Brittle Materials

Modified I Mohr

\[ \sigma_x = \frac{S_x}{n} \quad \sigma_y = \sigma_z = 0 \]
\[ \sigma_x = 0 \quad \sigma_y = \sigma_z \quad \text{and} \quad \frac{\sigma_x - \sigma_y}{\sigma_x} = 0 \]
\[ \frac{(S_x - S_y)}{S_x - S_z} \cdot \sigma_x = \frac{1}{n} \quad \sigma_y = 0 \quad \sigma_z \quad \text{and} \quad \frac{\sigma_x}{n} \quad \frac{\sigma_y}{n} \quad \frac{\sigma_z}{n} = 1 \]
\[ \sigma_x = -\frac{S_x}{n} \quad 0 \leq \sigma_x \leq \sigma_z \]

5-11 Selection of Failure Criteria

Problem 5-15 & 16

Extra Part of Problem 5-15

Determine the required yield strength of a material after considering the stress concentration factors.

\[ D/d = 1.33; \quad r/d = 0.133 \]
\[ \sigma'_x = K_x = 27.52 = 1.5 \times 27.52 = 41.28 \text{ksi} \]
\[ \tau_{xc} = K_y = 9.175 = 1.25 \times 9.175 = 11.47 \text{ksi} \]

Tresca: \[ C = \frac{41.28}{2} = 20.64; \quad S_{\text{tressa}} = 2 \times R = 2 \times \sqrt{20.64^2 + 11.47^2} = 47.23 \text{ksi} \]

Von Mises: \[ S_{\text{von Mises}} = \sqrt{41.28^2 + 3(11.47)^2} = 45.94 \text{ksi} \]
Problem 5-5

Determine the required yield strength of a material after considering the stress concentration factors.

\[
\frac{D}{d} = 1.1; \quad \frac{r}{d} = 0.05
\]

\[
\sigma_y = K_c, 27.52 + K_c, 0.095 = 1.85 \times 18.1 + 1.87 \times 0.095 = 33.66 \text{ksi}
\]

\[
\tau_{sc} = K_c, 9.175 = 1.25 \times 3.055 = 3.82 \text{ksi}
\]

Tresca: \( C = \frac{33.66}{2} = 16.83 \), \( S_c^T = 2 \times R = 2 \times \sqrt{16.83^2 + 3.82^2} = 34.555 \text{ksi} \)

von Mises: \( S_v^m = \sqrt{33.66^2 + 3(3.82)^2} = 34.3 \text{ksi} \)

---

5-12 Introduction of Fracture Mechanics

Rectangular Plate with Hole

\[
K_c = \frac{\text{actual maximum stress}}{\text{average stress}} \quad \frac{K_c}{ \frac{a}{h} }
\]

Rectangular plate with hole subjected to axial load. (a) Plate with cross-sectional plane; (b) one-half of plate with stress distribution; (c) plate with elliptical hole subjected to axial load.
Three Modes of Fracture

(a) Mode I, opening; (b) mode II, sliding; (c) Mode III, tearing.

Three Modes of Fracture

Infinite Plate with Crack

\[ \sigma_x = \frac{K_i}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[ \sigma_y = \frac{K_i}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left( 1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right) \]

\[ \tau_y = \frac{K_i}{\sqrt{2\pi r}} \sin \frac{\theta}{2} \cos \frac{\theta}{2} \cos \frac{3\theta}{2} \]

Crack Propagate when

\[ \sigma_z = \begin{cases} 
0 & \text{if } \sigma_z < \frac{\beta \sigma_y}{\sqrt{2}} \\
\sqrt{\sigma_x^2 + \sigma_y^2} & \text{if } \sigma_z \geq \frac{\beta \sigma_y}{\sqrt{2}} 
\end{cases} \]

Fracture Toughness

\[ K_I = \beta \sigma \sqrt{\pi a} \geq K_{IC} \]

Yield stress and fracture toughness data for selected engineering materials at room temperature.

Other geometries

\[ K_J = \beta \sigma \sqrt{\pi a} \]

Factor of Safety

\[ n = \frac{K_{IC}}{K_J} \]
Problem 5-43

5-43 Given: $a = 12.5$ in, $K_0 = 80$ MPa, $S_a = 1200$ MPa, $S_b = 1500$ MPa

$$r_i = \frac{125}{2} = 62.5 \text{ in}, \quad r_o = \frac{125 + 62.5}{2} = 93.75 \text{ in}$$

$$a(r_o - r_i) = 12.5 \sim 75\%$$

$$r_i/r_o = \frac{62.5}{93.75} = 0.667$$

Fig. 5-36

Eq. (5-37): $Q = 2.5$

$$K_u = \frac{\beta 4}{r_o} \sqrt{\frac{2}{S_a}}$$

$$80 = 2.5 \times \frac{r_o}{\sqrt{S_a}}$$

$$a = 100.5 \text{ MPa}$$

Eq. (5-50) at $x = r_i$:

$$\sigma_a = \frac{r^2 P_o}{r^2 - r_i^2} \left(1 + \frac{r^2}{r_o^2} \right)$$

$$\sigma_i = \frac{r_i^2 P_o}{r^2 - r_i^2} \left(1 + \frac{r^2}{r_o^2} \right)$$

$$\sigma_o = \frac{r_o^2 P_o}{r^2 - r_o^2} \left(1 + \frac{r^2}{r_o^2} \right)$$

$$\sigma_o = \frac{159.1(2)}{159.1 - 159.1} \left(1 + \frac{159.1}{159.1} \right)$$

$$\sigma_i = 159.1(2)$$

$$p_0 = 159.1 \text{ MPa}$$

$$p_0 = 159.1 \text{ MPa}$$