End-to-End Delay in Multi-Hop Wireless Networks With Random Relay Deployment

Yuan Liang Yu Zheng Jian Ren Tongtong Li

Department of Electrical & Computer Engineering, Michigan State University, East Lansing, MI 48824, USA.
Email: {liangy11, zhengy30, renjian, tongli}@egr.msu.edu

***Abstract*—This paper evaluates the end-to-end delay in multi-hop wireless networks with random relays using stochastic geometry. We model the nodes as Poisson Point Processes and calculate the spatial average of the delay over all potential geometrical patterns of the nodes. More specifically, first, under a simple automatic repeat request (ARQ) retransmission protocol, we derive the average end-to-end delay and show that the average delay scales super exponentially with the increase of routing distance. Then we apply the maximal ratio combining (MRC) technique in ARQ, and derive an upper bound of the average end-to-end delay. We show that a linear scaling law of the delay with respect to the routing distance can be obtained with the help of MRC. Our analysis is demonstrated through simulation examples.**

***Index Terms*—Multi-hop networks, delay, stochastic geometry, random relays.**

I. Introduction

Multi-hop communication with relay assistance has become a prominent scheme in todays hybrid network design. The main reason is that it can extend the communication distance in wireless networks without the deployment of wired backhaul facilities. In wireless networks, the geometric locations of the nodes play a key role in determining the signal to interference and noise ratio (SINR), and hence the probability of successful transmission. In large scale multi-hop wireless networks, the node locations, including the relay locations, are generally random. The spatial randomness in node locations raises significant challenges in network performance analysis.

An effective tool to characterize the spatial randomness in wireless networks is stochastic geometry, for which the basic idea is to model the nodes as Poisson Point Processes (PPPs) and calculate the spatial averages of network performance characteristics by averaging over all potential geometrical patterns of the nodes [1], [2].

Under the stochastic geometry modeling, the end-to-end delay of multi-hop wireless networks was studied in [3], [4]. However, in these approaches, the relay locations were assumed to be *deterministic and known*, so the randomness of relays was not fully characterized and taken into consideration. Assuming a linear pattern of random relays, the end-to-end delay was evaluated in [5] with different network setups. However, the scaling law of the average end-to-end delay with respect to the routing distance was not explicitly discussed there.

As an effort to further explore the effect of relay randomness on network performance, in this paper, we analyze the end-to-end delay of a general multi-hop route in a wireless network with randomly located relays. In our analysis, we model the relays as a linear PPP between the source and destination and model the external interferers as an independent PPP over the whole plane. We first show that under a simple automatic repeat request (ARQ) retransmission protocol, the average end-to-end delay scales super exponentially with the increase of routing distance. Then we apply the maximal ratio combining (MRC) technique in ARQ, where a linear scaling law of the average end-to-end delay with respect to the routing distance can be obtained.

II. System Description

A. Network Model

We consider a source node $\boldsymbol{S}$, and a destination node $\boldsymbol{D}$ located at a distance of R. A linear relay pattern is studied, where the relay nodes are distributed randomly along the line segment between $\boldsymbol{S}$ and $\boldsymbol{D}$. Without loss of generality, we assume $\boldsymbol{S}$ is at the origin and $\boldsymbol{D}$ is located at $(R,0)$. Thus the candidate relay nodes formulate a 1D point process $\Phi = \{\boldsymbol{X}_i, i = 1,2,...,N\}$, where N is the random variable (RV) denoting the number of relays, and $\boldsymbol{X}_i$ is the location of the i-th relay along the line segment between $(0,0)$ and $(R,0)$. In the remaining part of this paper, we model Φ as a 1D homogeneous PPP (HPPP) of intensity λ. The locations of the relays would remain static, in contrast to the high mobility model.

