Blocking Probability Analysis for Relay-Assisted OFDMA Networks using Stochastic Geometry

Ahmed Alahmadi, Yuan Liang, Run Tian, Jian Ren and Tongtong Li
Department of Electrical & Computer Engineering
Michigan State University, East Lansing, MI 48824, USA
Email: {alahmadi, liangy11, tianrun, renjian, tongli}@msu.edu

***Abstract*—Along with the emerging high density Internet of Things (IoT) networks, relay-assisted networks are attracting more research attention in recent years due to their abilities to extend the coverage area and improve the Quality of Service (QoS). Blocking probability (BP) has been used as a very important metric in evaluating the QoS of the network. In this paper, taking the spatial randomness of the IoT network into consideration, we investigate blocking probability in relay-assisted OFDMA networks using stochastic geometry. More specifically, first, we analyze the inter-cell interference from the neighboring cells at each typical node. Then, we derive the coverage probability in the downlink transmissions, including both the direct and relay-based transmissions. Finally, we classify the incoming users into different classes based on their data rate requirements, and calculate the blocking probability using the multi-dimensional loss model based on the Markov chains. We show that the blocking probability can be reduced by exploiting relay-assisted transmissions.**

***Index Terms*—OFDMA cellular networks, Relay, Blocking probability, Stochastic geometry.**

I. Introduction

In designing and analyzing wireless networks, blocking probability (BP) has been used as a very important metric in evaluating the Quality of Service (QoS) of the network. Blocking probability is the probability that an arriving user is denied of service due to insufficient network resources.

As a highly efficient communication scheme, orthogonal frequency division multiplexing (OFDM) has become a dominant transmission technique in wireless networks for supporting high data rate applications. In literature, blocking probability has been investigated for the OFDMA cellular networks in both single-hop and multi-hop networks.

Some representative work on blocking probability for the single-hop OFDMA networks can be found in [2]–[4]. In [2], the authors tried to model the subcarrier-allocation system using the batch-arrival model $M^X/M/c/c$. This model is not sufficiently accurate due to the assumption that the subcarriers are allocated in bulks but released one by one. Therefore, in [3], a more realistic model known as multiclass Erlang loss model was proposed. Motivated by the fact that power should be considered in addition to subcarriers as a system resource, a more general model was proposed in [4], where both the power and the subcarriers are regarded as the system resources.

Blocking probability for multi-hop OFDMA networks is attracting more research attention along with the emerging high density Internet of Things (IoT) networks. For example, in [5], the authors proposed a system model to evaluate the blocking probability for the relay-based OFDMA cellular networks. In their model, each cell consists of a base station, located at the center of the cell, and is surrounding by six relay stations with deterministic (known) locations. However, as pointed out in [6], results based on such highly idealized models generally may not be very accurate. On the other hand, it was also shown in [6] that modeling the locations of the base/relay stations stochastically according to a Poisson point process (PPP) depicts the reality and can achieve more reliable and accurate results compared to its idealized counterparts. More closely, stochastic geometry has been widely applied in modeling wireless networks in recent years [7]–[10]. For instance, in [7], stochastic geometry was used to analyze relay-aided two-hop cellular networks. In [8], it was used to analyze multi-hop transmission in ad-hoc networks. However, to the best of our knowledge, no work has been done in literature to investigate blocking probability in relay-assisted cellular OFDMA networks using stochastic geometry, which is the main contribution of this paper.

In OFDMA networks, users are assigned different number of resource elements (subcarriers) to meet their rate requirements. Therefore, the incoming users can be classified into different classes based on their subcarrier requirements. A very challenging problem in calculating the blocking probability is to obtain *the distribution of the group size of the subcarriers that an arriving user needs* to fulfill its data rate. In most of the existing work [2]–[4], this distribution is assumed to be known, usually assumed to be uniform. This may not be accurate, especially for large group sizes, which actually have very low request probability.

In this paper, we derive the distribution of user required subcarrier group size and then obtain the user blocking probability based on that. More specifically, in this paper, *first*, we model the inter-cell interference from the neighboring cells at a typical node. *Second*, we derive the coverage probability (i.e., the probability of successful transmission) in the downlink transmissions, including both the direct and relay-based transmissions. *Third*, we classify the incoming users into different classes based on their subcarrier requirements. The system under consideration can be modeled using multi-dimensional Markov chains. *Finally*, we calculate the blocking probability using the multi-dimensional loss model.

