Problem 1 (15 points): Given a circular piston surrounded by a rigid baffle with \( ka = 10 \) radians:

a) Compute the 'on axis' \( \% \text{error} \) at the near field/far field transition distance \( \frac{d^2}{4 \lambda} \) for the pressure field generated by this source, where \( d \) is the diameter of the circular piston. Use the following formula:

\[
\% \text{error} = \left| \frac{P_{\text{axial}} - P_{\text{far field}}}{P_{\text{axial}}} \right| \times 100\%.
\]

b) What is the directivity (\( D \)) of this circular source?

c) What is the directivity index (\( DI \))?

\[
P_{\text{axial}}(r) = 2 \rho c \mathcal{V}_0 \sin \left( \frac{\pi}{2} \right) K_r \left[ \sqrt{1 + \left( \frac{\pi}{r} \right)^2} - 1 \right] \frac{a}{r} = \frac{\rho c \lambda}{ka} \frac{2 \pi}{ka}
\]

\[
P_{\text{axial}}(\frac{d^2}{4 \lambda}) = 2 \rho c \mathcal{V}_0 \left( 1.983 \right)
\]

\[
P_{\text{far field}}(r) = \frac{\omega \rho c \mathcal{V}_0 a^2}{2 \pi r} \text{inc}(\theta) = \frac{\rho c \mathcal{V}_0}{2} \frac{ka}{a} \frac{a}{r}
\]

\[
P_{\text{far field}}(r = \frac{d^2}{4 \lambda}) = \rho c \mathcal{V}_0 \frac{ka}{ka} = \rho c \mathcal{V}_0 \pi
\]

\[
\% \text{error} = \left| \frac{\rho c \mathcal{V}_0 \left( 1.983 - \pi \right)}{\rho c \mathcal{V}_0 \left( 1.983 \right)} \right| \times 100\% = 58.4\%
\]

B) \( D = \frac{10^2}{1 - \text{inc}(\theta)} = 100.34 \)

C) \( DI = 10 \log_{10} 100.34 = 20 \text{dB} \)

Problem 2 (15 points): You have been asked to design a quarter-wave matching layer for an interface between a quartz (X-cut) transducer and castor oil (material properties for both are in appendix A10). The quartz operates in bulk mode at 50kHz. If the speed of sound in the matching layer is restricted to a value of 2000m/s,

a) what is the density of the matching layer?

b) what is the thickness of the matching layer?

\[
z_1 = 1.45 \times 10^6 \text{ rayl/s}
\]

\[
z_3 = 15.3 \times 10^6 \text{ rayl/s}
\]

\[
z_2 = \sqrt{z_1 z_3} = \sqrt{(1.45 \times 10^6 \text{ rayl/s})(15.3 \times 10^6 \text{ rayl/s})}
\]

\[
z_2 = 4.71 \text{ M rayl/s}
\]

A) \( \rho_2 = \frac{z_2}{c_2} = \frac{4.71 \text{ M rayl/s}}{2000 \text{ m/s}} = 2.355 \times 10^3 \text{ Kg/m}^3 \)

B) \( \lambda = \frac{c}{f} = \frac{2000 \text{ m/s}}{50 \text{ KHz}} = 0.04 \text{ m} \)

\( x/4 = 0.01 \text{ m} \)
Problem 3 (15 points): You have an irritating neighbor who lives on the other side of a 15cm thick concrete wall. Your neighbor plays loud, incredibly bad music all of the time, day and night. What frequency or frequencies in the audible range are perfectly transmitted through the concrete wall at normal incidence, assuming lossless bulk mode propagation through the concrete?

\[ \begin{align*}
z &= 8 \text{ Mm} \\text{yr} \\text{A}^5 \\
\frac{n}{c} &= 3100 \text{ m/s} \\
\frac{n^2}{2} &= 15 \times 10^{-2} \text{ m} \\
\frac{n}{2f} &= 15 \times 10^{-2} \text{ m} \\
f &= \frac{n}{2} \frac{3100 \text{ m/s}}{2 \cdot 15 \times 10^{-2} \text{ m}} = n \cdot 10,333 \text{ Hz} \\
&= 10,333 \text{ Hz}
\end{align*} \]

Problem 4 (15 points): A circular piston with a rigid baffle is sequentially excited at the fundamental and third harmonic frequencies. The piston, which has radius ‘a’ = 1cm, operates in fresh water at 20°C. The first sidelobe peak of the third harmonic is located at \( \theta \), and the first null of the fundamental is located at \( (\theta + 30^\circ) \).

a) Find the angle \( \theta \) (in degrees) at which the first sidelobe peak of the third harmonic occurs, and

b) Find the value of ‘k’, where ‘k’ is the wavenumber corresponding to the fundamental frequency in fresh water at 20°C.

