Influence of discharge polarity on streamer breakdown criterion of ambient air in a non-uniform electric field

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Abstract
Streamer breakdown of atmospheric air with non-uniform dc electric field in a needle-to-plate electrode configuration is studied using a semi-analytic model and experimental measurements. A high voltage (either positive or negative) is applied to a hollow needle with 0.51 mm outer diameter and 0.25 mm inner diameter separated from a planar ground electrode by a gap distance of 0.1–1.4 cm. Breakdown voltages are recorded for both positive and negative discharge polarities. Empirical relations between the critical avalanche size for streamer breakdown and the gap distance are proposed. Using these empirical relations, a semi-analytic model based on Meek’s criterion for streamer breakdown is developed to accurately predict the measured breakdown voltages. It is found that for \( pd > 380 \) Torr cm (or \( d > 0.5 \) cm at one atmosphere) streamer breakdown of ambient air occurs at a lower applied voltage for a positively biased needle compared to that with a negatively biased needle, referred as the polarity effect. For \( pd < 380 \) Torr cm breakdown is attained at a lower applied voltage with a negatively biased needle compared to that with a positively biased needle, and breakdown mode transits from the polarity effect to the so called inverted polarity effect.

Keywords: streamer breakdown, non-uniform electric field, needle-to-plate, discharge polarity, Meek’s criterion, polarity effect, inverted polarity effect

(Some figures may appear in colour only in the online journal)
1. Introduction

The streamer theory, developed by Raether [1], Loeb and Meek [2, 3] in the 1930s, proposed that in large inter-electrode gaps [4], the electrical breakdown mechanism depends on ionization by electrons and photoionization in the gas, instead of ionization avalanche combined with cathode emission caused by positive ions, as had previously been proposed [5, 6] by Townsend. For almost a century, this theory has been constantly employed by researchers, pioneering extensive research on streamer physics and applications [7, 8]. The theory has been particularly helpful to describe dielectric-barrier and lightning discharges [9, 10], transient luminous events in upper atmosphere [11], and electrical breakdown in liquid dielectrics and solid insulators [8, 12]. Streamers are employed in numerous modern applications including pollution control [13], air purification [14], water treatment [15], and medicine [16, 17]. Therefore, understanding the electrical breakdown phenomena caused by streamers is of great importance to their related applications.

Although streamer formation, propagation, and breakdown have been studied for many decades, most of these studies have focused on the homogeneous electric field condition. Meek’s criterion [2, 3, 18] prescribes a quantitative approach toward modeling streamer breakdown in a homogeneous electric field. However, the effects of a non-uniform electric field on streamer breakdown are still not well characterized. Studies show that non-uniform electric fields can produce rapid breakdown induced by highly energetic runaway electrons [19–22], which is the subject of many recent studies including particle-in-cell (PIC), PIC-Monte Carlo Collision (PIC-MCC) [23, 24] and hybrid [25] simulations as well as experimental investigations [26]. Despite these efforts, significant gaps still remain in the theoretical understanding of streamer breakdown caused by such runaway electrons as well as electrons with relatively low energy [27]. In addition, for such non-uniform electric fields, understanding the effects of the discharge polarity [28] on the critical avalanche size as well as the breakdown voltage is of fundamental importance, and yet rarely studied.

In this paper, we present experimentally measured breakdown voltages in various gap distances in a hollow needle-to-plate electrode configuration. Throughout this paper, the terms ‘hollow needle’ as well as ‘needle’ are used in reference to an electrode with a ring-shaped cross section and a large length-to-diameter aspect ratio (as shown in figure 1). We propose empirical formulae to calculate the critical avalanche size for streamer breakdown for both positive and negative needle voltages. Using these empirical equations, we develop a semi-analytic model based on Meek’s criterion [18] to predict breakdown voltages with a non-uniform dc electric field. We analyze the effects of discharge polarity on the breakdown voltage.

It is important to note the distinction between the discharge polarity [28] (i.e. positive/negative needle voltage) and the streamer polarity, i.e. the positive [29] (cathode directed) and negative [30] (anode directed) streamers described by Wagner [31]. The positive and negative streamers refer to the different directions of streamer propagation in a specific electrode configuration. For a fixed electrode polarity, depending on the regime of the applied voltage, a positive streamer may exist independently [32, 33] or coexist with a negative streamer to form a continuous channel [27, 31, 34]. In contrast, this paper focuses on the effect of different electrode polarities [35–39] on Meek’s criterion for breakdown due to positive streamers [2].

