Most OPP literature associates the PMU installation cost with the cost of the PMU unit. As a result, most of these techniques have been proposed to minimize the number of PMUs while considering the complete observability of the system [3]–[6]. Similarly, an integer linear programming (ILP)-based model was proposed for the OPP with minimum number of PMUs as an objective [7]. In [8], the OPP was solved for the minimum number of PMUs under controlled islanding conditions. Binary particle swarm optimization (BPSO) and exponential PSO were used in [6], [9] to obtain the optimum solution for the OPP problem. In [10], mixed integer linear programing was used to solve the OPP problem as a multi-objective problem. All the approaches mentioned above proposed minimizing the number of PMUs while achieving the desired observability conditions.

Application-based PMU placement, on the other hand, prioritizes buses for the installation of PMUs based on the chosen application. Most of these techniques are multistage approaches where PMU locations are determined in several stages to achieve complete observability of the network. Kumar and Thukaram [11] and Pal et al. [12] have suggested that the critical buses be ranked based on stability criteria, and then the PMUs be incrementally installed, enhancing the observability in the process. The OPP based on state estimation was solved in [13] using semidefinite programing and convex relaxation. In [14], ILP was used to solve the OPP problem and to detect bad data and critical measurements. In [15], the empirical observability Gramian was used to quantify and solve the OPP problem for dynamic state estimation. In [16], [17], ILP was used to enhance the reliability of observability by considering the probability of PMU failure in addition to the transmission line failure, while minimizing the number of PMUs. However, most of the application-based and multistage OPP approaches still use the traditional approach for OPP which assumes that minimizing the number of PMUs results in the lowest cost [11]–[21].

Realizing that the minimum number of PMUs does not guarantee the minimum installation cost, researchers started changing the scope of the OPP problem [22]–[25]. For instance, in [23], dual-use line relays were used to reduce the total cost of the OPP. Mohammadi et al. [24] incorporated the cost of the communication infrastructure into the OPP, where they have proposed to minimize the number of PMUs and the distance between the Phasor Data Concentrator (PDC) and PMUs. Nevertheless, they ignored the substation infrastructure and assumed that the minimum number of PMUs implies minimum cost. On the other hand, Rather et al. [26] highlighted the cost of substation infrastructure, and its effect on the OPP.
In [25], a channel-based OPP approach was used to enhance the voltage stability and measurement reliability with a flat cost per PMU.

Even with the optimization of PMU installation cost, the total investment cost required for complete observability is often quite significant. To address this issue, several multistage approaches were developed, where the PMUs are installed incrementally over time with a predetermined budget for each stage [20], [21], [27]. The predetermined budget can be based on the number of PMUs as in [20], [21] or on actual cost as in [27]. However, multistage PMU installation is likely to result in higher cost for the overall process. While the one-stage PMU installation treats the observability as a constraint, the multistage approach maximizes the observability at each stage, at the expense of the installation cost. In [27], the authors addressed this issue by relocating PMUs when needed after each stage. However, PMU relocation, at a flat rate, from a substation where the infrastructure has already been upgraded and prepared for PMU applications to a new substation, requires upgrading the new substation which can cost up to 95% of the PMU installation expense [28]. Moreover, other factors that affect the cost, in these multistage approaches such as the communication and substation infrastructures, were not considered.

In this paper, a multistage PMU placement strategy is proposed which considers two factors: substation cost (including PMU cost) and communication infrastructures. The prioritization of critical buses can be integrated with the proposed approach. This approach can be used in an incremental way where PMUs are installed in multiple stages under a constrained cost. Unlike most multistage approaches, this approach does not consider a predetermined number of PMUs at each stage. Instead, the multistage installation maximizes the network observability and prioritizes critical buses while remaining within a predetermined budget for each stage.

In OPP, buses having one or more of the following criteria: high voltage buses, high impact on transient stability, or sensitive loads, are considered critical buses [11], [12]. These buses are sometimes given higher priority to enhance system awareness from a stability perspective. Other researchers have proposed prioritizing buses based on different criteria such as reliability and state estimation [15]–[17]. Bus prioritization can be integrated with the proposed approach while considering both observability and the actual cost of the PMU installation. The major contributions of this work may be summarized as follows.