978-1-5386-9223-3/19/$31.00 ©2019 IEEE

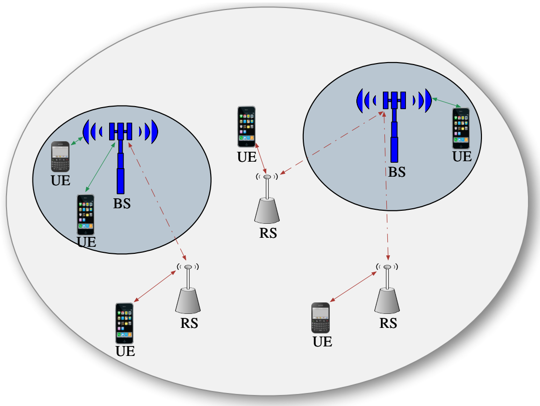

Fig. 1. The proposed relay-assisted network architecture.

II. System Model

A. Network Model

We consider a relay-assisted OFDM-based cellular network, as shown in Figure 1. In this model, the network consists of base stations, relay stations, and end users, such as Internet of Things (IoT) devices. To account for the spatial randomness of the network, the locations of the base stations are modeled by a homogeneous Poisson point process $\Phi_{\rm B} = \{X_i, i = 0, 1, 2, ...\}$ in $\mathbb{R}^2$ with intensity $\lambda_{\rm B}$, and the locations of the relay stations are modeled as another PPP $\Phi_{\rm R} = \{Y_i\}$ with intensity $\lambda_{\rm R}$, which is independent of $\Phi_{\rm B}$. It is further assumed that every user can establish a connection with its geographically closest base station, either through a direct single-hop path, or through a multi-hop path with assistance from the relay stations. More specifically, based on their locations, the users can be divided into two groups S_D and S_R, where S_D denotes the set of all EUs that are located at a distance less than r_{BS} from their closest base station, and can communicate directly with the BS. On the other hand, S_R represents the set of the users that are located outside that circle of radius r_{BS} of the BS, and only connect to the BS indirectly through the relay stations, as shown in Figure 1. As will be discussed in Section V, the optimal value of r_{BS} can be determined by minimizing the system blocking probability.

B. Transmission Protocol Design

Ideally, relays should lie on the line segment between the source and the destination to avoid the detour in routing. In addition, assuming that the source node and relay node use the same transmission power, then their intervals should be equal since the throughput of the path is bottlenecked by the hop with the longest distance. Motivated by this observation, we propose a routing protocol for relay-assisted transmission, as shown in Figure 2. More specifically, assume that a user EU_i is located at a distance $d_{BU} > r_{BS}$ from its base station, i.e, $EU_i \in S_R$. Typically, EU_i should first send its data to the RS located in the middle point between the EU_i and the BS, and then the RS decodes the information and forwards it to the BS. However, since the locations of the RSs are random, it is not guaranteed that there will be a RS in the halfway point between transmitter and the receiver. Therefore, intuitively, EU_i should selects its next hop to a RS that is located nearest to the optimal mid-point [11]. Let r denote the minimum distance between the optimal RS and any other arbitrary RS, then from the law of cosines, r can be expressed as:

$$r = \frac{1}{2}\sqrt{2(d_{BR}^2 + d_{RU}^2) - d_{BU}^2}\ , \tag{1}$$

where d_{BU}, d_{BR} and d_{RU} represent the distances between the BS to the user, the BS to the RS, and the RS to the user, respectively. Following the results in stochastic geometry [6], the probability density function (PDF) of the random variable r can be obtained as:

$$f_r(r) = 2\pi\lambda_{\rm R} r e^{-\lambda_{\rm R}\pi r^2}. \tag{2}$$

It should be emphasized that if $d_{BU} \leq r_{BS}$, user EU_i communicates directly with the base station, i.e, no hopping is required. This means, ideally, no relay stations should be deployed inside the circular areas of the BSs.

In this paper, all RSs are assumed to be half-duplex using decode-and-forward mechanism, which means that the 2-hop transmission is done through two different time slots (i.e., using TDD sharing mode). In the first time slot, BS/UE sends its information to the RS, then the RS decodes the information and forwards it to the UE/BS in the second slot. Moreover, a call will be dropped if it is wrongly decoded at the RSs.

