Supplementary Material for

Dynamic crossflow filtration with a rotating tubular membrane: Using centripetal force to decrease fouling by buoyant particles

Accepted for publication\(^1\) in Chemical Engineering Research and Design on November 11, 2015.
Published article DOI: 10.1016/j.cherd.2015.11.007

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June 8, 2015

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Experimental Apparatus

The membrane was housed in a stainless steel holder (Filtanium, TAMI Industries) that was modified with a concentrically mounted pulley. After the end of each filtration test, the membrane was cleaned following the manufacturer’s protocol, which included washing with 1 L of 20 g/L sodium hydroxide at 80 °C for 30 min followed by 1 L of 1g/L phosphoric acid at 50 °C for 15 min. Two swivels (Rotary Systems Inc.) were installed at the two ends of the membrane holder allowing for the free rotation of the entire membrane module. The module was rotated by an electric motor at 1725 rpm via a belt-and-pulley arrangement. A plywood shield was placed between the operator and the membrane for protection in case of system failure.

Pressure sensors (EW-68075, Cole Parmer) were installed immediately upstream and downstream from the membrane, and a flowmeter (M101, McMillan) was used to monitor retentate flow rate. A bypass line was added in parallel to the membrane module to control the crossflow rate within the membrane channel. Data from the pressure sensors and the flowmeter were processed by a National Instruments data acquisition system controlled via a program written in LabView. After each experiment, the membrane cake was removed in a two-step procedure. First, the membrane was disconnected from the retentate line and while disconnecting the membrane from the inlet line, the contents of the membrane channel were drained into a beaker. Second, a back flush with ~1 L of deionized water was applied to remove the remaining portion of the cake from the membrane surface for collection in the same container. The backflush employed a peristaltic pump whose outlet tubing connected to both outlets of the membrane holder. Particle size analysis was used to determine sizes of particles in the collected membrane cake. In several cases, the collected particles were also dried in an oven and weighed.
Reduced Grade Efficiency and Membrane Rejection

Standard particle separation methods, such as those based on hydrocyclones, often employ the terms total grade efficiency and reduced grade efficiency to describe the quality of a separation. In contrast, membrane techniques frequently employ rejection to characterize a separation. Thus, understanding the relationships between these quantities is vital for communication between researchers in the fields of membranes and hydrodynamic separations.

For a given particle size where the mass of all particles is the same, eq (S1) defines

\[ G_T = \frac{\dot{N}_R}{\dot{N}_f} \]  \hspace{1cm} (S1)

the total grade efficiency, \( G_T \), where \( \dot{N}_R \) is the number of particles of the given size entering the retentate per unit time and \( \dot{N}_f \) is the number of particles entering the membrane per unit time. The reduced grade efficiency, \( G_R \), includes the ratio, \( R_f \), of the retentate flow rate to the feed flow rate as shown in eq (S2).

\[ G_R = \frac{G_T - R_f}{1 - R_f} \]  \hspace{1cm} (S2)

If we define the concentration of particles in the feed and retentate as \( C_f \) and \( C_R \), respectively, and the volumetric flow rates of the feed and retentate as \( Q_f \) and \( Q_R \), respectively, we can rewrite eq (S2) as eq (S3).

\[ G_R = \frac{\frac{Q_R C_R}{Q_f C_f} - \frac{Q_R}{Q_f}}{1 - \frac{Q_R}{Q_f}} \]  \hspace{1cm} (S3)

Noting that at steady state the sum of the particles entering the permeate and the retentate must equal the particles entering the membrane from the feed leads to eq (S4) where \( C_P \) and \( Q_P \) are the concentration of particles and flow rate, respectively, for the permeate.

\[ G_R = \frac{\frac{Q_f C_f - Q_P C_P}{Q_f} - \frac{Q_f - Q_P}{Q_f}}{\frac{Q_f}{Q_f} - \frac{Q_P}{Q_f}} = 1 - \frac{C_P}{C_f} \]  \hspace{1cm} (S4)

Membrane scientists define rejection, \( R \), according to eq. (S5), so rejection and reduced grade efficiency are equivalent quantities.
\[ R = \frac{c_f - c_P}{c_f} = 1 - \frac{c_P}{c_f} \]  

(S5)

**Determining Reduced Grade Efficiency from Particle Size Distributions and Cake Mass**

We assume that for a specific particle size, the membrane cake captures a certain fraction of the feed particles, \( F_i \), in a single pass through the membrane (\( F_i = 1 - G_T \)), and that \( F_i \) is constant throughout the separation. Because the permeate returns to the feed tank without its particles, which form a cake in the membrane, and the retentate returns to the feed tank with its particles, eq (S6) describes the change in the particle concentration in the feed tank, \( C_{i,f} \), as a function of time. The subscript \( i \) refers to a specific particle size, \( V_f \) is the feed tank volume, and \( Q_f \) is the feed flow rate.

\[
\frac{dC_{i,f}}{dt} = -\frac{Q_f C_{i,f}}{V_f} F_i 
\]

(S6)

Solving this differential equation yields

\[
C_{i,f}(t) = C_{i,f}(t = 0) \exp \left( -\frac{Q_f}{V_f} F_i t \right) 
\]

(S7)

Assuming no other particle losses, eq (S8) gives the number of particles of a given size in the cake, \( N_{i,wall} \). Substituting eq (S7) into eq (S8) yields eq (S9).