- It develops a comprehensive cost model for the OPP problem, including the cost of the PMUs as well as the infrastructure upgrade costs.
- It presents a flexible, multistage deployment plan, implemented over a period of time depending on the budget of the utility company.
- It affords the ability to prioritize PMU placement based on specific criteria such as bus criticality, thereby enabling application-based deployment.

The OPP is solved using an opposition-based elitist binary genetic algorithm (O-BEBGA) along with multisource Dijkstra algorithm. Multisource Dijkstra algorithm is integrated into the O-BEBGA to optimize the network infrastructure cost. The Dijkstra algorithm is chosen for its efficiency in finding the shortest path compared to other graph algorithms [29]–[32]. This paper is organized as follows. Section II discusses the cost of installing PMUs and the communication infrastructure. The proposed approach is presented in section III. Sections IV and V present the case studies and the conclusion respectively.

II. PMU INSTALLATION COST

This section discusses the cost of PMU installation. It presents the cost model for upgrading a substation and the cost of the communication infrastructure for PMU installation.

A. Substation Infrastructure

Most of the cost is associated with the installation process of the PMU and not the PMUs themselves. In fact, the PMUs cost about 5% of the total installation cost [28]. Most of the cost is spent on upgrading the substation and communication infrastructures. A report recently published by the U.S. Department of Energy (DOE) showed that the PMU installation cost ranges from $40,000 to $180,000 per PMU [28].

The cost varies depending on the infrastructure support for the PMUs. Typically, PMUs need sufficient communication infrastructure to send the measurement data to the PDC. The substation infrastructure also needs to be sufficient to utilize the functionalities of PMUs. Formulating the installation cost for the PMUs is a complicated process. Although PMU installation requires the same infrastructure upgrades, such as communication, cyber-security, and other equipment upgrades, the approach to installing PMUs can differ depending on the utility and existing infrastructure support for the PMUs. For instance, a utility can install new stand-alone PMUs, or upgrade existing digital relays to enable PMU functionality [28]. Moreover, installing PMUs also depends on the availability of CTs and PTs [26]. Once the infrastructure of the substation is in place to support the PMUs, the installation cost can go down to 35% of the initial cost [28].

The proposed cost model of PMU installation in (1) considers the difference between prepared buses, where minimal upgrades are needed, and unprepared buses, by introducing $g_i$ index. The index takes the value of 4.5 for unprepared buses and 1 for prepared buses. The model also includes the cost of adding additional measurement channels by including the cost of PTs and CTs.

$$\text{Cost} = \sum_{i=1}^{N} g_i(aP_i + b_iP_i) + K(P)$$

where

- $N$ number of buses in the system;
- $g_i$ prepared bus index;
- $a$ cost of installing PMU and basic upgrades at the substation;
- $b_i$ cost for installing additional PT or CT at substation $i$; 
- $K(P)$ cost function for the communication infrastructure for PMUs;
- $P = [P_1, P_2, \ldots, P_i, \ldots, P_N]^T$.
where $P_i$ takes the value of zero or one and indicates if a PMU is to be installed at substation $i$.

B. Communication Infrastructure

The measurement data obtained by the PMUs are sent to the PDC, where the PDC sorts the data and processes it for other applications. Mohammadi et al. [24] have proposed to reduce the distance between the PMUs and the PDC to lower the total cost. In [24], it is also have proposed to place the PDC on a non-PMU bus to minimize the total communication distance. The work in [24], however, have not considered the cost of upgrading the substation for PMU installation. In this paper, the PDC is assumed to be installed at one of the substations where a PMU is to be placed. Then, the path connecting all PMUs is minimized to lower the communication infrastructure cost (2).

