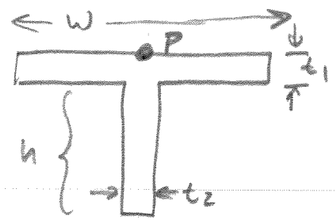
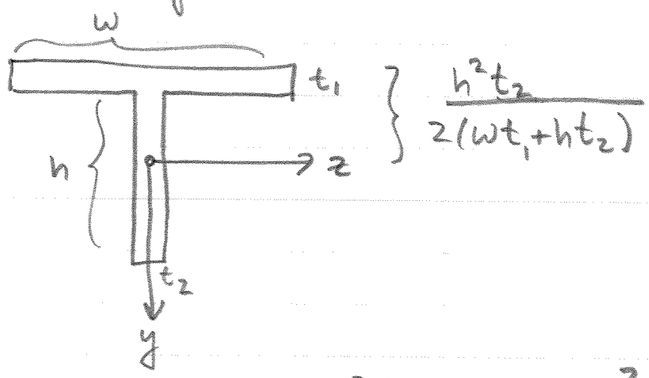
ME 423 Exam #8 Solution



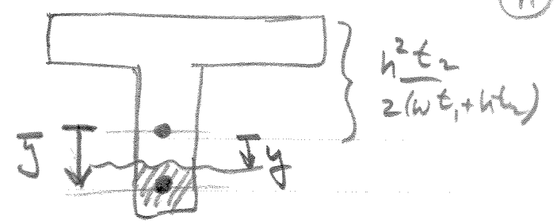
Centroid measured from point P $\quad \bar{y}_{\text{from P}} = \dfrac{\frac{1}{2}t_1 w t_1 + (t_1 + \frac{1}{2}h)t_2 h}{w t_1 + t_2 h} \approx \dfrac{h^2 t_2}{2(w t_1 + h t_2)}$

So locate origin as below



$$I = \tfrac{1}{12} w t_1^3 + w t_1 \left(\frac{h^2 t_2}{2(w t_1 + h t_2)} - \frac{t_1}{2} \right)^2$$
$$+ \tfrac{1}{12} t_2 h^3 + h t_2 \left(\frac{h^2 t_2}{2(w t_1 + h t_2)} - t_1 - \tfrac{1}{2}h \right)^2$$
$$\approx 0 + \frac{w h^4 t_1 t_2^2}{4(w t_1 + h t_2)^2} + \tfrac{1}{12} t_2 h^3 + h t_2 \left(\frac{h^2 t_2}{2(w t_1 + h t_2)} - \tfrac{1}{2}h \right)^2$$
$$= \frac{h^3 t_2}{12(w t_1 + h t_2)} (4 w t_1 + h t_2)$$

① $\tau_{xy} = \dfrac{QV}{Ib} = \dfrac{QV}{It_2}$



$Q = \bar{y} A^*$

$$A^* = t_2 \left(h - \left(\frac{h^2 t_2}{2(w t_1 + h t_2)} - t_1 \right) - y \right) \approx t_2 \left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} - y \right)$$

$$\bar{y} = y + \tfrac{1}{2} \left(h - \left(\frac{h^2 t_2}{2(w t_1 + h t_2)} - t_1 \right) - y \right) \approx \tfrac{1}{2} \left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} + y \right)$$

$$\Rightarrow \quad Q \approx \tfrac{1}{2} t_2 \left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} - y \right) \left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} + y \right)$$
$$= \tfrac{1}{2} t_2 \left(\left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} \right)^2 - y^2 \right)$$

$$\Rightarrow \quad \boxed{\tau_{xy} \approx \frac{V}{2I} \left(\left(h - \frac{h^2 t_2}{2(w t_1 + h t_2)} \right)^2 - y^2 \right)}$$

At bottom $\quad y = h - \dfrac{h^2 t_2}{2(w t_1 + h t_2)} \Rightarrow \tau_{xy} = 0$

At top $\quad y = \dfrac{h^2 t_2}{2(w t_1 + h t_2)} \Rightarrow \tau_{xy} = \dfrac{V}{2I} \left(\dfrac{w h^2 t_1}{w t_1 + h t_2} \right)$