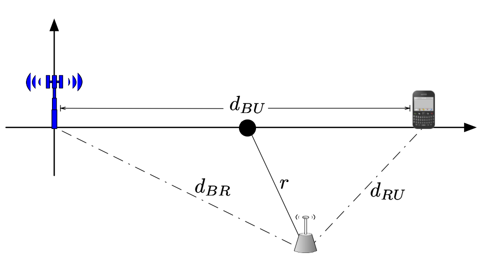

Fig. 2. The proposed 2-hop routing topology.

C. Channel Model

We consider a propagation channel model, where the received power at a typical node of a transmitted signal at a distance d is $P_r(d) = \frac{P_t h}{l(d)}$, where P_t is the transmit power, $l(d)$ is the path-loss attenuation function of distance d, and h is the channel (power) gain. A simplified model for the path-loss is:

$$l(d) = \kappa_0 \cdot d^\alpha, \tag{3}$$

where α is the path-loss exponent, $\kappa_0 = (4\pi/v)^2$, and v is the electromagnetic wavelength. h is exponentially distributed random variable with unit mean, i.e., Rayleigh fading is considered.

III. Interference Modeling and Achievable Rate Analysis

In this section, we first analyze the inter-cell interference from the neighboring cells at a typical node. Then, we calculate the coverage probability in the downlink transmissions. After that, we obtain the achievable transmission rate.

A. Interference Model

In OFDMA networks, the total available spectrum is divided into a number of orthogonal subcarriers such that no two users belonging to the same cell share the same set of subcarriers. As a result, the intra-cell interference at a typical receiver can be avoided. However, inter-cell interference due to the neighboring cells should be investigated.

Consider a relay-assisted OFDM system with N subcarriers, each of bandwidth $\Delta f = \frac{B}{N}$, where B is the system bandwidth. Define Φ_{T} as the set of active transmitters (BSs or RSs) occupying the same subcarrier k. Following the independent thinning property of the PPP [12], Φ_{T} can be modeled as a homogeneous PPP with intensity $p\lambda_{\mathrm{B}}$, where p is the activity factor of the BS. Note that $p = 1$ is the worst case scenario where all the transmitters are active.

The signal-to-interference-plus-noise ratio (SINR) experienced by a typical node (RS or UE) at a distance d_0 from a transmitter $X_0 \in \Phi_{\mathrm{T}}$ over subcarrier k can be expressed as:

$$\mathrm{SINR}_k = \frac{P_{t,k} h_k l^{-1}(d_0)}{I_k + W}, \quad (4)$$

where

$$I_k = \sum_{i \in \Phi_{\mathrm{T}} \setminus X_0} P_{t,k}\ h_{k,i}\ l^{-1}(d_i), \quad (5)$$

is the aggregate interference from all active transmitters over subcarrier k, except the serving transmitter X_0. $W = \Delta f N_0$ is the total noise power over subcarrier k sub-band, and N_0 is the power spectral density of the noise. In an interference limited scenario, W is small compared to the interferences from all the active transmitters, and it can be ignored. The Laplace transform (LT) of the PDF of the random variable I_k evaluated at s can be obtained as [12]:

$$\begin{aligned}&\mathcal{L}_{I_k}(s) = \mathbb{E}\{e^{-sI_k}\}\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\frac{s}{\kappa_0})^{2/\alpha}\frac{\pi}{\alpha\sin(2\pi/\alpha)}\},\end{aligned} \quad (6)$$

where it is assumed that $P_{t,k} = 1$. Hence, without loss of generality, we consider unit transmit power throughout this paper. Also, for notational simplicity, we omit the subscript k used to emphasize the explicit reference to a single subcarrier.

B. Coverage Probability

In this subsection, we study the probability of successful transmission (i.e., the coverage probability) in downlink cellular network including both the direct and relay-based transmissions.

The coverage probability is defined as the probability that the receive SINR at a typical node is above a certain threshold γ, which is the same as the complementary cumulative distribution function (CCDF) of the SINR, i.e.,

$$P_c(\gamma) = \Pr\{\mathrm{SINR} > \gamma\}. \quad (7)$$

1) Coverage Probability of Direct Transmission: Without loss of generality, we assume that the base station under consideration is located at the origin, as shown in Figure 2. Then, the coverage probability of the BS-UE link can be obtained as:

$$\begin{aligned}&P_{c,BU}(\gamma_{BU} \mid d_{BU}) = \Pr\{\mathrm{SINR}_{BU} > \gamma_{BU}\}\\ &= \mathbb{E}_I\Big\{\Pr\{h > \gamma_{BU} I l(d_{BU})\}\Big\}\\ &= \mathcal{L}_I(\gamma_{BU} l(d_{BU}))\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\gamma_{BU})^{2/\alpha} d_{BU}^2 \frac{\pi}{\alpha\sin(2\pi/\alpha)}\},\end{aligned} \quad (8)$$

where we used the assumption that $h \sim \mathrm{Exp}(1)$, $\mathcal{L}(\cdot)$ is the Laplace transform evaluated in (6).