\[
N_{i,wall} = \left( C_{i,f}(t = 0) - C_{i,f}(t) \right) V_f 
\]

(S8)

\[
N_{i,wall} = C_{i,f}(t = 0) \left( 1 - \exp \left( -\frac{Q_f}{V_f} F_i t \right) \right) V_f 
\]

(S9)

If we know the mass of the cake, \( m_C \), and the volume-based fraction \( f_{i,V} \) for each particle size bin, we can calculate the number of particles of a given size in the cake, \( N_{i,wall} \), using equation (S10). The calculation assumes all particles have the same density, \( \rho_p \), and \( d_p \) is the average particle diameter in the bin size.

\[
N_{i,wall} = \frac{f_{i,V} m_C}{\rho_p \pi d_p^3} 
\]

(S10)

An analogous equation with the mass of particles added to the feed and the volume fraction of those particles with a given size range gives the number of particles, \( N_{i,f}(t = 0) \), in the feed at the beginning of the experiment. With experimental values for \( N_{i,wall} \). 

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and $N_{i,f}(t = 0)$, and noting that $C_{i,f}(t = 0) = N_{i,f}(t = 0)/V_f$, we can solve eq (S9) for $F_i$ to obtain eq (S11).

$$F_i = -\frac{V_f}{Q_f} \ln \left( 1 - \frac{N_{wall}}{V_f \frac{N_{i,f}(t=0)}} \right) = -\frac{V_f}{Q_f} \ln \left( 1 - \frac{N_{wall}}{N_{i,f}(t=0)} \right) \quad (S11)$$

We can easily convert this value to $G_R$ (or equivalently rejection) using eq (S12).

$$G_R = \frac{1-F_i-R_f}{1-R_f} \quad (S12)$$

**Figure S1:** Values of the erosion parameter, $\kappa$, obtained from data represented in Figure 10. Squares present a point-by-point solution for $\kappa$ using the CFD data to obtain $G_T^{no \text{ erosion}}$ and equation (21) to find $\kappa$ from experiment data for $G_T^{\text{erosion}}$. (Equation 18 relates $G_T$ and $G_R$.) The dashed line shows a simultaneous fit of all the data in Figure 10 correcting the CFD data for erosion using the expression in equation (17) for $\kappa$. 

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure_s1.png}
\caption{Values of the erosion parameter, $\kappa$, obtained from data represented in Figure 10. Squares present a point-by-point solution for $\kappa$ using the CFD data to obtain $G_T^{no \text{ erosion}}$ and equation (21) to find $\kappa$ from experiment data for $G_T^{\text{erosion}}$. (Equation 18 relates $G_T$ and $G_R$.) The dashed line shows a simultaneous fit of all the data in Figure 10 correcting the CFD data for erosion using the expression in equation (17) for $\kappa$.}
\end{figure}
Figure S1 shows values for the erosion parameter, $\kappa$, as a function of particle size. The squares give the values of $\kappa$ that yield exact agreement (at each particle size) between the experimental $G_T$ or $G_R$ values and CFD $G_T$ or $G_R$ values corrected for erosion using eq. (21). The line shows the value of $\kappa$ obtained using the single parameter $\delta_e$ (eq. 17) to create the best fit of the CFD data corrected for erosion to the experimental data for all of the particle sizes. Clearly the single-parameter fit overpredicts $\kappa$ for larger values of $d_p$. This is also evident from Figure 10, where the fitted values give a value of $G_R$ that is too high at larger particle sizes. The deviation could result from either overprediction by the CFD model of the deposition rate for larger particles or additional factors (e.g. chemical or hydrodynamic) that decrease erosion as particle size increases. Nevertheless the important point from this fit is the value for $\delta_e$ of 5.7 μm, which is within 2 μm of the value we would obtain by extrapolating the point-by-point data for $\kappa$.

Figure S-2 shows a second set of experimental data for reduced grade efficiencies. The CFD-predicted data ($G_R^{no erosion}$) are the same as those in Figure 10, but the experimental data come from a different data set in a replicate experiment. The erosion correction is based on fitting the data set in this Figure. Fitting the data leads to a $\delta_e$ value of 6.9 μm, which is slightly larger than the value of 5.7 μm for the data set in Figure 10. The corresponding value of $\theta$ (angle of repose) is 14.6 ° compared to 13.2 ° for the first data set. If we assume that the azimuthal component of the shear stress dislodges particles, then $\delta_e=7.8$ um translates into $\theta = 35.6$ ° compared to 33.2 ° for the first data set. Recall that $\theta$ for hexagonal packing is 35.3 °. The biggest variation between the two data sets is not the size distributions, which are very reproducible, but the measured cake mass. Cake collection and drying are the biggest sources of uncertainty in these experiments, but they affect $\delta_e$ much more than $\theta$. 


Figure S2: Effect of particle size on the experimental (open circles) and CFD-predicted ($G_{R, no \text{ erosion}}$) reduced grade efficiency for crossflow filtration with rotation at 1725 rpm. The figure also shows the CFD value corrected for erosion. Feed flow rate = 0.36 L/min, retentate flow rate = 0.1 L/min, permeate flux = 3200 L/(m².hr), and transmembrane pressure is 0.1 MPa.