The nearest neighbor (NN) routing protocol is employed. Relay node $\boldsymbol{X}_i$ would transmit the packets originated from the source $\boldsymbol{S}$ to the next relay $\boldsymbol{X}_{i+1}$ along the direction to $\boldsymbol{D}$ in a *decode-and-forward* manner. *We define the end-to-end delay as the number of time slots it takes for a packet to travel from* $\boldsymbol{S}$ *to* $\boldsymbol{D}$. We assume that the source $\boldsymbol{S}$ will not transmit a new packet until the previous packet has been successfully received by the destination $\boldsymbol{D}$. Thus only one packet is being delivered on the route at any time. In this scenario, the queuing delay, as well as the intra-route interference, are avoided. We consider a slotted system, where the transmission of one packet takes one time slot.

We apply the *decoupling* technique in [5] to our network model, where all the other nodes that are not along the $\boldsymbol{S}$-$\boldsymbol{D}$ path are modeled as an independent 2D point process Ψ

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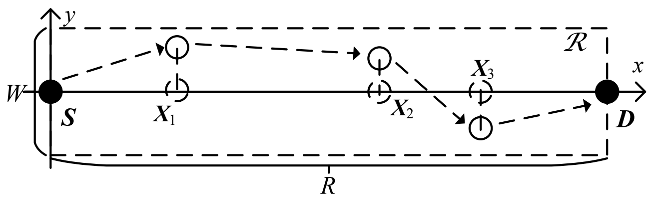

Fig. 1. An illustration of relays randomly deployed over a 2D area

over $\mathbb{R}^2$ from Φ. Potentially, these nodes can be the external interferers to the relays we study when they transmit over the same spectrum and time slot. For the remaining part of this paper, we model Ψ as a 2D HPPP of intensity μ. We assume that the transmissions of the nodes in Ψ follow the ALOHA protocol, where each node would transmit at each time slot independently with a probability of p_a.

For the tractability of the problem, we mainly consider the case where relays are deployed linearly between $\boldsymbol{S}$ and $\boldsymbol{D}$. However, the results obtained here actually provide an upper bound on the more practical scenario of a 2D case. For example, in Fig. 1, the relays are deployed randomly in a $R \times W$ rectangle $\mathcal{R}$ whose widths intersect $\boldsymbol{S}$ and $\boldsymbol{D}$. A simple routing protocol is that each relay would transmit to its nearest neighbor along the direction to $\boldsymbol{D}$ (the x-coordinate). By projecting the relays to the x-coordinate, we can find that the hop distances in the 2D case are lower bounded by the hop distances in the 1D case. Thus, the delay in 1D case is an upper bound for that of the 2D case.

B. Channel Model

Both large-scale path-loss and small-scale fading are considered. The received power of a signal transmitted at a distance of x meters with transmit power P_T is

$$P_R(x) = \frac{P_T \cdot H}{c \cdot x^\beta} , \qquad (1)$$

where H denotes channel gain, β is the path-loss exponent, and c is a constant determined by the antenna gains and signal wavelength. H is an exponentially distributed random variable with mean 1, i.e., Rayleigh fading is considered. Independent small scale fading is assumed for different transmitter-receiver pairs in different time slots. The small scale fading from location $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$ is represented by $H_{\boldsymbol{x}_1,\boldsymbol{x}_2}$.

C. SINR Characterization

A fixed rate coding scheme is employed in the physical layer, where a packet can be successfully decoded iff the received signal to interference and noise ratio (SINR) is above a given threshold θ. We consider an interference-limiting scenario, where the noise power is negligible compared with the interference power, so we use signal to interference ratio (SIR) and SINR interchangeably. Without loss of generality, we assume that each node in the network transmits with unit power. For $i = 1, 2, ..., N' + 1$ and let $\boldsymbol{X}'_{N'+1} = \boldsymbol{D}$, the received SIR of one transmission from $\boldsymbol{X}'_{i-1}$ to $\boldsymbol{X}'_i$ is

$$\mathrm{SIR}(\boldsymbol{X}'_i) = \frac{H_{\boldsymbol{X}'_{i-1},\boldsymbol{X}'_i} |\boldsymbol{X}'_i - \boldsymbol{X}'_{i-1}|^{-\beta}}{\sum_{\boldsymbol{Y}_j \in \Psi} B(\boldsymbol{Y}_j) H_{\boldsymbol{Y}_j,\boldsymbol{X}'_i} |\boldsymbol{Y}_j - \boldsymbol{X}'_i|^{-\beta}} , \qquad (2)$$

where for any $\boldsymbol{Y}_j \in \Psi$, the binary RV $B(\boldsymbol{Y}_j)$ indicates whether the "external" node $\boldsymbol{Y}_j$ would transmit under ALOHA protocol at the time slot of interest. The *coverage probability* for the transmission is $\Pr\{\mathrm{SIR}(\boldsymbol{X}'_i) > \theta\}$.