2) Coverage Probability of 2-hop Transmission: As mentioned earlier, based on the distance between the user and its associated base station, in case where $d_{BU} > r_{BS}$, the user may attempt to connect to the BS with the help of the RS. Therefore, the coverage probability of the downlink transmission from the base station to the relay station (i.e., BS-RS link) can be expressed as:

$$\begin{aligned}&P_{c,BR}(\gamma_{BR} \mid d_{BR}) = \Pr\{\mathrm{SINR}_{BR} > \gamma_{BR}\}\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\gamma_{BR})^{2/\alpha} d_{BR}^2 \frac{\pi}{\alpha\sin(2\pi/\alpha)}\}.\end{aligned} \quad (9)$$

Once the data is decoded correctly at the RS, the RS further forwards it to the intended user. The coverage probability of the RS-EU link can be expressed as:

$$\begin{aligned}&P_{c,RU}(\gamma_{RU} \mid d_{RU}) = \Pr\{\mathrm{SINR}_{RU} > \gamma_{RU}\}\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\gamma_{RU})^{2/\alpha} d_{RU}^2 \frac{\pi}{\alpha\sin(2\pi/\alpha)}\}.\end{aligned} \quad (10)$$

Assuming the two links are independent, then the coverage probability of the 2-hop transmission can be obtained as:

$$\begin{aligned}&P_{c,BRU}(\gamma_{BU}, \gamma_{RU} \mid d_{BR}, d_{RU})\\ &= \Pr\{\mathrm{SINR}_{BU} > \gamma_{BU}\}\Pr\{\mathrm{SINR}_{RU} > \gamma_{RU}\}\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\tilde{\gamma})^{2/\alpha}\frac{\pi}{\alpha\sin(2\pi/\alpha)}\\ &\quad \times \Big((\gamma_{BU})^{2/\alpha} d_{BR}^2 + (\gamma_{RU})^{2/\alpha} d_{RU}^2\Big)\}.\end{aligned} \quad (11)$$

Without loss of generality, let the required SINR thresholds for both links be the same, i.e., $\gamma_{BU} = \gamma_{RU} = \tilde{\gamma}$, then $P_{c,BRU}$ can be simplified to:

$$\begin{aligned}&P_{c,BRU}(\tilde{\gamma} \mid d_{BU}, r)\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\tilde{\gamma})^{2/\alpha}\frac{\pi}{\alpha\sin(2\pi/\alpha)}(d_{BR}^2 + d_{RU}^2)\}\\ &= \exp\{-2\pi p\lambda_{\mathrm{B}}(\tilde{\gamma})^{2/\alpha}\frac{\pi}{\alpha\sin(2\pi/\alpha)}(d_{BU}^2/2 + 2r^2)\},\end{aligned} \quad (12)$$

which can be averaged over r to yield:

$$\begin{aligned}\bar{P}_{c,BRU}(\tilde{\gamma} \mid d_{BU}) &= \int_0^\infty P_{c,BRU}(\tilde{\gamma} \mid d_{BU}, r) f_r(r) dr\\ &= \frac{\lambda_{\mathrm{R}}}{\beta + \lambda_{\mathrm{R}}} e^{-\pi\beta d_{BU}^2/4},\end{aligned} \quad (13)$$

where $\beta = 4p\lambda_{\mathrm{B}}(\tilde{\gamma})^{2/\alpha}\frac{\pi}{\alpha\sin(2\pi/\alpha)}$.