The experimental method for our study is described in section 2. Section 3 offers a detailed account of the semi-analytic model and numerical calculations. In section 4 we discuss the experimental results and compare them with the predictions of our model. In section 5, we summarize our work and discuss scopes of future work.

2. Experimental method

2.1. Experimental setup

The schematic of the experimental set-up is shown in figure 1. The electrode system consists of a stainless steel hollow needle, which has an inner diameter of 0.25 mm and outer diameter of 0.51 mm, and a ground plate that is made of copper. The end view (or cross section) of the hollow needle is shown in the insert of figure 1. The inter-electrode gap distance \(d\) is varied between 0.1 and 1.4 cm. A DC High-Voltage Power Source (Glassman PS/EH60R01.5), which has interchangeable positive and negative polarity modules, is used as the voltage source. The discharge voltage and current are measured with a high voltage probe (Tektronix P6015A) and current transformer (Ion Physics CM-100-L), connected to an oscilloscope. Note that the current monitor is based on a transformer and hence is not able to measure DC current. As this study is focused on streamer breakdown, aca. transition to a highly conductive discharge mode between the electrode gap, the voltage and current detectors are suitable for the application of transient measurements.

In atmospheric air and for a constant \(d\), the voltage applied to the needle electrode was increased gradually until breakdown occurred. Near the breakdown voltage, the voltage stayed constant for \(>1\) ms, so the voltage was assumed for direct current or quasi-static prior to the breakdown. Five repeated measurements were taken for each condition. After each breakdown occurrence, the voltage was immediately reduced to zero. The needle electrode was discharged with a grounded discharging rod in case of any residual charge and air-cooled for at least 3 min before the next trial. Ambient temperature and humidity were monitored with a thermometer (RadioShack Model 63-1090) and found to be an average of 21.9 °C and 37.1%, respectively. To ensure that the initial surface temperature of the needle electrode was kept constant for each breakdown measurement, separate temperature measurements were conducted using a fiber optic temperature sensor (Rugged Monitoring, model H201) in contact with the tip of the needle electrode. The surface temperature of the needle tip
was found increasing during breakdown, and the increase of the temperature varied with the gap distance and needle polarity. A maximum temperature change of 11.2 °C was detected for the positive needle voltage and \( d = 1.4 \text{ cm} \). Nevertheless, the temperature increase was highly transient; it took less than 10 s for the surface temperature of the needle dropping back to the ambient room temperature. It is hence safe to assume that the initial surface temperature was kept constant for each breakdown trial following aforementioned protocol that included 3 min cooling time between trials.

2.2. Determination of streamer breakdown with voltage and current measurements

At a relatively large \( d \), e.g. \( d > 0.3 \text{ cm} \), streamer discharges were observed before the breakdown occurrence or streamer-to-spark transition. For the positive polarity, the cathode-directed streamer is unstable and accompanied with tens of mA current pulses while the charge in the DC voltage is small, e.g. <5% for \( d = 0.9 \text{ cm} \). For the negative polarity, stable DC corona discharges were observed around the needle. For smaller \( d \), breakdown may happen directly without any initiation of streamers. This is similar to what was observed in pulsed discharges with repetitive impulse voltages \([38]\). Once the breakdown occurs, the waveforms of the voltage and current demonstrate a similar behavior regardless of the needle polarity or the gap distance; the collapse of the voltage to \( \leq 5\% \) of its original value is accompanied by a rapid increase of the current. Typical voltage and current waveforms of the breakdown for both positive and negative polarities at \( d = 0.9 \text{ cm} \) are shown in figure 2. For the negative polarity, the hold-off voltage was observed at \(-17.7 \text{ kV}\) and the peak current reached \(-652 \text{ A}\) during breakdown. At the same gap distance (\( d = 0.9 \text{ cm} \)), the lower hold-off voltage was observed for the positive polarity, about \(12.6 \text{ kV}\), and the peak current was measured to be \(378 \text{ A}\). The maximum allowed voltage of the power supply used in our experiments is \(30 \text{ kV}\), which limits the gap distance to be no greater than \(1.4 \text{ cm}\).

3. Semi-analytic streamer breakdown model

3.1. Negative needle voltage case

Figure 3 shows a simplified schematic of the semi-analytical streamer breakdown model for a needle-to-plate geometry in atmospheric air when a negative voltage is applied to the needle and the flat surface is grounded. In this case, seed electrons originated from the needle cathode propagate towards the flat-surface anode, creating a positive ions channel along its way through ionization induced electron/ion avalanche.

