3.3

(a) Equations (3.6) become

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0 \qquad \frac{\partial \tau_{xy}}{\partial x} = 0$$

Substituting the given stresses, we have

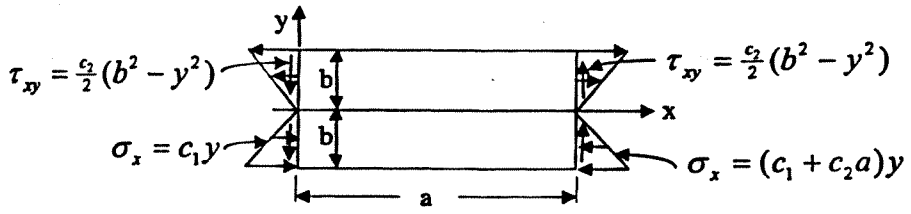
$$c_2 y - 2c_3 y = 0$$

Thus

$$c_2 = 2c_3 \qquad c_1 = \text{arbitrary}$$

(b) $\sigma_x = c_1 y + c_2 xy \qquad \tau_{xy} = \frac{c_2}{2}(b^2 - y^2)$

Assume $c_1 > 0$ and $c_2 > 0$.



3.7

SOLUTION (3.7)

Equations of equilibrium,

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad -2axy + 2axy = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0, \qquad ay^2 - ay^2 = 0$$

are satisfied. Equation (3.12) gives

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = -4ay \neq 0$$

Compatibility is violated; solution is *not* valid.

3.18

SOLUTION (3.18)

Substituting the stresses from Eqs. (3.10) into Eqs. (3.6) and taking $F_x = F_y = 0$:

$$\frac{E}{1-\nu^2}\left(\frac{\partial \varepsilon_x}{\partial x} + \nu \frac{\partial \varepsilon_y}{\partial x}\right) + G\frac{\partial \gamma_{xy}}{\partial y} = 0$$

$$\frac{E}{1-\nu^2}\left(\frac{\partial \varepsilon_y}{\partial y} + \nu \frac{\partial \varepsilon_x}{\partial y}\right) + G\frac{\partial \gamma_{xy}}{\partial x} = 0$$

or

$$\frac{E}{2(1+\nu)}\left[\frac{2}{1-\nu}\frac{\partial^2 u}{\partial x^2} + \frac{2\nu}{1-\nu}\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}\right] = 0$$

$$\frac{E}{2(1+\nu)}\left[\frac{2}{1-\nu}\frac{\partial^2 v}{\partial y^2} + \frac{2\nu}{1-\nu}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2}\right] = 0$$

The foregoing become

$$\frac{\partial^2 u}{\partial x^2} + \frac{1+\nu}{1-\nu}\frac{\partial^2 u}{\partial x^2} + \frac{1+\nu}{1-\nu}\frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{1+\nu}{1-\nu}\frac{\partial^2 v}{\partial y^2} + \frac{1+\nu}{1-\nu}\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial x^2} = 0$$

or

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{1+\nu}{1-\nu}\frac{\partial}{\partial y}\left(\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\right) = 0$$

note misprint in problem statement. Final $\frac{\partial v}{\partial x}$ should be $\frac{\partial u}{\partial x}$