Hence, the coverage probability of the relay-assisted system can be expressed as:

$$P_c = \begin{cases} P_{c,BU}(\gamma_{BU} \mid d_{BU}), & d_{BU} \leq r_{BS}, \\ \bar{P}_{c,BRU}(\tilde{\gamma} \mid d_{BU}), & d_{BU} > r_{BS}. \end{cases} \tag{14}$$

C. Achievable Transmission Rate

According to Shannon-Hartley theorem, given SINR $= \gamma$, the maximum rate of information that can be reliably sent over a given link with unit bandwidth (i.e., spectral efficiency in bit/s/Hz) is $\log_2(1+\gamma)$. As a result, the successful transmission per subcarrier at rate $\log_2(1+\gamma_{BU})$ for the BS-EU link can be obtained as:

$$\begin{aligned} R_{b,BU} &= \Delta f \log_2(1+\gamma_{BU}) P_{c,BU}(\gamma_{BU} \mid d_{BU}) \\ &= \Delta f \log_2(1+\gamma_{BU}) e^{-2\pi p \lambda_{\mathrm{B}}(\gamma_{BU})^{2/\alpha} d_{BU}^2 \frac{\pi}{\alpha\sin(2\pi/\alpha)}}. \end{aligned} \tag{15}$$

Similarly, the achievable rate for the 2-hop transmission can be obtained as:

$$\begin{aligned} R_{b,BRU} &= \frac{1}{2}\Delta f \log_2(1+\tilde{\gamma}) \bar{P}_{c,BRU}(\tilde{\gamma} \mid d_{BU}) \\ &= \frac{\lambda_{\mathrm{R}}\Delta f}{2(\beta+\lambda_{\mathrm{R}})} \log_2(1+\tilde{\gamma}) e^{-\pi\beta d_{BU}^2/4}. \end{aligned} \tag{16}$$

Note that the rate for the relay-based transmission is half that of the direct transmission because the data requires two time slots to reach the destination for the 2-hop transmission.

The achievable transmission rate for the relay-assisted system can be expressed as:

$$R_b = \begin{cases} R_{b,BU}, & d_{BU} \leq r_{BS}, \\ R_{b,BRU}, & d_{BU} > r_{BS}. \end{cases} \tag{17}$$

IV. Blocking Probability Analysis

In this section, we investigate the blocking probability of the relay-assisted system. First, we model the traffic of the users and classify them into different classes based on their required subcarriers. We consider the queueing system to be multi-dimensional with multiple classes. Then, we calculate the blocking probability using the multi-dimensional loss model.

A. Resource Allocation and Traffic Modeling

In OFDMA networks, users are assigned different number of subcarriers to fulfill their rate requirements. In fact, even if all users request the same predefined rate R, due to the relative distances between the users and the base station, some users would experience poor SINR levels and hence would require more subcarriers to satisfy their transmission requirements. To see this, let R be the rate achieved using M subcarriers on the BS-UE link. Then, from (15), we have:

$$R = \Delta f \sum_{i=1}^{M} \log_2(1+\gamma_{BU,i}) P_{c,BU}(\gamma_{BU,i} \mid d_{BU}). \tag{18}$$

Assume that the SINR targets are equal for all M subcarriers, then, M can be expressed as:

$$\begin{aligned} M &= \frac{R}{\Delta f \log_2(1+\gamma_{BU}) P_{c,BU}(\gamma_{BU} \mid d_{BU})} \\ &= \frac{R}{\Delta f \log_2(1+\gamma_{BU}) e^{-2\pi p \lambda_{\mathrm{B}}(\gamma_{BU})^{2/\alpha} d_{BU}^2 \frac{\pi}{\alpha\sin(2\pi/\alpha)}}}. \end{aligned} \tag{19}$$

It is clear that, for given R and γ_{BU}, as the distance between the base station and the user increases, M also increases. That is, under the same data rate requirement R, users that are close to the base station need fewer subcarriers, while users that are far away need more subcarriers to compensate for the low RSS. This explains why cell-edge users consume more resources than the cell-center users.

Motivated by the observation above, we can classify the incoming users into different classes based on their subcarrier requirements, or alternatively based on their relative distances to the base station, as follows. *First*, let the entire distance range be divided into L non-overlapping consecutive intervals/classes, denoted by C_j, $j = 1, 2, \ldots, L$. Thus, a user at a distance d_{BU} from its serving base station in the range $[d_{BU,j-1}, d_{BU,j})$ belongs to class C_j. We refer to this user as class-j user. *Second*, each class will be allocated a certain number of subcarriers to satisfy its rate requirement. Without loss of generality, we assign M_j subcarriers to class C_j, where M_j is the mean number of subcarriers required to achieve the rate R for d_{BU} in the range $[d_{BU,j-1}, d_{BU,j})$.