According to Meek’s criterion for electrical breakdown of gases \([2, 3, 18]\), when the avalanche reaches the anode, the positive space charge due to ions produces an electric field (\(E_{SC}\)) towards the anode of the same order as the external applied field (\(E_a\)), so as to retain electrons in the ion channel from the anode for streamer formation. For simple analytical treatment, we assume that the positive space charge required to cause breakdown, \(Q_{x0}\), is generated through ionization at a distance \(x = x_0\) from the tip of the needle and is concentrated in a spherical volume \([18]\) (figure 3). Calculation procedure of the distance \(x = x_0\) will be discussed in section 4. Mathematically, the number of electrons in this spherical volume is given by \(n_{ex} = e^{x_0}\), where \(k_{ct}\) is a constant representing the critical avalanche size for breakdown \([40–42]\). Once the positive space charge reaches the critical value of \(Q_{x0}\) at \(x = x_0\), further propagation of the electrons towards the anode is assumed not to cause more ionization and the total space charge \(Q_{x0}\) is kept constant for \(x > x_0\). The value of \(k_{ct}\) can be obtained from

\[k_{ct} = \int_{0}^{x_0} (\alpha - \eta) \, dx.\] (1)

Here, \(\alpha\) and \(\eta\) are Townsend’s first ionization coefficient and the attachment coefficient \([43]\), respectively. We set
Figure 2. Voltage and current waveforms of the streamer breakdown across a 0.9 cm gap in air with (a) negative and (b) positive voltages applied to the needle electrode.

Figure 3. Schematic of the semi-analytical streamer breakdown model for negative needle-voltage.

\[ \eta = 0 \] in our model. For air, the first Townsend ionization coefficient \( \alpha \) is expressed as [43–45],

\[ \alpha (x) \, [\text{cm}^{-1}] = A \rho^{-\eta/\alpha} \quad (2) \]

where \( \rho \, [\text{Torr}] \) is the gas pressure, \( \alpha \, [\text{Vcm}^{-1}] \) is the non-uniform electric field, \( A \) and \( B \) are gas dependent constants. In our model we use, \( A = 15 \text{Torr}^{-1} \text{cm}^{-1} \) and \( B = 365 \text{V} \text{Torr}^{-1} \text{cm}^{-1} \) for air [46]. As the avalanche progresses from the cathode needle to the anode plate, the radius of the space-charge sphere increases due to the radial diffusion of electrons. Based on the continuity equation of diffusion, the radius of the space-charge sphere \( r_x \) at a distance \( x \) from the point of origin can be calculated from [2, 18],

\[ r_x \, [\text{cm}] = (2D_r t_x)^{0.5} \quad (3) \]

where \( t_x \, [\text{s}] \) is the time required for the avalanche to reach the distance \( x \) from the origin and \( D_r \, [\text{cm}^2 \text{s}^{-1}] \) is the radial diffusion coefficient. Based on the experimental evaluation of the radial diffusion coefficient of electrons in the presence of an axial electric field \( (E_x) \) by Masek [47], Novak and Bartnikas proposed that \( D_r \) can be approximated as [48],

\[ D_r (x) \, [\text{cm}^2 \text{s}^{-1}] = 232 \times \left( \frac{E_x \, [\text{Vcm}^{-1}]}{p \, [\text{Torr}]} \right)^{0.5} \quad (4) \]

The time of advance of the avalanche, \( t_x \), can be calculated from \( t_x = \int_0^x dx'/v_e (x') \), where \( v_e (x) \) is the electron drift velocity in the axial direction. In our model, we adopt the following empirical relation between the electron drift velocity and the applied axial electric field \( (E_x) \) given by Ali [49],

\[ v_e (x) \, [\text{cm} \text{s}^{-1}] = 6 \times 10^6 + 2.3 \times 10^5 \left( \frac{E_x \, [\text{Vcm}^{-1}]}{p \, [\text{Torr}]} \right) \]

for \( \frac{E_x}{p \, [\text{Torr}]} < 120 \),

\[ v_e (x) \, [\text{cm} \text{s}^{-1}] = 3.38 \times 10^6 \left( \frac{E_x \, [\text{Vcm}^{-1}]}{p \, [\text{Torr}]} \right)^{0.5} \]

for \( \frac{E_x}{p \, [\text{Torr}]} \geq 120 \).
When the space-charge sphere reaches the anode plate, the electric field produced by positive space-charge in the sphere can be obtained as [18],

\[ E_{SC} = \frac{Q_0}{r_{anode}^2} = \frac{1.44 \times 10^{-7} e_{no}}{r_{anode}^2} \text{ [V cm}^{-1}\text{]}, \]  

