SOLUTION (2.18)

(a) Applying Eqs. (2.17),

$$J_1 = 200 - 100 - 400 = -300\ \mu$$

$$J_2 = (-2 - 8 + 4 - 9 - 4 - 25)(10^4) = -44(10^4)\ (\mu)^2$$

and

$$J_3 = \begin{vmatrix} 200 & 300 & 200 \\ 300 & -100 & 500 \\ 200 & 500 & -400 \end{vmatrix} = 58(10^6)\ (\mu)^3$$

(b) Table of direction cosines:

	x	y	z
x'	$\sqrt{3}/2$	$1/2$	0
y'	$-1/2$	$\sqrt{3}/2$	0
z'	0	0	1

Thus, using Eqs. (2.15a),

$$\varepsilon_{x'} = \varepsilon_x l_1^2 + \varepsilon_y m_1^2 + \gamma_{xy} l_1 m_1$$

$$= 200\left(\tfrac{\sqrt{3}}{2}\right)^2 - 100\left(\tfrac{1}{2}\right)^2 + 600\left(\tfrac{\sqrt{3}}{2}\right)\left(\tfrac{1}{2}\right) = 385\ \mu$$

(c) Use Table B.1 (with $\sigma \to \varepsilon$ and $\tau \to \gamma/2$):

$$\varepsilon_1 = 598\ \mu \qquad \varepsilon_2 = -126\ \mu \qquad \varepsilon_3 = -772\ \mu$$

(d) $$\gamma_{max} = 598 + 772 = 1370\ \mu$$

SOLUTION (2.13)

We have $u_B = 3\ mm$ and $v_B = 1.5\ mm$.

(a) Take: $u = c_1 xy \qquad v = c_2 xy$

Hence,

$$3(10^{-3}) = c_1(3 \times 2), \qquad c_1 = 500(10^{-6})$$

$$1.5(10^{-3}) = c_2(3 \times 2), \qquad c_2 = 250(10^{-6})$$

Thus,

$$u = 500(10^{-6})xy \qquad v = 250(10^{-6})xy$$

(b) Using Eqs. (2.3),

$$\varepsilon_x = 500\ \mu y \qquad \varepsilon_y = 250\ \mu x \qquad \gamma_{xy} = 250\ \mu(2x + y)$$

which satisify Eq. (2.8): the strain field is possible.

At point B, we thus have

$$(\varepsilon_x)_B = 1000\ \mu \qquad (\varepsilon_y)_B = 750\ \mu \qquad (\gamma_{xy})_B = 2000\ \mu$$

(c) We obtain $\theta = \tan^{-1}\frac{2}{3} = 33.69^\circ$.

Equation (2.11a) is therefore

$$\varepsilon_{x'} = 875 + 125\cos 67.38^\circ + 1000\sin 67.38^\circ = 1846\ \mu$$

SOLUTION (2.25)

Using Eqs. (2.36):

$$\varepsilon_{1,2} = \tfrac{1}{2}\{(-100 + 100) \pm [(-200)^2 + 100^2]^{\frac{1}{2}}\} = \tfrac{1}{2}(0 \pm 224)$$

or

$$\varepsilon_1 = 112\ \mu \qquad \varepsilon_2 = -112\ \mu$$

$$\sigma_{1,2} = \frac{200(10^3)}{2}\left[0 \pm \left(\frac{224}{1.3}\right)\right]$$

or

$$\sigma_1 = 17.2\ MPa \qquad \sigma_2 = -17.2\ MPa$$

and

$$\theta_p = \tfrac{1}{2}\tan^{-1}\frac{100}{-200} = -13.28^\circ$$

Equations (2.34),

$$\varepsilon_x = \varepsilon_a \qquad \varepsilon_y = \varepsilon_c \qquad \gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$$

Equation (2.11a) gives

$$\varepsilon_{x'} = 0 - 100\cos(-26.56^\circ) + 50\sin(-26.56^\circ) = -112\ \mu$$

Thus,

$$\theta_p = -13.28^\circ$$