The distribution of external interferers in a given time slot can be viewed as an independent thinning of Ψ with a retention probability of p_a, i.e., an HPPP with intensity $\mu' = p_a\mu$. Following the same assumption made in [6], we make the approximation that packet successes are independent across different hops and time slots, i.e., the distribution of external interferers are independent across different time slots. Under such approximation, the coverage probability for an individual hop of distance l can be calculated as follows.

Lemma 1. *Given a hop distance l, the coverage probability for the hop is*

$$P_s(l) = \exp(-\kappa l^2), \qquad (3)$$

where $\kappa = 2\pi\mu' \frac{\pi}{\beta \sin(2\pi/\beta)} \theta^{2/\beta}$.

Proof. See [7]. □

D. Retransmission Scheme

Once a node on the route receives a new packet from the previous hop, it will transmit the packet to its next hop in the following time slot. However, because of the SINR limitation, the reception may fail and retransmission will be needed. Here, we employ the *automatic repeat request* (ARQ) [8] protocol, where the receiver will send a retransmission request to the transmitter at the end of the time slot if the reception fails. The transmitter will keep retransmitting the packet until the packet is successfully received.

In a simple implementation of ARQ, the received signals in different retransmission attempts of a packet are demodulated individually, and the packet is successfully received iff the SINR of one transmission is above the threshold θ. However, since the received signal will still contain certain information of the original packet even if the SINR is below the threshold, fewer retransmissions can be required by jointly demodulating the received signals in retransmissions.

In this paper, we apply *maximal ratio combining* (MRC) in ARQ for joint demodulation of retransmitted signals. MRC [9] is a technique that is widely used in the demodulation of multi-antenna receiver. The basic idea of MRC is that, if we can have multiple received signals of the same packet, where the interferences/noises are not fully correlated among the received signals, we can estimate the transmitted packet by demodulating the multiple received signals jointly. In MRC, the received signals are weighted according to their SINRs and the weighted signals are summed to generate a combined signal which can maximize its SINR. We utilize MRC to jointly demodulate retransmitted signals. A similar idea was also proposed in [10].

In the following part of this paper, we first derive the average end-to-end delay under random relays using a simple ARQ without MRC, and then analyze the case with MRC.

III. The End-to-End Delay under Simple ARQ

We denote the number of time slots it takes to successfully transmit a packet from $\boldsymbol{X}_{i-1}$ to $\boldsymbol{X}_i$ by $D(\boldsymbol{X}_i)$, i.e., the local delay at $\boldsymbol{X}_i$, and define the end-to-end delay as

$$D_{\text{end}} \triangleq \sum_{i=1}^{N+1} D(\boldsymbol{X}_i) \ . \tag{4}$$

Under a simple ARQ, assume that the successful packet transmission are independent across different hops and time slots. For a given the hop distance L_i, $D(\boldsymbol{X}_i)$ follows a geometric distribution, i.e.,

$$\Pr\{D(\boldsymbol{X}_i) = n \mid L_i\} = [1 - P_s(L_i)]^{n-1} \ P_s(L_i) \ , \tag{5}$$

and the average local delay is $1/P_s(L_i)$. So the distribution of the local delay at a specific hop depends on its hop distance and is independent of the distribution of other relays in Φ.