(6)

where \( r_{anode} \) (cm) is the radius of the space-charge sphere at the anode plate (\( x = d \)). According to Meek’s criterion for breakdown [18], \( E_{SC} \) will be of the same order as the applied electric field at the anode \( E_{x, anode} \), i.e.,

\[ E_{x, anode} \text{ [V cm}^{-1}\text{]} = \frac{1.44 \times 10^{-7} e_{no}}{r_{anode}^2} \text{ [V cm}^{-1}\text{].} \]  

(7)

3.2. Numerical calculation procedure

We first obtain the vacuum non-uniform axial electric field \( (E_x) \) across the gap distance \( (d) \) for an applied voltage \( (V_d) \) using an electrostatic COMSOL simulation for the hollow needle-to-plate geometry used in experiments (figure 4(a)). Figure 4(b) shows an example of the axial electric field profile across a gap distance, \( d = 1 \text{ cm} \), when a negative voltage, \( V_d = 19.9 \text{kV} \), is applied to the needle. Then we obtain the first Townsend coefficient \( (\alpha) \) profile using equation (2), as shown in figure 4(c). We also obtain the electron diffusion coefficient \( (D_e) \) and the electron drift velocity \( (v_e) \) profiles by employing equations (4) and (5), respectively, shown in figures 5(a) and (b). Figure 5(c) shows the time elapsed \( t_x \) as an electron travels across the gap, with respect to the distance traveled \( x \), calculated numerically from \( t_x = \int_0^x \frac{dx'}{v_e (x')} \). It is noteworthy that \( V_d = 19.9 \text{kV} \) is the measured negative breakdown voltage for the needle-to-plate geometry. For this applied voltage, we observe from figure 5(c) that a space-charge sphere originated at the cathode reaches the anode after \( t_x = 1.134 \times 10^{-2} \text{s} \) and then causes breakdown (or streamer formation). This estimated formative time lag is in excellent agreement with the measured conventional value of formative time lag for breakdown of air, which is \( 10^{-7} \text{s} \) [2].

Because of the spatial dependence of diffusion coefficient and drift velocity, the radius of the ion space charge sphere is calculated as follows. From equation (3), the radial expansion speed of the ion sphere head at position \( x \) is,

\[ \frac{dr_x}{dx} = \sqrt{\frac{D_e (x')}{2 \tau_x}}. \]  

(8)

Thus, the radius of the sphere at \( x \) is,

\[ r_x = \int_0^x \frac{1}{v_e (x')} \sqrt{\frac{D_e (x')}{2 \tau_x}} dx', \]  

(9)

where \( t_x = \int_0^x \frac{dx'}{v_e (x')} \). Equation (9) is integrated numerically to obtain the radius of the ion space charge sphere.

The electric field produced by the positive space charge inside the ion sphere head at \( x \) is,

\[ E_{SC, i} \text{ [V cm}^{-1}\text{]} = \frac{1.44 \times 10^{-7} n_{e,x} x}{r_{anode}^2} \text{ [V cm}^{-1}\text{]}, \]  

(10)

where \( n_{e,x} \) represents the number of positive ions contained in the spherical head, calculated as follows,

\[ n_{e,x} = \int_0^\alpha (x') dx', \text{ for } \int_0^x \alpha (x') dx' \leq k_{ct}; \]  

\[ n_{e,x} = e^{k_x}, \text{ for } \int_0^x \alpha (x') dx' > k_{ct}. \]  

(11)

The distance \( x = x_0 \) described in section 3.1 is the distance from the tip of the needle at which \( n_{e,x} \) reaches the critical value \( e^{k_x} \). In our numerical calculation, when the total positive charge contained in the spherical volume reaches the saturation value \( (e^{k_x}) \) at \( x = x_0 (< d) \), the total charge inside the ion sphere head is kept fixed whereas its radial expansion continues for \( x > x_0 \) following equation (9) as it advances towards the anode \((x = d)\).

3.3. Positive needle voltage case

The streamer breakdown mechanism changes when a positive voltage is applied to the needle instead of a negative voltage (figure 6(a)). Firstly, the seed electrons in this case are not accelerated towards the plate, instead they are accelerated towards the positively biased needle. Secondly, in contrast to the case with negative needle-voltage, the primary conducting ion channel cannot be originated near the planar cathode for positive needle-voltage. This is because the electric field near the sharp needle is extremely nonuniform, being significantly enhanced closer to the needle, but substantially smaller when away from the needle, such that first Townsend coefficient, \( \alpha \), becomes less than unity beyond a distance \( x_0 \) from the tip of the needle. Therefore, ionization does not take place in the region where \( x > x_0 \) (with \( x = 0 \) being the needle tip location).

