SOLUTION (**1.46**)

(a) Equation (1.27) becomes

$$\begin{vmatrix} (30-\sigma_p) & 0 & 20 \\ 0 & -\sigma_p & 0 \\ 20 & 0 & -\sigma_p \end{vmatrix} = 0$$

Expanding,

$$-\sigma_p[\sigma_p(\sigma_p - 30) - 400] = 0$$

or

$$\sigma_p = 0, \qquad \sigma_p = -10, \qquad \sigma_p = 40$$

Thus

$$\sigma_1 = 40\ MPa, \qquad \sigma_2 = 0, \qquad \sigma_3 = -10\ MPa$$ ◀

(b) For $\sigma_1 = 40\ MPa$:

$$(30-40)l_1 + (0)m_1 + 20n_1 = 0, \qquad l_1 = 2n_1$$
$$(0)m_1 = 0, \qquad m_1 = 0$$

The condition $l_1^2 + 0 + n_1^2 = 1$ gives

$$(2n_1)^2 + n_1^2 = 1, \qquad n_1 = \tfrac{1}{\sqrt{5}}$$

Thus

$$l_1 = \tfrac{2}{\sqrt{5}}, \qquad m_1 = 0, \qquad n_1 = \tfrac{1}{\sqrt{5}}$$ ◀

SOLUTION (**1.6**)

Use Eqs. (1.11),

$$(2x) + (-2xy) + (x) + F_x = 0$$
$$(-y^2) + (-2yz + x) + (0) + F_y = 0$$
$$(z - 4xy) + (0) + (-2z) + F_z = 0$$

Solving, we have (in MN/m^3):

$$F_x = -3x + 2xy \qquad F_y = -x + y^2 + 2xz \qquad F_z = 4xy + z$$ (a) ◀

Substituting x=-0.01 m, y=0.03 m, and z=0.06 m, Eqs. (a) yield the following values

$$F_x = 29.4\ kN/m^3 \qquad F_y = 14.5\ kN/m^3 \qquad F_z = 58.8\ kN/m^3$$

Resultant body force is thus

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = 67.32\ kN/m^3$$ ◀

SOLUTION (**1.58**)

(a)
$$\begin{vmatrix} (-100-\sigma_p) & 0 & -80 \\ 0 & (20-\sigma_p) & 0 \\ -80 & 0 & (20-\sigma_p) \end{vmatrix} = 0$$

Expanding,

$$(20-\sigma)[(\sigma+100)(\sigma-20) - 6400] = 0$$

and

$$\sigma_1 = 60\ MPa, \qquad \sigma_2 = 20\ MPa, \qquad \sigma_3 = -140\ MPa$$ ◀

(b) Apply Eqs. (1.38), (1.39), and (1.40):

$$\tau_{oct} = \tfrac{1}{3}[(60-20)^2 + (20+140)^2 + (-140-60)^2]^{\frac{1}{2}} = 86.41\ MPa$$ ◀
$$\sigma_{oct} = \tfrac{1}{3}(60 + 20 - 140) = -20\ MPa$$
$$\tau_{\max} = \tfrac{1}{2}(60 + 140) = 100\ MPa$$