SOLUTION (**2.46**)

(a)

$$U = \frac{1}{2E}\int dx \int \sigma^2 dA = \frac{1}{2E}\int dx \int (\frac{P}{A} + \frac{My}{I})^2 dA$$

Since $\int y dA = 0$, this becomes:

$$U = \int \frac{P^2 dx}{2AE} + \int \frac{M^2 dx}{2EI} = U_a + U_b$$

Here

$$U_b = \frac{1}{2EI}\left[\int_0^a M_{AD}^2 dx + \int_0^b M_{DB}^2 dx\right]$$

$$= \frac{1}{2EI}\left[\int_0^a (-\frac{M_0 x}{L})^2 dx + \int_0^b (\frac{M_0 x'}{L})^2 dx'\right]$$

$$= \frac{M_0^2}{6EIL^2}(a^3 + b^3)$$

Thus,

$$U = \frac{P^2 L}{2AE} + \frac{M_0^2}{6EIL^2}(a^3 + b^3)$$

(b) Substituting the given data:

$$U = \frac{(8\times10^3)^2(1.2)}{2(7.5\times10^{-3})(70\times10^9)} + \frac{(2\times10^3)^2(0.027+0.729)}{6(70\times10^9)(0.075\times0.1^3/12)(1.2)^2}$$

$$= 0.0731 + 0.8 = 0.8731\ N\cdot m$$

SOLUTION (**2.30**)

Using Eq. (2.29),

$$G = \frac{79(10^9)}{2(1+0.3)} = 26.92\ GPa$$

For x=1 m, y=2 m, and z=4 m, we obtain $[\tau_{ij}]$:

$$\begin{bmatrix} 34 & 48 & 8 \\ 48 & 5 & 1 \\ 8 & 1 & 5 \end{bmatrix} MPa$$

Then, Eqs. (2.28) yield $[\varepsilon_{ij}]$:

$$\begin{bmatrix} 443 & 892 & 149 \\ 892 & -96 & 19 \\ 149 & 19 & -96 \end{bmatrix} \mu$$

SOLUTION (**2.43**)

Stress field is described by

$$\sigma_x = \sigma_y = \sigma_z = -p, \qquad \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

(a) Equation (2.31) reduces to

$$e = -\frac{3(1-2\nu)p}{E}; \qquad -0.005 = -\frac{3[1-(2/3)]p}{110(10^9)}$$

or

$$p = 550\ MPa$$

(b) Equation (2.42) becomes

$$U_0 = \frac{1}{2E}(p^2 + p^2 + p^2) - \frac{\nu}{E}(p^2 + P^2 + p^2) = \frac{3p^2(1-2\nu)}{2E}$$

$$= \frac{3(550\times10^6)^2}{2(110\times10^9)}(1 - \frac{2}{3}) = 1.375\ MPa$$

Thus,

$$U = U_0 V_0 = 1.375(10^6)(\frac{4\pi}{3}\times0.15^3)$$

$$= 19.439\ kN\cdot m$$