Define function

$$D(r) \triangleq \mathbb{E}\{D_{\text{end}} \mid R = r\} \text{ for } r > 0, \tag{6}$$

as the average end-to-end delay given routing distance $R = r$. Basing on the Palm theory of PPP[1], conditioned on the location of the first relay $|\boldsymbol{X}_1| = x$, the remaining relay nodes within the interval (x, r) is still a 1D PPP of the same intensity λ. So the delay from the second relay to the destination satisfies

$$\mathbb{E}\{\sum_{i=2}^{N+1} D(\boldsymbol{X}_i) \mid R = r, |\boldsymbol{X}_1| = x\} = D(r - x). \tag{7}$$

Conditioned on $\boldsymbol{X}_1$, we can derive that

$$D(r) = \int_0^{r^+} f_{|X_1|}(x) \mathbb{E}\{D_{\text{end}} \,|\, R = r, |X_1| = x\} \mathrm{d}x$$
$$= \int_0^{r^+} f_{|X_1|}(x) \left[\mathbb{E}\{D(X_1) \,|\, |X_1| = x\} + D(r-x)\right] \mathrm{d}x. \tag{8}$$

Then, we have the following equation

$$D(r) = \int_0^{r^-} \lambda e^{-\lambda x} [\frac{1}{P_s(x)} + D(r-x)] \ \mathrm{d}x + e^{-\lambda r} \frac{1}{P_s(r)}. \tag{9}$$

Basing on (9), we have the following results on $D(r)$.

Theorem 1. *The average end-to-end delay given routing distance r using ARQ is*

$$D(r) = \frac{e^{-\lambda r}}{P_s(r)} + 2\lambda \int_0^r \frac{e^{-\lambda x}}{P_s(x)} dx + \lambda^2 \int_0^r (r - x) \frac{e^{-\lambda x}}{P_s(x)} dx \ , \tag{10}$$

[1]For a PPP, given that one node is located at a particular point, the conditional distribution of all other nodes is still a PPP, which is known as Slivnyak-Mecke Theorem [11, Theorem 1.4.5].

where $P_s(x)$ is the single hop transmission success probability given hop distance x, as defined in Lemma 1.

Proof. It can be verified that (10) satisfies (9). As (9) represents an AR system, and the initial state of average delay is 0 for $r = 0$, the solution (10) is unique. □

Basing on *Theorem 1* and *Lemma 1*, we have the following corollary on the scaling law of the end-to-end delay $D(r)$.

Corollary 1. *As $r \to +\infty$, the average end-to-end delay using ARQ scales with $\mathcal{O}(e^{\kappa r^2 - \lambda r})$.*

So, under a simple ARQ, the average end-to-end delay scales super exponentially with the increase of routing distance. Note that for the equidistant relays with a fixed hop distance, the delay scales linearly with the increase of routing distance. This demonstrates the performance loss introduced by the randomness of relays.

IV. The End-to-End Delay Analysis using MRC

To improve the communication efficiency, in this section, we apply MRC in ARQ. We first evaluate the average local delay and then analyze the average end-to-end delay.

A. The Local Delay Analysis

Let D denote the local delay of a packet at an arbitrary hop where the hop distance is l, SIR_i denote the received SIR at the i-th transmission of the packet, and $\widetilde{\text{SIR}}_d$ denote the equivalent SIR of the combined signal by applying MRC to the received signals of the first d transmissions. Assume that the signals transmitted by the interferers are independent across different time slots, then we have [12]

$$\widetilde{\text{SIR}}_d = \sum_{i=1}^{d} \text{SIR}_i \ . \tag{11}$$

So the delay D satisfies

$$\Pr\{D \geq d\} = \Pr\{\widetilde{\text{SIR}}_{d-1} < \theta\}, \quad d \geq 2 \ . \tag{12}$$

In this subsection, we aim to evaluate the average local delay $\mathbb{E}\{D\}$ w.r.t. different hop distance l.

As what we have assumed before, the channel gains and the locations of the interferers are modeled as independent across different time slots. In this case, the SIR_i's are i.i.d. From *Lemma 1*, the complementary cumulative probability function (CCDF) of SIR_i is

$$\Pr\{\text{SIR}_i > x\} = \exp\{-\eta x^{2/\beta}\} \text{ for } x \geq 0, \tag{13}$$

where $\eta = \frac{2\pi^2 \mu'}{\beta \sin(2\pi/\beta)} l^2$. The average local delay $\mathbb{E}\{D\}$ is hard to derive because of the difficulty in analyzing the distribution of $\widetilde{\text{SIR}}_d$. So we derive an upper bound of $\mathbb{E}\{D\}$ instead. First, we introduce the following lemmas.