There can be several communication paths connecting all PMUs at different substations. Consider the 9-bus system in Fig. 1; the PMUs are placed at buses 4 and 7 to make the system observable. However, there are two communication paths to connect both PMUs with the PDC located at bus 7. As seen in Fig. 1, the first communication path is about 32 miles, and the second one is about 24 miles. Therefore, in order to minimize the communication infrastructure cost, the path connecting all PMUs with PDC needs to be minimized. The communication infrastructure is assumed to be passive optical network (PON) with optical ground wire (OPGW). The cost model in (2) is derived from [33], [34].

$$\min K(P) = \sum_{j=1}^{n} \text{len}_{i,j} \cdot cc_{i,j} + N_c \cdot P_j + N_b \cdot P_j + \text{PDC}$$

subject to

$$O = H \times P$$

III. PROPOSED APPROACH

This section presents the approach for the multistage OPP. As discussed in the previous sections, the PMU installation cost plays a critical role in determining the installation process. In the multistage approach, maximizing the benefits of PMU installation takes higher priority over the cost function. This approach assumes that the utility sets a budget for the installation of PMUs and the first objective is to maximize the observability and priority buses while minimizing the installation cost and not exceeding the predetermined budget for the current stage.

A. Problem Statement

As mentioned earlier, the optimal placement for PMUs highly depends on the installation cost and available budget. In the proposed multistage approach, the observability is maximized, subject to the observability constraint described below, and the cost function is minimized. The observability in (3) needs to have enough redundancies for the desired observability conditions. For instance, under normal operating conditions, the observability constraint in (4) must be satisfied.

$$O = H \times P$$

$$O \geq I'$$

where $P$ is a vector of length equal to the number of buses $N$, as described in section II-A; $I'$ is a vector of length $N$ with all its elements equal to 1; and $H$ is an $N \times N$ connectivity matrix. The entries for $P$ and $H$ are defined in (5) and (6) respectively.

$$P_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{if no PMU is at bus } i \end{cases}$$

$$h_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases}$$

In the proposed approach, the observability function in (7) is treated as a higher level objective function, and is subjected to minimizing the cost function. The cost function in (9) is treated as the lower level objective function, subjected to the higher level objective (observability function). This setup allows maximizing the observability while minimizing the cost without violating the budget constraint, thereby reaching the optimal solution for the given budget. It should be noted that during the multistage process complete observability in (4) cannot be achieved; therefore the observability constraint is changed to the multistage condition in (8).

$$\max \sum_{i=1}^{N} O_{i}'$$

subject to

$$O \leq 1 \quad \text{“multistage observability condition”}$$

$$\min \text{Cost} = \sum_{i=1}^{N} g_i(a_i P_i + b_i P_i) + K(P)$$

subject to

$$C \leq C_{\text{budget}}$$

Fig. 1. Sample 9-bus system with possible communication paths
1) Priority Buses: For application-based OPP schemes, some buses are prioritized for PMU installation regardless of their contribution to the overall observability. These schemes range from stability criteria to reliability, and many others [11], [12], [14], [16]. In the proposed OPP scheme, priority buses can be chosen using any criterion.

The priority buses for the network are embedded in the observability function as bias using a priority vector $R$ (10). The $R$ vector has the length of the number of buses $N$. If all buses are treated equally then all elements of $R$ are set to zero. The higher priority buses are determined based on the utility criteria, then arranged in descending order in a vector $L$. Then the priority bias is assigned using the following algorithm.

**Procedure 1 Priority Vector $R$**

Initialize priority buses $(L)$, $R = \text{zeros}_{1 \times N}$

- $N$ = number of buses
- $M$ = maximum number of branches in (H)
- $k_k$ = length of $L$

for $j = 1 : k_k$

- $i = l(j)$
  - if $i \neq 0$ then
    - $r_i = M(k_k - j + 1)$
  - else
    - $r_i = 0$

endfor

Maximizing the modified observability vector $O'$ gives bias to the higher priority buses. However, this vector cannot be used to test the observability of the network since the $R$ vector skews the observability. As a result, the skewed observability in (7) is used for optimizing the OPP, and the original observability in (3) is used as a constraint for observability testing.

$$h_{ij}^{'} = \begin{cases} 1 + r_i, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus}(i) \text{ and bus } (j) \\ 0, & \text{otherwise} \end{cases}$$ (10)

where

- $O' = H' \times P$
- $r_i$ = bus $(i)$ priority index
- $N$ = number of buses.

