5.2.4:

A) \[ M = (0.78084)(28.0134) + (0.20946)(31.9988) + (0.00934)(39.948) + (0.00031)(44.010) \]

\[ M = 28.9638 \]

B) \[ g_0 = \frac{8.314 \text{ J/}(\text{Kg} \cdot \text{K})}{28.9638} = \frac{287.05 \text{ J/}(\text{Kg} \cdot \text{K})}{(\text{Kg} \cdot \text{K})} \]

In Al, p.509, \( \gamma \) for dry air is

\[ 287.06 \text{ J/}(\text{Kg} \cdot \text{K}) \]

\[ \frac{\gamma - g_0}{g_0} \times 100\% = 0.0044\% \text{ error} \]
5.6.1: A) \( c = \sqrt{\frac{8 \rho_0}{\rho}} = \sqrt{(1.41)(1.013 \times 10^5 \text{Pa})} \cdot 0.090 \text{ kg/m}^3 \)

\[ c = 1259.8 \text{ m/s} \]

B) p. 528, \( c = 1269.5 \text{ m/s} \)

To 2 sig. figs., \( c = \sqrt{(1.4)(1.0 \times 10^5 \text{ Pa})} \cdot 0.090 \text{ kg/m}^3 \)

\[ c = 1200 \text{ m/s} \]

clearly, yes, within error

\[ \frac{1269.5 - 1259.8}{1269.5} \times 100\% = 0.76\% \]

\[ \frac{1269.5 - 1200}{1269.5} \times 100\% = 5.47\% \]

C) \( c = \sqrt{8 \pi r \gamma \kappa} \)

\[ c^2 = (1.41) \frac{8314 \text{ J/(kg} \cdot \text{K})}{2} (T+273.15) \]

\[ T = \frac{(1259.8)^2 \cdot 2}{(1.41)(8314)} - 273.15 \degree = -2.38 \degree \]
5.6.8: \( P_\theta = 0 \), \( T = 30^\circ C \), \( t = \frac{T}{100} = .3 \)

A) \( c = 1402.7 + 488t - 482t^2 + 135t^3 \)

\[ c = 1509.4 \text{ m/s} \]

B) \( \frac{dc}{dt} = 488 - 2.482t + 3.135t^2 = 235 \)

\( \frac{dt}{dT} = \frac{1}{100} \)

\( \frac{dc}{dT} = \frac{dc}{dt} \cdot \frac{dt}{dT} = \frac{235}{100} = 2.35 \text{ m per } \circ C \)
5.7.1: \[ \bar{\mathbf{v}} = \hat{x} U e^{j(\omega t - kx)} \]

\[ \frac{\partial \bar{\mathbf{v}}}{\partial t} = \hat{x} U j\omega e^{j(\omega t - kx)} \]

\[ (\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}} = \left[ \hat{x} U e^{j(\omega t - kx)} \right] \cdot \hat{x} U(-jk)e^{j(\omega t - kx)} \]

\[ (\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}} = U^2(-jk)e^{2j(\omega t - kx)} \]

\[ \frac{|(\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}}|}{|\frac{\partial \bar{\mathbf{v}}}{\partial t}|} = \frac{|U^2(-jk)e^{2j(\omega t - kx)}|}{|U(j\omega)e^{j(\omega t - kx)}|} \]

\[ = \frac{UK}{\omega} = \frac{U\omega}{\omega c} = \frac{U}{c} \]

so \( U \ll c \) when \( |(\bar{\mathbf{v}} \cdot \nabla) \bar{\mathbf{v}}| \ll |\frac{\partial \bar{\mathbf{v}}}{\partial t}| \)
5.7.4:  \[ \vec{p} = A e^{j(\omega t - k_x x - k_y y - k_z z)} \]

\[ \vec{p} = -\rho_0 \frac{\partial \vec{\Phi}}{\partial t} \]

\[ \vec{E} = \vec{\Phi} e^{j(\omega t - k_x x - k_y y - k_z z)} \]

\[ \frac{\partial \vec{\Phi}}{\partial t} = j\omega \vec{\Phi} e^{j(\omega t - k_x x - k_y y - k_z z)} \]

\[ \vec{A} = -\rho_0 j\omega \vec{\Phi} \]

\[ \vec{\Phi} = \frac{\vec{A}}{-j\omega \rho_0} \]

\[ \vec{E} = \frac{\vec{A}}{-j\omega \rho_0} e^{j(\omega t - k_x x - k_y y - k_z z)} \]

\[ \vec{u} = \nabla \vec{\Phi} = \frac{\vec{A}}{\omega \rho_0} \left[ k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \right] \cdot e^{j(\omega t - k_x x - k_y y - k_z z)} \]

\[ \vec{K} = k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \]