Lemma 2. *Consider RVs $Z_i \sim \Gamma(2/\beta, \eta')$ that are i.i.d. over i, where $\eta' = \left[\eta \frac{2}{\beta} \Gamma(2/\beta)\right]^{\beta/2}$. That is,*

$$f_{Z_i}(x) = \frac{\eta'^{2/\beta}}{\Gamma(2/\beta)} x^{2/\beta - 1} e^{-\eta' x} \ \text{for } x > 0. \tag{14}$$

Then, SIR_i has first-order stochastic dominance (FSD) over Z_i, i.e., $\Pr\{SIR_i \geq x\} \geq \Pr\{Z_i \geq x\}$ for all x and $\Pr\{SIR_i \geq x\} > \Pr\{Z_i \geq x\}$ for some x.

Proof. This lemma can be proved by analyzing the monotonicity of function $G(x) \triangleq \Pr\{\mathrm{SIR}_i \geq x\} - \Pr\{Z_i \geq x\}$. We skip the process for brevity. □

Lemma 3. *For any given $a > 0$, function $u(x) \triangleq \gamma(x, a)/\Gamma(x)$ is a decreasing function w.r.t. $x > 0$, where $\Gamma(\cdot)$ is the gamma function and $\gamma(\cdot,\cdot)$ is the lower incomplete gamma function.*

Proof. Note that $u(x)$ is the probability $\Pr\{U \leq 1\}$, where RV $U \sim \Gamma(x, a)$. For any $x_1 < x_2$, consider three RVs, $U_1 \sim \Gamma(x_1, a)$, $U_2 \sim \Gamma(x_2 - x_1, a)$ and $U_3 = U_1 + U_2$, where $U_3 \sim \Gamma(x_2, a)$ according to the property of gamma distribution. So U_3 has FSD over U_1, where $\Pr\{U_3 > 1\} > \Pr\{U_1 > 1\}$. Thus $u(x_1) > u(x_2)$, which completes the proof. □

With the lemmas above, we have the following theorem.

Theorem 2. *For a hop distance of l, the average local delay at the hop is upper bounded by*

$$\mathbb{E}\{D\} \leq \lceil \frac{\beta}{2} \rceil (1 + \omega l^\beta) \ , \tag{15}$$

where $\omega = \theta \left[\frac{\mu'}{\sin(2\pi/\beta)} \left(\frac{2\pi}{\beta}\right)^2 \Gamma(2/\beta) \right]^{\beta/2}$.

Proof. From the definition of RVs Z_i in Lemma 2, define RVs $\widetilde{Z}_d = \sum_{i=1}^{d} Z_i$. Basing on the property of gamma distribution, we have $\widetilde{Z}_d \sim \Gamma(\frac{2}{\beta}d, \eta')$. Since SIR_i has FSD over Z_i, $\widetilde{\mathrm{SIR}}_d$ also has FSD over $\widetilde{Z}_d$. So we have

$$\Pr\{\widetilde{\mathrm{SIR}}_d < \theta\} \leq \Pr\{\widetilde{Z}_d < \theta\} = \frac{\gamma(\frac{2}{\beta}d, \eta'\theta)}{\Gamma(\frac{2}{\beta}d)}, \ d \geq 1 \ . \tag{16}$$

Note that the average local delay is

$$\mathbb{E}\{D\} = \sum_{d=1}^{+\infty} d \Pr\{D = d\} = \sum_{d=1}^{+\infty} \Pr\{D \geq d\} \ , \tag{17}$$

which is upper bounded by

$$1 + \sum_{d=2}^{+\infty} \Pr\{\widetilde{Z}_{d-1} < \theta\} \leq \lceil \frac{\beta}{2} \rceil + \sum_{d=\lceil \frac{\beta}{2} \rceil}^{+\infty} \frac{\gamma(\frac{2}{\beta}d, \eta'\theta)}{\Gamma(\frac{2}{\beta}d)}. \tag{18}$$