The cost of the overall PMU installation can be reduced by considering the effect of zero-injection buses (ZIB). Considering the effect of ZIBs improves the overall observability of the system, thereby reducing the number of PMUs needed to achieve the observability constraint. The effect of ZIB can be summarized into two points. If all buses connected to a ZIB are observable, the ZIB is considered observable by applying KCL. Also, an unobservable bus, when connected to an observable ZIB, is considered observable only if all of the other buses connected to the ZIB are observable.

B. Algorithm

The optimal placement problem in subsection III-A is a discontinuous bi-level problem. It also involves optimizing the communication infrastructure cost $K(P)$ within the installation cost in (9). A multisource Dijkstra algorithm is used to obtain the shortest path connecting all PMUs and PDC. Since the proposed model involves optimizing three objective functions (2), (7) and (9), evolutionary algorithms are the appropriate tools for solving such a problem. The proposed algorithm uses an opposition-based elitist binary genetic algorithm (O-BEBGA) to solve the bi-level OPP in subsection III-A. The opposition element is added to the algorithm to enhance the overall performance since opposition-based methods have proven their superiority in terms of convergence speed and results [37]–[39].

1) Multisource Dijkstra: The design of the communication infrastructure involves finding the most cost effective path (2) between the PMUs and PDC. There are several algorithms that can be used to find this path such as Ford-Warshall, Bellman-Ford, Johnson and Dijkstra algorithms. The Dijkstra algorithm is among the most efficient algorithms for single source undirected weighted graphs [29]. However, the communication network design problem is not a single source/destination problem; rather, it is a multisource single destination problem, or a single source/destination with a must pass nodes. The traditional Dijkstra algorithm can still be used to solve this problem. This entails using the Dijkstra algorithm $n$ times to establish one communication line between two source nodes out of $n$ source nodes. The multisource Dijkstra algorithm, on the other hand, can be used to pair up source nodes in one run.

The multisource Dijkstra is used to find the shortest paths $P_{x_1}$ connecting every source node $s_i$ to the nearest source node $s_j$, where $(s_i, s_j) \in S$. This step generates a set $G = \{\phi_1, \phi_2, \ldots, \phi_{n_1}\}$ with $n_1$ subsets, where $n_1 = \text{floor}(N/s)$ and $N$ is the number of source nodes. Each subset $\phi$ has at least two connected source nodes. The next step is to connect the subsets in $G$ to each other. First, the weights for the paths in $P_{z_1}$ are set to zero. The multisource Dijkstra is then used to obtain new paths $P_{x_2}$ that connect the subsets in $G$. It should be noted that the new paths $P_{x_2}$ may have redundant routes, however these redundant routes have zero weight. The process of finding new paths and updating their weights is repeated until all the subsets in $G$ are connected with one path. This path is the union of all paths $P_{x_1}$ obtained from the multisource Dijkstra algorithm.

The sample graph in Fig. 2 demonstrates the implementation of multisource Dijkstra algorithm for the communication network design. The source nodes in $S$ are $a, d, f$ and $j$. The first loop of the multisource Dijkstra generates the $G$ set with subsets $\phi_1$ and $\phi_2$. The $\phi_1$ subset contains the source nodes $a$ and $j$; the $\phi_2$ subset contains source nodes $d$ and $f$. The $P_{x_1}$ paths for the subsets in $G$ are $\{a-k-j, d-f\}$. The next step is to connect the subsets $\phi_1$ and $\phi_2$, which produce the path $P_{x_2} = \{a-b-c-d\}$. The union of the paths $P_{x_1}$ and $P_{x_2}$ produces the shortest path connecting all the source nodes in $S$. 
2) O-BEBGA: As discussed previously, multistage approaches can lead to higher cost for the overall installation of PMUs, since the solutions for each stage are often sub-optimal for the complete observability. This is because maximizing the observability at each stage increases the installation cost [27]. To overcome this issue, the O-BEBGA solves the OPP for the desired observability condition first. Then, the optimal solution for the complete observability ($X_s$) is used as optimal solution for the multistage installation. To enhance the performance further, the search space multistage installation is reduced to include only optimal location of PMUs in $X_s$.