For instance, for a gap distance \( d = 1 \text{ cm} \) and an applied positive needle-voltage, \( V_d = 13.4 \text{kV} \), we find that \( \alpha < 1 \) for \( x > 0.0911 \text{ cm} \) (figure 6(c)). As a result, seed electrons are generated through ionization only in the region where \( x < 0.0911 \text{ cm} \). Therefore, in our simple semi-analytical model for positive needle-voltage, the primary conducting ion channel is represented by a spherical volume of space-charge originated at a distance \( x = x_0 \) from the tip of the needle, which propagates towards the anode needle located at \( x = 0 \) (figure 6(a)).

Except these differences, the model and calculation steps for the positive needle-voltage remain exactly the same as those for negative needle-voltage described in sections 3.1 and 3.2.
Figure 4. (a) COMSOL multiphysics electrostatic model of the hollow needle-to-plate geometry for gap distance \( d = 1 \) cm and negative needle-voltage, \( V_d = 19.9 \) kV. Both needle and plate are considered as perfect conductors in the simulation. The color bar represents the spatial profile of the magnitude of the non-uniform electric field along the axial direction, \( |E_x| \). One side of the needle-to-plate geometry is shown here, split along the needle electrode’s central axis. (b) Non-uniform electric field profile along the axial direction at the radial position of the inner radius of the needle \( (r = 0.125) \). (c) The first Townsend ionization coefficient \( \alpha \) profile calculated from equation (2) corresponding to the electric field shown in (b).

Figure 5. (a) Radial diffusion coefficient, \( D_r \), (b) electron drift velocity, \( v_e \), and (c) time elapsed as an electron travels along the gap, with respect to the axial distance \( x \) from the tip of the needle. Here, we use \( d = 1 \) cm and negative needle-voltage \( V_d = 19.9 \) kV.

4. Results and discussion

First, we employ the technique described in section 3 with Meek’s criterion (equation (7)) to determine the breakdown voltage for an applied uniform electric field in a parallel plate-to-plate geometry and compare it with measured conventional values. For \( d = 1 \) cm and \( p = 760 \) Torr, the constant \( k_{cr} \) representing the critical avalanche size for breakdown of air is calculated as \( k_{cr} = 17.55 \) from the empirical formula given by Malik [42]. This value represents a critical avalanche size of \( 4.1867 \times 10^9 \) positive ions. Using these parameters in our calculation for the plate-to-plate geometry for different applied voltages \( V_d \) resulting in different values of \( E_v_{anode} = V_d/d \) and \( r_{anode} \), we find through a process of trial and error that equation (7) is satisfied for \( V_d = 31.5 \) kV. This result is in excellent agreement with the measured conventional value of the breakdown voltage of air at atmospheric pressure with uniform electric field \( (31.5 \) kV) [18].

Figure 7(a) shows the increase of the radius of the space-charge sphere calculated for the applied uniform electric fields with \( V_d = 25 \) kV (dashed blue curve), \( 31.5 \) kV (solid black curve), and \( 40 \) kV (dashed red curve), as it travels from the cathode plate \( (x = 0) \) to the anode plate \( (x = d) \). Figure 7(b) shows the corresponding applied electric fields \( E_x = V_d/d \) (dotted curves) as well as the electric fields produced by the positive space charge inside the spherical volume \( E_{SC} \) (dashed curves) calculated for \( k_{cr} = 17.55 \). From figures 7(b) and (c), we observe that if the applied voltage is small \( (25 \) kV), the space charge field due to the positive ions at the anode is larger than the applied field, \( E_{SC} (x = d) / E_x = 1.24 > 1 \); if the applied voltage is large \( (40 \) kV), the space charge field at the anode is smaller than the applied field, \( E_{SC} (x = d) / E_x = 0.82 < 1 \). When the applied voltage is \( 31.5 \) kV, \( E_{SC} (x = d) = E_x = 31.5 \) kV \( \text{cm}^{-1} \), satisfying Meek’s criterion [2, 18].

Unlike the case with the uniform electric field, the critical avalanche size for breakdown of air is not extensively researched and well documented for non-uniform electric fields. Therefore, we use the experimentally measured breakdown voltages (see below in figure 10) in our numerical calculation and employ Meek’s criterion (equation (7)) to calculate the critical avalanche size for breakdown of air with non-uniform electric fields.

