice tasks and 2-AC tasks with recency effect

Future Directions:
1. relaxing the continuous communication assumption
2. heterogeneous individuals
3. general decision-making tasks, e.g., multi-armed bandits