The proposed algorithm solves the optimal placement problem in a parallel manner by initializing random candidates where each candidate $x_i$ has the length of the number of buses $N$, thereby evaluating the candidate buses simultaneously instead of using systematic increments. By maximizing the observability function (higher objective), while minimizing the cost of PMU installation, the predetermined budget $C_{budget}^k$ for each stage ($k$) is optimally utilized.

The higher and lower objectives share the same decision variables, meaning there are no decision variables exclusive to one objective or the other. The proposed approach exploits this advantage to evaluate both objectives simultaneously without using different search spaces for each objective. The proposed approach uses a sorting function to handle the simultaneous evaluation in the same search space. This function sorts all candidates according to their feasibility and fitness of the higher and lower objectives, as seen in Fig. 3. As a result, the algorithm is guided towards the optimal solution where the cost is minimized and the observability is maximized.

The overall flowchart for the proposed algorithm is shown in Fig. 4. The crossover and mutation probabilities are $P_c = 0.7$ and $P_m = 0.3$ respectively. The $O_{cn}$ donates the opposition random variable; the crossover and mutation random variables are denoted by $C_{cn}$ and $M_{cn}$ respectively. Double point crossover is used to generate the offspring population $X_c$, and single point mutation is used to generate the mutated population $X_m$. The algorithm uses dynamic opposition with probability of $P_o = 0.4$.

Fig. 2. Sample graph to demonstrate the implementation of multisource Dijkstra algorithm.

Fig. 3. Sorting function.

Fig. 4. Flow chart of the proposed O-BEBGA algorithm.
There are many variations of opposition techniques in the literature. The two most common are the global opposition and the dynamic opposition. In the proposed algorithm, a modified dynamic opposition is used to generate the opposition population when applicable. Instead of generating the total opposite of the chosen individual \( X_i \), only a third of the variables in \( X_i \) are selected for the opposition process (11).

\[
X_i^{\text{opp}} = X_{\text{min}} + X_{\text{max}} - X_i
\]  

The algorithm is terminated if the conditions in (12) are met. The terms \( \alpha \) and \( \beta \) are constants; where \( \gamma(1) \) indicates how much of the current population is feasible and \( \gamma(2) \) indicates if the population is converging to an optimum. The variance of the cost \( (\text{Var}[C]) \) is used to determine \( \gamma(2) \), where \( k \) is the index for the current population.

\[
\gamma = \begin{cases} 
\gamma(1) = \left| \sigma - \text{max}(O) \right| \leq \alpha \\
\gamma(2) = \text{Var}[C^{k-1}] - \text{Var}[C^k]
\end{cases}
\]  

\[
\sigma = \sum_{i=1}^{n} s_i
\]  

\[
H \times P = \mathcal{O}_p \geq \mathbf{1}
\]  

\[
\mathcal{O}_{s,i} = \begin{cases} 
\sum (a_{ij} \times p_i) \\
2, & \text{if a PMU is installed at bus } i
\end{cases}
\]

where \( \mathcal{O}_s \) is the observability vector for all buses in the system and \( \mathbf{1} \) is a vector of length equal to the number of buses, with all entries equal to 2.

3) Single PMU outage: In a single PMU outage, every bus needs to have two independent measurements, either by two different PMUs or if ZIB effect is considered through KCL and a PMU. Therefore, the observability constraint in (8) is changed to the following:

\[
H \times P = \mathcal{O}_p \geq \mathbf{1}
\]  

\[
\mathcal{O}_{p,i} = \sum (a_{ij} \times p_i)
\]

IV. SIMULATION AND RESULTS

In this section, the proposed approach is tested on the IEEE reliability test system (RTS), IEEE 14-bus, 30-bus and 118-bus test systems. In subsection IV-A all buses are treated equally and no prioritization is given to any bus. The higher priority buses and other observability conditions are tested in subsection IV-B. The buses are divided into two categories: prepared buses and unprepared buses. The prepared buses are assumed to have sufficient infrastructure, require basic security, network upgrades, and cost 75% less than the unprepared buses [28].