Basing on Lemma 3, (18) is upper bounded by

$$\lceil \frac{\beta}{2} \rceil + \sum_{d=\lceil \frac{\beta}{2} \rceil}^{+\infty} \frac{\gamma(\lfloor \frac{2}{\beta}d \rfloor, \eta'\theta)}{\Gamma(\lfloor \frac{2}{\beta}d \rfloor)} \leq \lceil \frac{\beta}{2} \rceil \left[1 + \sum_{d=1}^{+\infty} \frac{\gamma(d, \eta'\theta)}{\Gamma(d)} \right]. \tag{19}$$

Note that for an integer d, from the property of gamma distribution, we have

$$\frac{\gamma(d, \eta'\theta)}{\Gamma(d)} = e^{-\eta'\theta} \sum_{i=d}^{+\infty} \frac{(\eta'\theta)^i}{i!}. \tag{20}$$

So

$$\sum_{d=1}^{+\infty}\sum_{i=d}^{+\infty} \frac{(\eta'\theta)^i}{i!} = \sum_{i=1}^{+\infty}\sum_{d=1}^{i} \frac{(\eta'\theta)^i}{i!} = \sum_{i=1}^{+\infty} \frac{(\eta'\theta)^i}{(i-1)!} = \eta'\theta e^{\eta'\theta} \tag{21}$$

By replacing η' with its definition, the result is obtained . □

The effect of correlated SIRs: In the analysis above, the channel gains and the locations of the interferers are i.i.d. across different time slots. Here, we discuss the case where such i.i.d. assumption is not satisfied. The correlation of interference in Poisson network and its influence on network performance were explicitly addressed in [13]. We consider an extreme example where the channel gains and the interferer locations keep static during the retransmissions of a packet. That is, $\mathrm{SIR}_i \equiv \mathrm{SIR}_1$ for all the possible i's and $\widetilde{\mathrm{SIR}}_d = d\mathrm{SIR}_1$. So the CCDF of $\widetilde{\mathrm{SIR}}_d$ satisfies

$$\Pr\{\widetilde{\mathrm{SIR}}_d > x\} = \Pr\{\mathrm{SIR}_1 > \frac{x}{d}\} = \exp\left\{ -\eta \left(\frac{x}{d}\right)^{\frac{2}{\beta}} \right\}, \ x > 0. \tag{22}$$

From (17), The corresponding average local delay is

$$\mathbb{E}\{D\} = 1 + \sum_{d=1}^{+\infty} \Pr\{\widetilde{\mathrm{SIR}}_d < \theta\} \ . \tag{23}$$

Because $\beta > 2$, the summation of series in (23) does not converge. So the average local delay is infinitely large in this case. Even though the assumption of fully correlated SIRs is not very practical, this example demonstrates the possible performance loss introduced by the correlation of SIRs.

B. The End-to-End Delay Analysis

Note that in the simple ARQ case, given a hop distance of l, the average local delay is $1/P_s(l)$. By replacing $1/P_s(\cdot)$ in (10) with the upper bound of average local delay of MRC in (15), we have the following theorem on the upper bound of the average end-to-end delay using MRC.

Theorem 3. *The average end-to-end delay given routing distance r using MRC is upper bounded by*

$$\lceil \frac{\beta}{2} \rceil \left[(\lambda r + 1)\left(\frac{\omega r^\beta}{e^{\lambda r}} + 1 \right) + \frac{\omega}{\lambda^\beta}(\lambda r + 2 - \beta)\gamma(\beta + 1, \lambda r) \right] \tag{24}$$

where $\gamma(\cdot,\cdot)$ is the incomplete gamma function.

Note that $\frac{r^\beta}{e^{\lambda r}}$ and $\gamma(\beta + 1, \lambda r)$ are both bounded with $r \to +\infty$. Since the average number of relays is a linear function w.r.t. routing distance r, the average end-to-end delay scales, at least, linearly w.r.t. r. So basing on Theorem 3, the following Corollary is obtained.