To show \( \vec{u} \) is parallel to \( \vec{K} \),

can note either that directions

are exactly the same, or
evaluate the dot product (parallel if

\[
\frac{\hat{k} \cdot \vec{v}}{|\hat{k}| |\vec{v}|} = 1
\]

\[
\frac{|\vec{A}| \left[ k_x^2 + k_y^2 + k_z^2 \right]}{\omega p_0 \sqrt{k_x^2 + k_y^2 + k_z^2}} = 1
\]

\[
\frac{\sqrt{k_x^2 + k_y^2 + k_z^2} \cdot |\vec{A}|}{\omega p_0 \sqrt{k_x^2 + k_y^2 + k_z^2}}
\]

\[\Rightarrow \text{parallel}\]
5.9.2: A) \( \frac{\rho}{\rho_0} = \left( \frac{\rho}{\rho_0} \right)^\gamma \)

\[ \rho = \rho_0 T \Rightarrow \frac{\rho}{\rho_0} = T = \frac{T}{T_0} \]

\[ \Rightarrow \frac{\rho}{T} = \frac{T_0}{\rho_0} = \frac{\rho_0}{T_0} \]

Eliminate \( \rho, \rho_0 \):

\[ \frac{\rho}{\rho_0} = \left( \frac{\rho_0 T_0}{\rho_0 T} \right)^\gamma = \left( \frac{T_0}{T} \right)^\gamma \]

\[ \left( \frac{\rho}{\rho_0} \right)^{1-\gamma} = \left( \frac{T_0}{T} \right)^\gamma \]

\[ \left( \frac{\rho}{\rho_0} \right)^{\frac{1-\gamma}{\gamma}} = \frac{T_0}{T} \]

\[ \rho = \rho_0 + \rho \]

\[ T = T_0 + \Delta T \]

\[ \left( \frac{\rho_0 + \rho}{\rho_0} \right)^{\frac{1-\gamma}{\gamma}} = \frac{T_0}{T_0 + \Delta T} \]
\[ T_0 + \Delta T = T_0 \left(1 + \frac{\rho}{\rho_0}\right)^{\frac{\gamma - 1}{\gamma}} \]

\[ \Delta T = T_0 \left(1 + \frac{\rho}{\rho_0}\right)^{\frac{\gamma - 1}{\gamma}} - T_0 \]

Use binomial expansion to linearize:

\[ \left(1 + \frac{\rho}{\rho_0}\right)^{\frac{\gamma - 1}{\gamma}} \approx 1 + \frac{\gamma - 1}{\gamma} \frac{\rho}{\rho_0} + \ldots \]

\[ \Delta T = T_0 \left(1 + \frac{\gamma - 1}{\gamma} \frac{\rho}{\rho_0}\right) - T_0 \]

\[ \Delta T = \frac{\gamma - 1}{\gamma} \frac{T_0 \rho}{\rho_0} \]

B) \[ I = 10 \text{ W/m}^2 \], assume plane wave

\[ I = \frac{p^2}{2 \rho c} \Rightarrow p = \sqrt{2 \rho c I} \]

\[ p = \sqrt{2 \left(1.21 \text{ kg/m}^3 \right) \left(343 \text{ m/s} \right) \left(10 \text{ W/m}^2 \right)} \]

\[ p = 91.11 \text{ Pa} \]
\[ \Delta T = \frac{(1.402 - 1)}{1.402} \left( \frac{293.15 \, ^\circ K \cdot 91.11 \, \text{Pa}}{1.013 \times 10^5 \, \text{Pa}} \right) \]

\[ \Delta T = 0.0756 \, ^\circ K \]