The base cost per PMU is assumed to be $40,000, and the cost per additional PT or CT is assumed to be $2,380 [40]. The cost of PDC is assumed to be $7,500. The length of the transmission lines are obtained from [41]. The communication links are assumed to be running along the transmission lines where the cost of the communication links is assumed to be $2,414 per mile [35] or $0 if the communication link already exists.
The proposed algorithm is used with a population size of $3 \times N$, where $N$ is the number of buses. The performance of the O-BEBGA is shown in Fig. 5. Although the algorithm maximizes the observability, the minimization of the PMU installation cost drives the observability to a cost effective solution. It should be noted that maximizing observability often increases the cost. However, there exist cases where the same or better observability can be achieved at a better cost, as is the case for generations 5 and 12 in Fig. 5.

### A. No Priority Buses

The model in section II is used and no priority is given to any bus. The multistage OPP is performed as a two-stage process for the IEEE 14-bus and a three-stage process for the IEEE RTS, the IEEE 30-bus and the IEEE 118-bus. Each stage is treated independently budget-wise, meaning the remainder of the budget from each stage is not added to the next stage budget. Complete observability is achieved for all systems within three stages. It should be noted that the number of stages in which complete observability is achieved depends on the budget specified by the utility.

The OPP is performed in two different cases. In the first case, ZIBs are treated as normal buses. The result of the multistage OPP is shown in Table I. The OPP solution for the IEEE 14-bus is shown in Fig. 6 and Fig. 7. The ZIB effect is considered in the second case as shown in Table II.

The proposed approach is compared with some of the recent approaches in OPP literature, as seen in Table III. These approaches include classical and evolutionary methods, mainly particle swarm optimization (PSO), Cellular Learning Automata (CLA) and binary imperialistic competition algo-

### TABLE I

**MULTISTAGE APPROACH, NOT CONSIDERING EFFECT OF ZIB**

<table>
<thead>
<tr>
<th>System</th>
<th>PMU locations</th>
<th>Stage budget $C_{\text{budget}}$</th>
<th>Cost</th>
<th>Remaining</th>
<th>Unprepared buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Stage ($k = 1$); Effect of ZIB Is Not Considered</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IEEE 14-bus</td>
<td>4, 5, 11</td>
<td>$300,000</td>
<td>$252,189</td>
<td>$47,811</td>
<td>7, 9</td>
</tr>
<tr>
<td>IEEE RTS</td>
<td>15, 16, 21</td>
<td>$350,000</td>
<td>$213,522</td>
<td>$136,478</td>
<td>10, 11, 17, 24</td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>15, 16, 26</td>
<td>$450,000</td>
<td>$407,745</td>
<td>$42,255</td>
<td>9, 12, 25, 27, 28</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>2, 5, 15, 19, 21, 30, 34, 45, 66, 68, 77, 84, 89, 92, 105</td>
<td>$4,000,000</td>
<td>$3,910,268</td>
<td>$89,732</td>
<td>—</td>
</tr>
</tbody>
</table>

| Second Stage ($k = 2$); Effect of ZIB Is Not Considered |
| IEEE 14-bus  | 4, 5, 8, 11, 13 * | $200,000                          | $121,656   | $78,344   | 7, 9             |
| IEEE RTS     | 5, 6, 8, 15, 16, 21 | $350,000                          | $286,227   | $63,773   | 10, 11, 17, 24   |
| IEEE 30-bus  | 6, 7, 11, 15, 16, 22, 26 | $350,000                          | $235,776   | $114,224  | 9, 12, 25, 27, 28|
| IEEE 118-bus | 2, 5, 9, 15, 19, 21, 27, 30, 34, 40, 45, 49, 52, 56, 59, 66, 68, 71, 77, 80, 84, 89, 92, 105, 110 | $2,000,000 | $1,987,498 | $11,932   | —                |

| Third Stage ($k = 3$); Effect of ZIB Is Not Considered |
| IEEE RTS     | 5, 6, 8, 9, 15, 16, 21, 23 * | $350,000                          | $266,245   | $83,755   | 10, 11, 17, 24   |
| IEEE 30-bus  | 3, 6, 7, 11, 13, 15, 16, 20, 22, 26, 29 * | $350,000                          | $317,741   | $132,259  | 9, 12, 25, 27, 28|
| IEEE 118-bus | 2, 5, 9, 12, 15, 19, 21, 27, 30, 31, 32, 34, 36, 40, 45, 49, 52, 56, 59, 63, 66, 68, 70, 71, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110, 118 * | $2,000,000 | $1,022,043 | $977,957   | —                |

