Web Homework

(1) on $\theta = \frac{\pi}{2}$ $\quad \sigma_\theta = \frac{P}{2A}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\right]$

so $\sigma_\theta = \frac{2P}{A} \Rightarrow 2 = \frac{1}{2}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\right]$

$\Rightarrow 3\frac{a^4}{r^4}+\frac{a^2}{r^2}-2=0$ quadratic in $\frac{a^2}{r^2} \Rightarrow \frac{a^2}{r^2} = -1 \text{ or } 2/3$ ✓ X

$\Rightarrow r = \sqrt{3/2}\, a = 1.22a$

(2) on $r=a$ $\quad \sigma_\theta = \frac{P}{A}(1-2\cos 2\theta)$

so $\sigma_\theta = \frac{2P}{A} \Rightarrow (1-2\cos 2\theta)=2 \Rightarrow \cos 2\theta = -1/2$

$\Rightarrow \theta = 60^\circ$ and also $120^\circ, -60^\circ, -120^\circ$

(3)

$10 = h = \frac{P/A\,(1+2b/a)}{P/A} \Rightarrow \frac{b}{a} = 4.5$

SOLUTION (3.24)

Assume a stress distribution:

$$\tau_{xy} = \tau_{yz} = \tau_{xz} = \sigma_y = \sigma_z = 0 \qquad \sigma_x = \text{constant} \tag{a}$$

which satisfy Eqs. (3.6) and (3.24). Then, Hooke's law becomes

$$\varepsilon_x = \tfrac{\sigma_x}{E} + \alpha T, \qquad \varepsilon_y = \varepsilon_z = -\tfrac{\nu\sigma_x}{E} + \alpha T \tag{b,c}$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0 \tag{d}$$

Due to constraint imposed by the walls and because of the uniformity of the temperature distribution: we take $\varepsilon_x = 0$. Equations (a) and (b) give

$$\sigma_x = -\alpha E T$$

Equation (c) is then

$$\varepsilon_y = \varepsilon_z = \alpha T(1+\nu) = \text{constant}$$

The compressive force P on the tube is

$$P_x = \sigma_x A = -E\alpha A t$$

Substituting the data given

$$P_x = -120(10^9)(16.8\times10^{-6})\times(800\times10^{-6})(100)$$
$$= -161.3\ kN$$

SOLUTION (3.31)

NOTE: (In Probs. 3.31, 3.32, and 3.33), the P, L, and α are constants. It can readily be verified that, the maximum values of the functions in parentheses occur:

$\frac{d}{d\theta}(\sin\theta\cos^3\theta)=0$, or $\tan^2\theta = \frac{1}{3}$ when $\theta = \pm 30^\circ$

$\frac{d}{d\theta}(\sin^2\theta\cos^2\theta)=0$, or $\tan^2\theta = 1$ when $\theta = \pm 45^\circ$

Maximum stresses, using Eqs. (3.34) and (3.40):

$$(\sigma_x)_{elast.} = \tfrac{P}{L(\alpha+\frac{1}{2}\sin 2\alpha)} \tag{a}$$

$$(\tau_{xy})_{elast.} = \tfrac{P\sin\theta\cos^3\theta}{L(\alpha+\frac{1}{2}\sin 2\alpha)}$$

Elementary solution of maximum stresses are (b)

$$(\sigma_x)_{elem.} = \tfrac{P}{2L\tan\alpha}, \qquad (\tau_{xy})_{elem.} = 0$$

(a) For $\alpha = 15^\circ$, $(\alpha + \frac{1}{2}\sin 2\alpha = 0.512)$:

$(\sigma_x)_{elast.} = P/0.512L$ at $\theta = 0^\circ$

$(\tau_{xy})_{elast.} = P/2.195L$ at $\theta = 15^\circ$

$(\sigma_x)_{elem.} = P/0.536L$ at any θ

Thus,

$$(\sigma_x)_{elast.} = 1.047(\sigma_x)_{elem.}$$