Figure 8(a) shows the increase of the radius of the space-charge sphere calculated from our model, as it travels from the cathode needle \( (x = 0) \) to the anode plate \( (x = d = 1) \) cm when the applied negative voltage to the needle is equal to the measured breakdown voltage, \( V_d = 19.9 \) kV (figure 10). With this applied voltage, we obtain the vacuum electric field at the
Figure 6. (a) Schematic of the semi-analytical streamer breakdown model for positive needle-voltage. (b) Non-uniform electric field profile obtained from the electrostatic COMSOL simulation for \(d = 1\) cm and applied positive needle-voltage, \(V_d = 13.4\) kV. (c) The first Townsend ionization coefficient (\(\alpha\)) profile corresponding to the electric field shown in (b).

Figure 7. (a) The radius (\(r_x\)) of the spherical volume of space-charge traveling across the gap distance vs the distance traveled (\(x\)) in a parallel plate-to-plate geometry for \(V_d = 25\) kV (dashed blue curve), 31.5 kV (solid black curve), and 40 kV (dashed red curve). (b) The external applied uniform electric fields, \(E_x\) (dotted curves), and the electric fields produced by the positive space charge inside the spherical volume, \(E_{SC}\) (dashed curves), vs the distance between electrodes, for \(V_d = 25\) kV (blue curves), 31.5 kV (black curves), and 40 kV (red curves). (c) Zoomed in view of plot (b) near the anode (i.e. \(0.9 \text{ cm} < x < 1 \text{ cm}\)). For all cases, we use \(d = 1\) cm and \(k_{cr} = 17.55\).

planar anode \(E_{x,\text{anode}} = 3.7\text{ kV cm}^{-1}\) and \(r_{\text{anode}} = 0.0154\text{ cm}\) at the anode plate from figures 4(b) and 8(a), respectively. Using these values in equation (7), we obtain \(k_{cr} = 15.65\), corresponding to a critical avalanche size of \(6.26 \times 10^6\) positive ions within the sphere head. Figure 8(b) shows the corresponding applied electric field \(E_x\) (blue dotted curve) as well as the electric field produced by the positive space charge inside the spherical volume \(E_{SC}\) (black solid curve). We observe from figure 8(b) that at the anode (\(x = 1\) cm), \(E_{SC} = E_x = 3.7\text{ kV cm}^{-1}\), as expected from Meek’s criterion.

On the other hand, when the measured positive breakdown voltage \(V_d = 13.4\) kV (figure 10) is applied to the needle, the space-charge sphere is originated at a distance \(x_c = 0.0911\) cm from the tip of the needle (figure 8(c)). As the sphere advances towards the anode needle, its radius increases rapidly (figure 8(c)). However, due to very low values of \(\alpha\) near \(x = x_c\) (\(\alpha \sim 1\) as shown in figure 6(c)), the positive space-charge produced through ionization inside the sphere increases slowly at the beginning, resulting in an initial decrease of \(E_{SC}\) (figure 8(d)). As the sphere head approaches closer to the anode, it experiences increasingly larger values of \(\alpha\) (figure 6(c)), leading to a rapid increase of \(E_{SC}\). Finally, we observe from figure 8(d) that at the anode (\(x = 0\)), \(E_{SC} = E_x = 1.71 \times 10^6\text{ kV cm}^{-1}\), as expected from Meek’s criterion.
Figure 8. Top row: for negative needle-voltage, (a) the radius ($r_x$) of the spherical volume of space-charge traveling across the gap distance ($x$), and (b) the external applied non-uniform electric field, $E_x$ (blue dotted curve), and the electric field produced by the positive space charge inside the spherical volume, $E_{SC}$ (black solid curve), vs the distance from the cathode. Here, we use $d = 1 \text{ cm}$ and $V_d = 19.9 \text{ KV}$, and $k_c = 15.65$. Bottom row: (c), (d) the same plots as (a), (b) for positive needle-voltage. Here, we use $d = 1 \text{ cm}$ and $V_d = 13.4 \text{ KV}$, and $k_c = 18.83$. Note that in (c), (d) the space-charge sphere is originated at a distance $x = x_c = 0.0911 \text{ cm}$ from the tip of the needle ($x = 0$) as described in section 3.3. Also note that the direction of the formation of the primary conducting ion channel for the figures in the top row and the bottom row are opposite to each other for negative and positive needle voltages.

With positive needle-voltage $V_d = 13.4 \text{ kV}$, we obtain $r_{anode} = 0.0035 \text{ cm}$ from figure 8(c) and the corresponding critical avalanche size is calculated from equation (7) as $k_c = 18.81$ or $1.50 \times 10^8$ positive ions.

Figure 9 shows the critical values of several important parameters corresponding to the measured breakdown voltages for different gap distances (shown below in figure 10). We observe from figures 9(a) and (d) that the critical radius of the spherical volume of space-charge at the anode plate ($r_{anode,cr}$) increases as the gap distance ($d$) increases. However, for negative needle-voltage, the sphere experiences a greater radial diffusion over its longer traveling distance compared to that for positive needle-voltage. Therefore, the critical radii for negative needle-voltage (figure 9(a)) for different gap distances are greater than those for positive needle-voltage (figure 9(d)).