---

*aComplete observability is achieved.*

---

Fig. 6. OPP solution for the IEEE 14-bus; not considering effect of ZIB.

Fig. 7. OPP solution for the IEEE 14-bus; considering effect of ZIB.
The multistage PMU installation is performed on the IEEE 14-bus test system with predetermined priority buses. The higher priority buses for the IEEE 14-bus are chosen to be the high voltage buses, $L = [1, 2]$. The installation is performed as a four-stage process.

The first stage has a budget limit of $400,000$, and the remaining stages have budget limits of $500,000$ each. The first and second stages are used to achieve complete observability under normal conditions (14). The third stage is used to achieve observability for single line outage contingencies (16). The single PMU outage in (18) is chosen as the desired observability for the final stage. The results in Table IV show the optimal PMU installation at each stage. The results show a comparison between treating all buses equally. The results

<table>
<thead>
<tr>
<th>System</th>
<th>PMU locations</th>
<th>Stage budget $C_{budget}$</th>
<th>Cost</th>
<th>Remaining</th>
<th>Unprepared buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 14-bus</td>
<td>4, 5, 11</td>
<td>$300,000$</td>
<td>$252,189$</td>
<td>$47,811$</td>
<td>7, 9</td>
</tr>
<tr>
<td>IEEE RTS</td>
<td>5, 20</td>
<td>$400,000$</td>
<td>$352,366$</td>
<td>$47,634$</td>
<td>10, 11, 17, 24</td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>3, 4, 10, 15, 20</td>
<td>$500,000$</td>
<td>$426,236$</td>
<td>$73,764$</td>
<td>9, 12, 25, 27, 28</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>45, 49, 53, 72, 80, 84, 86, 94</td>
<td>$2,500,000$</td>
<td>$2,357,860$</td>
<td>$142,140$</td>
<td>—</td>
</tr>
</tbody>
</table>

Second Stage ($k = 2$): Effect of ZIB Is Considered

<table>
<thead>
<tr>
<th>System</th>
<th>PMU locations</th>
<th>Stage budget $C_{budget}$</th>
<th>Cost</th>
<th>Remaining</th>
<th>Unprepared buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 14-bus</td>
<td>4, 5, 11, 13*</td>
<td>$150,000$</td>
<td>$73,448$</td>
<td>$76,552$</td>
<td>7, 9</td>
</tr>
<tr>
<td>IEEE RTS</td>
<td>2, 5, 14, 20, 21</td>
<td>$350,000$</td>
<td>$222,514$</td>
<td>$127,486$</td>
<td>10, 11, 17, 24</td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>3, 4, 10, 13, 15, 29</td>
<td>$350,000$</td>
<td>$238,861$</td>
<td>$111,139$</td>
<td>9, 12, 25, 27, 28</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>2, 8, 12, 19, 21, 27, 34, 37, 43, 45, 53, 56, 68, 72, 75, 77, 80, 84, 86, 92, 94</td>
<td>$2,000,000$</td>
<td>$1,924,513$</td>
<td>$75,487$</td>
<td>—</td>
</tr>
</tbody>
</table>

