Chapter 7: Shafts

When a man has a vision, he cannot obtain the power from that vision until he has performed it on the Earth for the people to see.

Black Elk

7-1 Introduction

- A shaft is a rotating member, usually of circular cross section, used to transmit power or motion.
- It provides the axis of rotation, or oscillation, of elements such as gears, pulleys, flywheels, cranks, sprockets.
- Topics
  - Material selection
  - Geometric layout
  - Stress and strength
  - Deflection and rigidity
  - Vibration

7-2 Shaft materials

- Low carbon, cold-drawn or hot-rolled steels.
- High alloy content and strengthening from heat treatment are not recommended as it reduces fatigue strength.
- Started to examine the low-cost low- & medium carbon steels
- Alloy steels – ANSI 1340-50, 3140-50, 4140, 4340, 5140 and 8650.
- Surface harden only at the location where the bearings sit. – ANSI 1020, 4320, 4820 & 8620.
- Cold drawn steels for diameter less than 3 inches
- Hot rolled steel should be machined all over.
- Cast iron for high production
- Stainless steels for certain applications.

7-3 Shaft layout

- Torque transmission - Keys, Splines, Setscrews, Pins, Press or shrink fits and Taper fits
Assembly and Disassembly

Techniques for reducing stress concentration

Shaft Design Equations

\[
\sigma_n = K_f \frac{M_c}{I} = K_r \frac{32M_n}{\pi d^2}; \quad \tau_n = K_f \frac{T_c}{I} = K_r \frac{32T_n}{\pi d^2}
\]

\[
\sigma_n = \sqrt{\sigma_n^2 + 3\tau_n^2} = \left( \frac{32K_r M_n}{\pi d^2} \right)^2 + \left( \frac{16K_r T_n}{\pi d^2} \right)^2^{1/2}
\]

\[
\sigma_n = \sqrt{\sigma_n^2 + 3\tau_n^2} = \left( \frac{32K_r M_n}{\pi d^2} \right)^2 + \left( \frac{16K_r T_n}{\pi d^2} \right)^2^{1/2}
\]

DE - Goodman

\[
\frac{1}{n} = \frac{\sigma_n}{S_y} + \frac{\sigma_n}{S_y}
\]

\[
d = \frac{16n}{\pi} \left[ \frac{1}{S_y} \left[ 4K_r M_n \right]^2 + \frac{1}{K_r T_n} \left[ 3K_r T_n \right] \right]^{1/2}
\]

7-4 Shaft Design for Stress

Table 7-1

<table>
<thead>
<tr>
<th></th>
<th>Bending</th>
<th>Torsional</th>
<th>Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoulder fillet—sharp (r/d = 0.02)</td>
<td>2.7</td>
<td>2.2</td>
<td>3.0</td>
</tr>
<tr>
<td>Shoulder fillet—well rounded (r/d = 0.1)</td>
<td>1.7</td>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td>End-mill keyseat (r/d = 0.02)</td>
<td>2.2</td>
<td>3.0</td>
<td>—</td>
</tr>
<tr>
<td>Sleeve runner keyseat</td>
<td>1.7</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Retaining ring groove</td>
<td>3.0</td>
<td>3.0</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Note: Values in the table are not exactly calculable.
Example 7-2

Estimate $K_a = 1.7$ and $K_{ae} = 1.5$ from Table 7-1

Choose 1020 Steel with $S_{ue} = 68$ ksi

$S_e = k_s k_d 0.5 S_{ue} = (0.883)(0.9)(0.5)(68) = 27$ ksi

$k_s = 0.9(\text{assumed})$

$k_d = k_j = k_a = 1$

$d = \left( \frac{16n}{\pi} \left[ \frac{1}{S_e} \left[ \left( 4K_a M_2 \right)^{1/3} + \frac{1}{S_{ue}} \left( 4K_a M_2 \right)^{1/3} \right] \right] \right)^{1/3}$

$= \left( \frac{16(1.5)}{\pi} \left[ \frac{1}{27000} \left[ (4(1.7)(3651))^{1/3} + \frac{1}{68000} \left( 1.5(3240) \right)^{1/3} \right] \right] \right)^{1/3}$

$= 1.625 in \times 1.625 in$

At the shoulder $D/d = 1.2; \quad D = 1.2(1.625)$

$r/d = 0.1; \quad r = 0.16 in$

$K_a = 1.6(\text{Fig.A} - 15 - 9), q = 0.82(\text{Fig.6 - 20})$

$K_j = 1 + 0.82(1.6 - 1) = 1.49$

$K_a = 1.35(\text{Fig.A} - 15 - 8), q_s = 0.95(\text{Fig.6 - 21})$

$K_j = 1 + 0.95(1.35 - 1) = 1.33$

7.5 Deflection Consideration

Determining a new diameter

$d_{new} = d_{old} \frac{n_f}{\sqrt{\frac{y_{ad}^{6/4}}{y_{ad}}}}$

Angular deflection

$\theta = \sum \theta_i = \sum \frac{T_i l_i}{G_i J_i}$

Torsional Stiffness

$\frac{1}{k} = \sum \frac{1}{k_i}$

7-7 Miscellaneous Shaft Components

- Setscrews

Holding power – the resistance to axial motion of the collar relative to the shaft

Table 7-4

<table>
<thead>
<tr>
<th>Setscrew</th>
<th>Head Type</th>
<th>Nut Size</th>
<th>Set Screw Size</th>
<th>Holding Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1/4</td>
<td>6</td>
<td>0.301</td>
<td>25000</td>
</tr>
<tr>
<td>B</td>
<td>1/8</td>
<td>60</td>
<td>0.301</td>
<td>25000</td>
</tr>
<tr>
<td>C</td>
<td>1/8</td>
<td>70</td>
<td>0.301</td>
<td>25000</td>
</tr>
<tr>
<td>D</td>
<td>1/8</td>
<td>100</td>
<td>0.301</td>
<td>25000</td>
</tr>
<tr>
<td>E</td>
<td>1/8</td>
<td>120</td>
<td>0.301</td>
<td>25000</td>
</tr>
<tr>
<td>F</td>
<td>1/8</td>
<td>140</td>
<td>0.301</td>
<td>25000</td>
</tr>
</tbody>
</table>

Socket Setscrews (a) flat point (b) cup point (c) oval point (d) cone point (e) half-dog point
### Keys and Pins

(a) Square key

(b) Round key

(c) Round pins

(d) Round pins

(e) Taper pins

(f) Split tubular pin

### Woodruff key

### Gib-head key

### Retaining ring

- External ring
- Internal ring

### 7-8 Limits and Fits

<table>
<thead>
<tr>
<th>Type of Fit</th>
<th>Description</th>
<th>Symbol</th>
</tr>
</thead>
</table>
| Clearance   | Interference fits | $D_{\text{max}} = D + \Delta D$  
$D_{\text{min}} = D$ |
| See Tables A-11 and A-13 |

For the hole

$d_{\text{max}} = d + \delta_e$

$d_{\text{min}} = d + \delta_e - \Delta d$

For shafts with clearance fits c, d, f, g and h

Interference fits

$p = \frac{d}{2d} \left[ \frac{d^2 + d_1^2}{E_1} + \frac{d^2 + d^2}{E_2} \right] - \frac{d}{d_1}$

$\delta = d_{\text{sh}} - d_{\text{sw}}$

$\delta_{\text{sw}} = d_{\text{sh}} - D_{\text{sw}}$

$\delta_{\text{sh}} = d_{\text{sw}} - D_{\text{sh}}$