Define $P(C_j \mid d_{BU} \leq r_{BS})$ as the probability that an arriving user belongs to class C_j and requires M_j subcarriers given that it is located within the base station coverage region, which can be obtained as:

$$\begin{aligned} &P(C_j \mid d_{BU} \leq r_{BS}) \\ &= P(d_{0,j-1} \leq d_{BU} < d_{0,j} \mid d_{BU} \leq r_{BS}) \\ &= \frac{\int_{\min(d_{0,j-1}, r_{BS})}^{\min(d_{0,j}, r_{BS})} f_{d_{BU}}(d_{BU}) dd_{BU}}{\int_0^{r_{BS}} f_{d_{BU}}(d_{BU}) dd_{BU}} \\ &= \frac{(e^{-\lambda_{\mathrm{B}}\pi\min(d_{0,j-1}^2, r_{BS}^2)} - e^{-\lambda_{\mathrm{B}}\pi\min(d_{0,j}^2, r_{BS}^2)})}{(1 - e^{-\lambda_{\mathrm{B}}\pi r_{BS}^2})}, \end{aligned} \tag{20}$$

where $f_{d_{BU}}(d_{BU}) = 2\pi\lambda_{\mathrm{B}} d_{BU} e^{-\lambda_{\mathrm{B}}\pi d_{BU}^2}$ is the PDF of the distance between a user and its nearest base station. $d_{0,j-1}$ and $d_{0,j}$, which can be calculated from (19), represent the distances between the user and the base station to achieve the rate R with M_{j-1} and M_j, respectively. $P(C_j \mid d_{BU} > r_{BS})$ can be calculated similarly.

Using the law of total probability, the probability that a new user belongs to class C_j and requires M_j subcarriers can be obtained as:

$$\begin{aligned} &P(C_j) \\ &= e^{-\lambda_{\mathrm{B}}\pi\min(d_{0,j-1}^2, r_{BS}^2)} - e^{-\lambda_{\mathrm{B}}\pi\min(d_{0,j}^2, r_{BS}^2)} \\ &\quad + e^{-\lambda_{\mathrm{B}}\pi\max(d_{1,j-1}^2, r_{BS}^2)} - e^{-\lambda_{\mathrm{B}}\pi\max(d_{1,j}^2, r_{BS}^2)}. \end{aligned} \tag{21}$$

B. Blocking Probability Calculation

We assume that users arrive to the network according to a Poisson process with mean arrival rate λ, and depart with service rate μ, in which the service times (holding times) are independently and exponentially distributed with mean $\frac{1}{\mu}$.

As discussed earlier, based on the users rate requirements, each user is allocated, by the base station, a certain number of subcarriers from the total N subcarriers. That is, a class-j user would be assigned M_j subcarriers. If the available number of subcarriers is less than M_j, then this user will be blocked from accessing the network and may try later. Therefore, a blocking occurs when a user is denied of service due to the insufficient network resources. Since each user can request multiple subcarriers and release them simultaneously upon the completion of the service, then the relay-assisted system can be modeled using multi-dimensional Markov chains. More specifically, let λ_j denote the arrival rate of the users belonging to class C_j and requesting M_j subcarriers to meet their rate requirements. In this case, $\lambda_j = P(C_j)\lambda$, where $P(C_j)$ obtained in (21). Note that $\lambda = \sum_j \lambda_j$ for $j = 1, 2, \ldots, L$.

The state space (i.e., the set of all allowable states) of the existing users in the system can be expressed as:

$$S = \{\boldsymbol{n} : \sum_{j=1}^{L} n_j M_j \leq N\}, \tag{22}$$

where $\boldsymbol{n} = (n_1, n_2, \ldots, n_L)$ represents a state with n_1 users from class C_1, n_2 users from class C_2, and so on, that satisfies the inequality in (22).