Third Stage ($k = 3$): Effect of ZIB Is Considered

<table>
<thead>
<tr>
<th>System</th>
<th>PMU locations</th>
<th>Stage budget $C_{budget}$</th>
<th>Cost</th>
<th>Remaining</th>
<th>Unprepared buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE RTS</td>
<td>2, 5, 8, 14, 20, 21*</td>
<td>$350,000$</td>
<td>$142,932$</td>
<td>$207,068$</td>
<td>10, 11, 17, 24</td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>3, 4, 7, 10, 13, 15, 16, 20, 29*</td>
<td>$350,000$</td>
<td>$230,720$</td>
<td>$119,280$</td>
<td>9, 12, 25, 27, 28</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>2, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 68, 72, 75, 77, 80, 84, 86, 92, 94, 100, 105, 110, 110*</td>
<td>$2,000,000$</td>
<td>$1,924,513$</td>
<td>$75,487$</td>
<td>—</td>
</tr>
</tbody>
</table>

*Complete observability is achieved.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 14-bus</td>
<td>2, 8, 10, 15</td>
<td>$354,650$</td>
<td>$870,400$</td>
<td>$870,400$</td>
<td>$870,400$</td>
<td>$873,849$</td>
<td>4, 5, 8, 11, 13</td>
</tr>
<tr>
<td>IEEE RTS</td>
<td>3, 4, 7, 10, 13, 16, 20, 21</td>
<td>$342,300$</td>
<td>$1,222,100$</td>
<td>$1,222,100$</td>
<td>—</td>
<td>—</td>
<td>5, 6, 8, 9, 15, 16, 21, 23</td>
</tr>
<tr>
<td>IEEE 30-bus</td>
<td>3, 6, 7, 11, 15, 17, 20, 21, 24, 26, 30</td>
<td>$286,400$</td>
<td>$1,825,100$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>3, 6, 7, 11, 13, 15, 16, 20, 26, 29</td>
</tr>
<tr>
<td>IEEE 118-bus</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

*Has two optimal solutions, one for minimum number of PMUs and the other for a cost model.

*Has multiple optimal solutions, only the solution with the minimum cost is presented.
show that prioritizing buses can drive up the PMU installation cost as seen in Table IV.

<table>
<thead>
<tr>
<th>Stage One</th>
<th>Stage Two</th>
<th>Stage Three</th>
<th>Stage Four</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma \leq 1$</td>
<td>$\sigma &gt; 1$</td>
<td>$\sigma_h \geq 1$</td>
<td>$\sigma_p \geq 1$</td>
</tr>
<tr>
<td>4, 6, 8</td>
<td>4, 5, 6, 8, 11, 13, 14</td>
<td>2, 4, 5, 6, 8, 10, 11, 13, 14</td>
<td>2, 4, 5, 6, 8, 10, 11, 13, 14</td>
</tr>
<tr>
<td>$\leq 6$</td>
<td>$\leq 9$</td>
<td>$\leq 9$</td>
<td>$\leq 9$</td>
</tr>
<tr>
<td>$219,799$</td>
<td>$300,966$</td>
<td>$133,860$</td>
<td>$90^b$</td>
</tr>
<tr>
<td>1, 2</td>
<td>1, 2, 8, 10, 13</td>
<td>1, 2, 4, 6, 8, 10, 11, 13, 14</td>
<td>1, 2, 4, 6, 8, 10, 11, 13, 14</td>
</tr>
<tr>
<td>$\geq 185,777$</td>
<td>$\geq 491,476$</td>
<td>$\geq 255,764$</td>
<td>$\geq 255,764$</td>
</tr>
<tr>
<td>Unprepared buses</td>
<td>7, 9</td>
<td>7, 9</td>
<td>7, 9</td>
</tr>
</tbody>
</table>

*No additional installation of PMUs.

Observability already achieved at the previous stage.

V. CONCLUSION

In this paper, a new approach for multistage OPP is presented, where a real-time model for the installation of PMUs and the communication infrastructure is considered. This approach considers a practical OPP installation process, where the PMUs are installed over the span of a number of years in a multistage manner, which is determined by the budget of the utility. Unlike the multistage OPP in the literature, the proposed approach can be scaled to achieve different observability conditions such as a single PMU outage. Moreover, this approach can be used to handle application-based OPP problems, where certain buses are given higher priority based on the utility criteria. The effectiveness of the proposed approach is demonstrated on the IEEE 14-bus, IEEE RTS, IEEE 30-bus and IEEE 118-bus test systems.

REFERENCES


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