Corollary 2. *As $r \to +\infty$, the average end-to-end delay using MRC scales with $\mathcal{O}(r)$.*

That is, with the help of MRC, the end-to-end delay can scale linearly with the increase of routing distance. It should be noted that, for the case of equidistant relays where the hop distance is fixed, even though the local delay can be reduced by applying MRC, the linear scaling law of the end-to-end delay will keep unchanged as the hop number scales linearly with r. This shows that MRC can play a more important role in reducing the average end-to-end delay for the random relays.

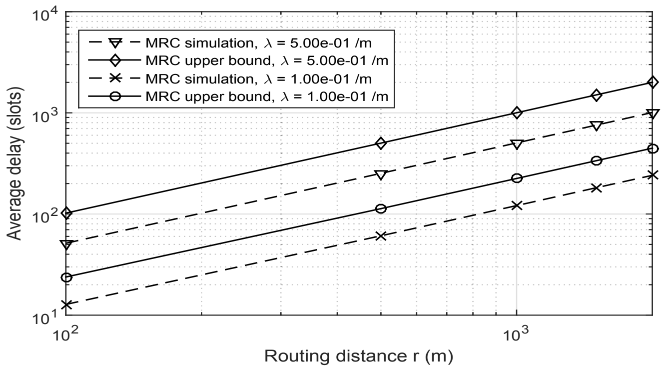

Fig. 2. The average end-to-end delay versus different routing distances

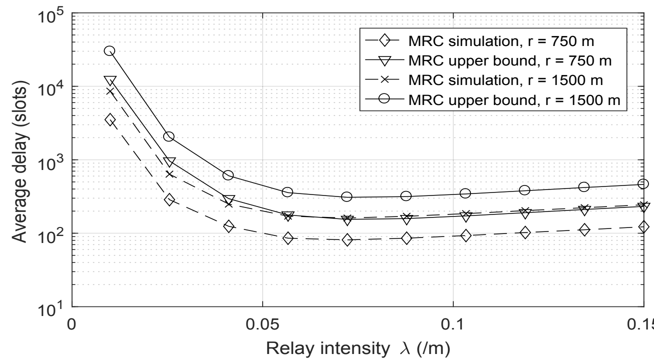

Fig. 3. The average end-to-end delay versus different relay intensities

V. Simulations

We verify our results through simulations. The following parameters are employed: the interferer intensity $\mu = 5 \times 10^{-4}\ /m^2$, the ALOHA access probability $p_a = 0.1$, the path-loss exponent $\beta = 4$ and the SINR threshold $\theta = 10$ dB.

Using MRC, the simulated values and the upper bounds derived in Theorem 3 of the average end-to-end delay are shown in Fig. 2 for relay intensities $\lambda = 0.1$ /m and 0.5 /m respectively versus different routing distances. First, in the simulation, a linear scaling law of the average end-to-end delay with the increase of routing distance is observed, which is consistent with Corollary 2. Second, for the selected parameters in simulation, the actual values of end-to-end delay are around 3 dB lower than the bounds derived in Theorem 3.

Fig. 3 plots the average end-to-end delay versus different relay intensities for routing distances $r = 750$ m and 1500 m using MRC. The effect of relay intensity on the end-to-end delay is two-fold. A higher relay intensity is able to shorten the hop distances between neighboring relays, decreasing the local delay at each hop, while it will also increase the number of hops on the route, introducing unnecessary relaying. So an optimal relay intensity exists. Fig. 3 shows that the optimal relay intensities are between 0.05/m and 0.1/m for the selected parameters, and the optimal relay intensity is not sensitive to the change of routing distances.

VI. Conclusions

In this paper, we evaluated the end-to-end delay in multi-hop wireless networks with a random relay deployment using stochastic geometry. We modeled the nodes as Poisson point processes. We first showed that, under a simple automatic repeat request retransmission protocol, the average delay scales super exponentially with the increase of routing distance. This revealed that, under a simple network setup, the performance loss introduced by the randomness of relays is severe. Then, to improve the communication efficiency, we applied maximal ratio combining in the retransmission scheme, and proved that the average delay can scale linearly with the increase of routing distance, which is a significant improvement over the general scheme. The possible performance loss introduced by the correlated SIRs was also discussed. It was found that during the retransmissions, the average local delay becomes infinite under fully correlated SIRs.

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