Previous studies [41, 42] have shown that the critical avalanche size ($k_c$) for breakdown of air is a function of $pd$ for uniform applied electric field. From figures 9(b) and (e), we observe a similar relation for applied non-uniform electric field in the needle-to-plate setup as well. We can express the values of $k_c$ for negative and positive needle-voltages ($k_{c,neg}$ and $k_{c,pos}$, respectively) observed from figures 9(b) and (e), respectively, as functions of $pd$ by the following approximate expressions:

$$k_{c,neg} = 12.82 + 0.3787 \ln (pd [\text{Torr cm}]); \quad 76 \leq pd [\text{Torr cm}] \leq 380$$

$$k_{c,neg} = 10.43 + 0.7871 \ln (pd [\text{Torr cm}]); \quad 380 < pd [\text{Torr cm}] \leq 1064 \quad (12)$$

$$k_{c,pos} = 14.58 + 0.6107 \ln (pd [\text{Torr cm}]); \quad 76 \leq pd [\text{Torr cm}] \leq 380$$

$$k_{c,pos} = 11.99 + 1.0310 \ln (pd [\text{Torr cm}]); \quad 380 < pd [\text{Torr cm}] \leq 1064. \quad (13)$$
Figure 9. Top row: For negative needle-voltage, at breakdown, (a) the critical radius, \( r_{anode, cr} \), of the spherical volume of space-charge at the anode plate, (b) the constant representing the critical avalanche size for breakdown, \( k_{cr, neg} \), and (c) the critical space charge density at the anode calculated from \( \rho_{anode, cr} = \frac{Q_{x0}}{\frac{4}{3} \pi r_{anode}^3} \), vs gap distance, \( d \). Bottom row: (d)–(f) the same plots as (a)–(c) for positive needle-voltage. For all calculations, we use \( p = 760 \) Torr.

The fitted curves corresponding to equations (12) and (13) (solid blue curves) are shown in figures 9(b) and (e), respectively, along with the values of \( k_c \) obtained from numerical calculations (blue square points). We observe from figures 9(c) and (f) that the critical space charge density at the anode (\( \rho_{anode, cr} \)) is on the order of \( 10^{12} \) ions cm\(^{-3} \) and \( 10^{14} \) ions cm\(^{-3} \) for negative and positive needle-voltage, respectively. It is notable that for negative needle-voltage, the ratio \( e^{k_{cr, neg}}/\rho_{anode}^3 \) and therefore the critical space-charge density at the anode (\( \rho_{anode, cr} \)) decreases with increasing gap distance (figure 9(c)). However, for positive needle-voltage, the ratio \( e^{k_{cr, pos}}/\rho_{anode}^3 \) and therefore the critical space-charge density at the anode (\( \rho_{anode, cr} \)) increase with the increase of the gap distance (figure 9(f)).

Figure 10 shows the comparison of the measured breakdown voltages (red circular points for negative needle voltage and blue circular points for positive needle voltage) with error bars along with the predictions of the classical Townsend theory (black triangular points) and the numerical calculations from our model (green diamond-shaped points for negative needle voltage and black diamond-shaped points for positive needle voltage). We observe that for \( pd = 760 \) Torr \( \times 0.1 \) cm (the left-most data points of figure 10), the predicted values of the breakdown voltage from the Townsend theory are in good agreement with the measured breakdown voltages. However, for \( pd \geq 760 \) Torr \( \times 0.2 \) cm the breakdown mechanism transitions from Townsend breakdown to non-uniform electric field induced streamer breakdown [2, 4]. Therefore, we observe from figure 10 that in this regime, the Townsend theory significantly underestimates the breakdown voltage. On the other hand, predictions from our semi-analytical model based on Meek’s criterion are in excellent agreement with the measured breakdown voltages for both negative and positive needle voltages.
In figure 10, we observe the manifestation of the well-known polarity effect [35] for \( 380 < pd \lesssim 1064 \) Torr cm \((0.5 < d \lesssim 1.4 \) cm). In this regime, for the positive needle voltage, as the primary conducting ion channel is formed toward the tip of the needle, seed electrons converge in the radial direction. Therefore, Meek’s criterion, i.e. the development of a space charge field of the same order as the external applied field about the positive space charge in this converging seed electron avalanche, can be attained with a lower applied voltage. On the other hand, for the negative needle voltage, as the primary conducting ion channel propagates toward the planar anode, seed electrons diverge in the radial direction. Therefore, a larger applied voltage is required for satisfying Meek’s criterion.