The stationary distribution of $\boldsymbol{n}$ can be expressed in a product form as [13]:

$$\pi(\boldsymbol{n}) = \frac{1}{G(N,L)} \prod_{j=1}^{L} \frac{\rho_j^{n_j}}{n_j!}, \tag{23}$$

where

$$G(N,L) = \sum_{\boldsymbol{n}\in S} \prod_{j=1}^{L} \frac{\rho_j^{n_j}}{n_j!}, \quad \rho_j = \frac{\lambda_j}{\mu}. \tag{24}$$

Now, since the system has multiple classes and each class requires a certain number of subcarriers, then each class experiences different blocking probability. Let $P_{B,j}$ denote the class C_j blocking probability, which is the probability that a class-j user arrives and finds less than M_j subcarriers available in the system. $P_{B,j}$ can be obtained as:

$$P_{B,j} = \sum_{\boldsymbol{n}\in S_j} \pi(\boldsymbol{n}), \tag{25}$$

where $S_j = \{\boldsymbol{n} : N - M_j < \sum_{j=1}^{L} n_j M_j \leq N\}$ denotes the set of blocking states for class-j user.

The entire system blocking probability P_B can be obtained as:

$$P_B = \frac{1}{\rho} \sum_{j=1}^{L} \rho_j P_{B,j}, \tag{26}$$

where $\rho = \frac{\lambda}{\mu} = \sum_j \rho_j$ is the utilization factor.

Due to the large state space of the system, which is generally in the order of $\mathcal{O}(N^L)$, computing the blocking probability in (25) directly is computationally inefficient. Therefore, to reduce the computational complexity, for numerical evaluation, we use the well-known Kaufman-Roberts algorithm to calculate the blocking probability.

TABLE I
SYSTEM PARAMETERS

Number of subcarriers N	256
Number of classes L	200
Predefined rate R	0.2 Mbps
Subcarrier bandwidth Δf	180 KHz
Base/Relay stations intensity λ_B/λ_R	$1 \times 10^{-5}/\text{m}^2$
Base station activity factor p	1
SINR thresholds $\gamma_{BU} = \tilde{\gamma}$	5 dB
Mean arrival rate λ	3
Mean service rate μ	1
Path loss exponent α	4

V. SIMULATION RESULTS

In this section, we carry out the QoS performance analysis of relay-assisted system based on the proposed method for BP evaluation. More specifically, first, we obtain the optimal value of r_{BS}, denoted by $r_{BS,Opt}$, that minimizes the system blocking probability P_B. Then, using $r_{BS,Opt}$, we calculate the class distribution as well as the class blocking probability. The simulation parameters are listed in Table I.

Example 1: The optimal base station range In this example, we obtain $r_{BS,Opt}$ that achieves minimum blocking probability P_B, as shown in Figure 3. It can be seen that for small values of r_{BS}, the blocking probability is relatively high since most of the users communicate with the base station indirectly through the relay stations. As r_{BS} starts increasing, more users will be located inside the base station coverage region and establish a direct connection with the base station, which reduces the blocking probability. When r_{BS} becomes larger, P_B starts increasing again, since the users that are far away would connect to the base station directly, but have low coverage probability. Thus, more network resources would be released for such users to meet their service requirements. It can also be observed that, when r_{BS} is very large, P_B stays constant. In this case, all users are located withing the base station coverage area.

From the discussions above, it is concluded that deploying relay stations along with the base stations extend the coverage area and improve the QoS, i.e., reduce the blocking probability.

Example 2: Class distribution and class blocking probability Using $r_{BS,Opt}$ obtained in **Example 1**, first, we evaluate the distribution of the number of subcarriers that an arriving user requires to meet its rate requirement. Then, based on this distribution, we calculate the class blocking probability $P_{B,j}$, as depicted in Figure 4. It can be noted from Figure 4 that as the group size of the number of subcarriers increases, the class probability decreases. This is in contrast with most

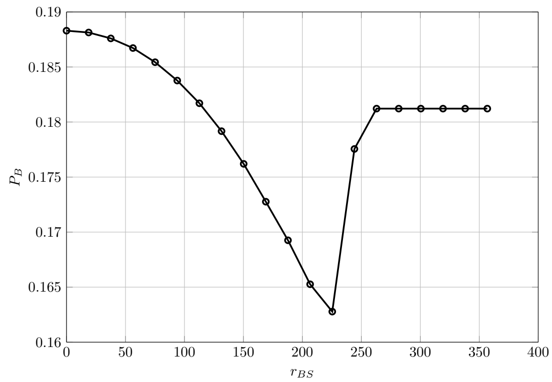

Fig. 3. Example 1: The system blocking probability P_B versus r_{BS}. Here, $r_{BS,Opt} = 225.36$ m.