In figure 10, we also observe the inverted polarity effect [36, 38, 39] in the regime \( pd < 380 \) Torr cm \((d < 0.5 \) cm). This effect was previously observed for subnanosecond pulsed breakdown of SF\(_6\) in the regime 487.54 \( \leq pd \) [Torr cm] < 975 [38] and of ambient air in the regime 126.75 \( < pd \) [Torr cm] \( \leq 2925 \) [36]. Shao et al showed (figure 4 of [38]) that for positive needle voltage, in the low \( pd \) regime, when the ionization wavefront arrives at the flat cathode, the explosive emission centers are not formed easily there and as a result efficient electron emission from the cathode is retarded. Therefore, in this regime, a higher applied voltage is required for breakdown with a positively biased needle compared to that with a negatively biased needle.

From a quantitative perspective, equation (7) shows that the breakdown voltage depends on the critical avalanche size \( (e^{\kappa r}) \) and the radius of the space charge sphere at the anode \( (r_{\text{anode}}) \). Therefore, in our semi-analytic model, the effect of discharge polarity is accounted for through the values of \( r_{\text{anode}} \) and \( e^{\kappa r} \) calculated from equations (9), (12) and (13), respectively. We note that although the breakdown mode transition from the inverted polarity effect to the polarity effect was previously recorded for subnanosecond pulsed discharge in SF\(_6\) (i.e. figure 7 of [38]), to our knowledge, our current study offers the first observation of such mode transition for ambient air under dc discharge condition.

5. Conclusion

In summary, breakdown voltage of atmospheric air in a non-uniform dc electric field in a needle-to-plate setup was measured as the gap distance varies from 0.1 to 1.4 cm. It was found that the classical Townsend’s theory fails to accurately predict the breakdown voltage for \( pd \geq 760 \) Torr \( \times 0.2 \) cm. A semi-analytic model was developed based on Meek’s criterion for streamer breakdown. By empirically estimating the critical avalanche size required to satisfy Meek’s criterion for streamer breakdown, our semi-analytic model was able to accurately predict breakdown voltages for the hollow needle-to-plate electrode configuration. It was also demonstrated that the discharge polarity in a non-uniform electric field plays an important role in streamer breakdown. For gap distance, \( pd > 360 \) Torr cm, breakdown voltage was measured to be lower with a positive needle compared to that with a negative needle due to the polarity effect. For \( pd < 360 \) Torr cm, the inverted polarity effect was observed which is attributed to the impeded formation of the explosive emission centers on a flat cathode after the arrival of the ionization wavefront.

It is important to note that although our semi-analytic model successfully explains the trend of the breakdown curves due to different discharge polarities and accurately predicts the corresponding breakdown voltages, the flexibility of this early model is limited due to a number of simplifying assumptions. The model does not yet explicitly account for electron inertia, electron emission physics, humidity, gas pressure, etc. Instead, these effects are assumed to be included in the empirical expressions of the critical avalanche size, electron drift velocity, and radial diffusion coefficient.

Future works include improving the proposed simple semi-analytic model by incorporating the effects of different emission mechanisms, electron inertia, different gas species, humidity, and gas pressure, etc. In addition, the critical avalanche sizes for breakdown \((k_{\text{r, pos}} \) and \( k_{\text{r, neg}}\) could be experimentally determined to validate or amend the proposed semi-analytic model. The role of discharge polarity on fundamental breakdown diagnostics such as the formative time lag and the partial discharge inception voltage (PDIV) [50–52] during streamer formation remains an open question. Exploring varying gap distances together with different needle-morphology under different voltage polarity [53] might be of great interest from a breakdown voltage control perspective. Detailed numerical simulations, such as using fluid calculations or PIC methods, are needed to verify the proposed theory. Studies with applied pulsed voltages of different time scales (e.g. changing from millisecond to nanosecond pulses) might elucidate important breakdown physics.

It is important to note from figure 10 that the effect of discharge polarity, i.e. the convergence or divergence of seed electrons in the radial direction, is not prominent for small gap distances \((d \leq 7 \) mm). However, for \( d > 7 \) mm, this effect becomes quite significant. In this regime, the breakdown voltage for a negative needle becomes higher than that for a positive needle as discussed above, and the difference between the breakdown voltages for different discharge polarities increases as the gap distance increases (figure 10). Further studies may examine the effects of fields in radial direction near the needle tip (ignored in this work) to better understand the physics of polarity effects for streamer breakdown.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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