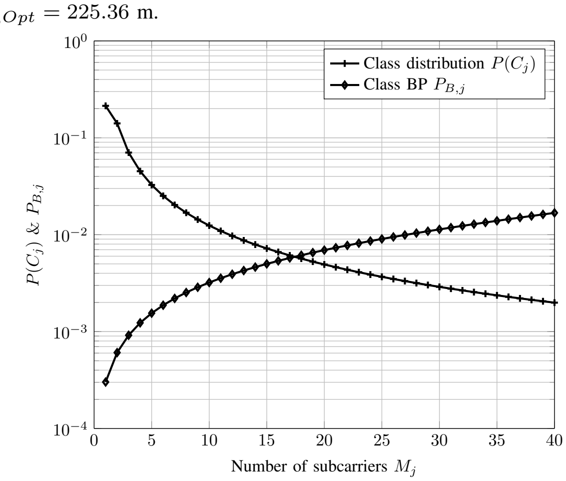

Fig. 4. Example 2: Class distribution and class blocking probability versus M_j using $r_{BS,Opt} = 225.36$ m.

of the existing work, which assumes uniform distribution [2]–[4]. It can also be seen from Figure 4 that the class blocking probability $P_{B,j}$ is a monotonically decreasing function with the class sizes, since users that are requiring large number of subcarriers are more likely to be blocked than those that are requiring fewer subcarriers.

VI. Conclusions

In this paper, we studied blocking probability in relay-assisted cellular OFDMA networks using stochastic geometry. First, we modeled the inter-cell interference from the neighboring cells at a typical node. Second, we derived the coverage probability in the downlink transmissions, including both the direct and relay-based transmissions. Third, we classified the incoming users into different classes based on their subcarrier requirements, and calculated the system blocking probability. It was shown that, with the relay-assisted system, the blocking probability can be reduced and therefore the network performance can be improved.

References

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, "What will 5G be?" *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1065–1082, June 2014.

[2] J.-C. Chen and W.-S. E. Chen, "Call blocking probability and bandwidth utilization of OFDM subcarrier allocation in next-generation wireless networks," *IEEE Communications Letters*, vol. 10, no. 2, pp. 82–84, Feb 2006.

[3] V. Pla, J. Martinez-Bauset, and V. Casares-Giner, "Comments on "call blocking probability and bandwidth utilization of OFDM subcarrier allocation in next-generation wireless networks"," *IEEE Communications Letters*, vol. 12, no. 5, pp. 349–349, May 2008.

[4] C. Paik and Y. S. Suh, "Generalized queueing model for call blocking probability and resource utilization in OFDM wireless networks," *IEEE Communications Letters*, vol. 15, no. 7, pp. 767–769, July 2011.

[5] M. Mehta, R. B. Jain, and A. Karandikar, "Analysis of blocking probability in a relay-based cellular OFDMA network," *Wireless Personal Communications*, vol. 84, no. 4, pp. 2467–2492, 2015.

[6] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Transactions on Communications*, vol. 59, no. 11, pp. 3122–3134, November 2011.

[7] W. Lu and M. D. Renzo, "Stochastic geometry modeling and system-level analysis & optimization of relay-aided downlink cellular networks," *IEEE Transactions on Communications*, vol. 63, no. 11, pp. 4063–4085, Nov 2015.

[8] P. H. J. Nardelli, P. Cardieri, and M. Latva-aho, "Efficiency of wireless networks under different hopping strategies," *IEEE Transactions on Wireless Communications*, vol. 11, no. 1, pp. 15–20, January 2012.

[9] H. ElSawy, E. Hossain, and M. Haenggi, "Stochastic geometry for modeling, analysis, and design of multi-tier and cognitive cellular wireless networks: A survey," *IEEE Communications Surveys Tutorials*, vol. 15, no. 3, pp. 996–1019, Third 2013.

[10] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An aloha protocol for multihop mobile wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, Feb 2006.

[11] Z. Lin, Y. Li, S. Wen, Y. Gao, X. Zhang, and D. Yang, "Stochastic geometry analysis of achievable transmission capacity for relay-assisted device-to-device networks," in *2014 IEEE International Conference on Communications (ICC)*, June 2014, pp. 2251–2256.

[12] F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks: Volume I Theory (Foundations and Trends in Networking)*. Now Publishers Inc, 2009.

[13] R. R. Mazumdar, "Performance modeling, loss networks, and statistical multiplexing," *Synthesis Lectures on Communication Networks*, vol. 2, no. 1, pp. 1–151, 2009.