A) \( k_1 = \frac{\omega}{c} = k \)

\[ k_3 = \frac{3\omega}{c} = 3k_1 = 3k \]

\[ 3.83 = k_3 \sin (\theta + 30^\circ) \]

\[ 5.14 = 3k_3 \sin \theta \]

\[ 3.83 = k_3 \sin \theta \cos 30 + k_3 \cos \theta \sin 30 \]

\[ \sqrt{3} \]

\[ 3.83 = \frac{5.14}{3} \sqrt{3} + k_3 (\cos \theta) \frac{1}{2} \]

\[ 2(3.83) - \frac{5.14}{3} \sqrt{3} = k_3 \cos \theta = 4.69 \]

\[ (k_3)^2 = (k_3)^2 \sin^2 \theta + (k_3)^2 \cos^2 \theta = 4.69^2 + \left( \frac{5.14}{3} \right)^2 = 24.95 \]

\[ k_3 = 5.00 \]

\[ \theta = \sin^{-1} \left( \frac{5.14}{5.3} \right) = 20.04^\circ \]

B) \( k = \frac{5.00}{1 \times 10^{-2} \text{ m}} = 500 \text{ m}^{-1} \)
Problem 5 (20 points): A line source in air at 20°C has been amplitude shaded according to the triangular waveform shown below. You should note that a triangle is the convolution of two rect functions, that the Fourier transform of the rect function is a sinc, that the Fourier transform of two convolved functions is the product of the Fourier transforms, and therefore the far field beam pattern of this line source is then proportional to sinc²(1/2 kL sinθ). The wavelength is 10cm, and the length of the line source is 200cm (figure not to scale).

![Diagram of triangular waveform]

a). What is the −3dB beamwidth of the main lobe (in degrees) of the amplitude shaded line source?
b). What is the peak amplitude of the first sidelobe (in dB, relative to the peak of the main lobe) of the amplitude shaded line source?
c). How does the −3dB beamwidth of a uniformly excited line source compare to the above amplitude shaded line source? Be sure to justify your answer by including the numerical value of the −3dB beamwidth (in degrees) for the uniformly excited line source.
d). How does the peak amplitude of the first sidelobe (in dB, relative to the peak of the main lobe) of a uniformly excited line source compare to the above amplitude shaded line source? Be sure to justify your answer by including the numerical value of the peak amplitude of the first sidelobe (in dB) for the uniformly excited line source.

A) \[ -3 \text{dB} = 20 \log_{10} \left| \text{sinc}^2 \left( \frac{1}{2} kL \sin \theta \right) \right| \]
\[ -3 = 40 \log_{10} \left| \text{sinc} \left( \frac{1}{2} kL \sin \theta \right) \right| \]
\[ 10^{-3/40} = 0.8414 = \left| \text{sinc} \left( \frac{1}{2} kL \sin \theta \right) \right| \]

P.517: sinc(1) = 0.8415 (close enough)

\[ 1 = \frac{1}{2} \frac{2\pi}{10 \text{cm}} (200 \text{ cm}) \sin \theta \]
\[ \theta = \sin^{-1} \left( \frac{1}{20 \pi} \right) = 0.9119^\circ, \quad 2\theta = 1.8239^\circ \]

B) peaks of \( |\text{sinc}(x)| \) are in same location as peaks
of \( |\text{sinc}^2(x)| \)

\[ 20 \log_{10} \left| \text{sinc}^2 \left( 4.49 \right) \right| = 40 \log_{10} \left| \text{sinc}(4.49) \right| \]
\[ = -26.52 \text{ dB} \]

C) \[ \theta = \sin^{-1} \left( \frac{1.3915}{\pi \cdot 20} \right) = 1.269^\circ, \quad 2\theta = 2.538^\circ \]

\[ \therefore \text{shaded beamwidth is smaller than uniform beamwidth} \]

when sinc²(1/2 kL sinθ) is compared to sinc(1/2 kL sinθ) (Are both sources the same length? See notes from class).

D) \[ 20 \log_{10} \left| \text{sinc}(4.49) \right| \]
\[ = -13.26 \text{ dB} \]

\[ \therefore \text{shaded sidelobes are smaller than uniformly excited sidelobes} \]
Problem 6 (20 points): In the figure below, the power transmission coefficient is plotted as a function of the angle of incidence for two unknown gas mixtures separated by an infinitesimally thin membrane. The incident plane pressure wave originates in medium 1, and the transmitted plane pressure wave propagates in medium 2, where $T_x(\theta=0^\circ) = 0.845$, $T_x(\theta=68.8^\circ) = 1.0$, and $T_x(\theta=90^\circ) = 0$.

a) Find the ratio of impedances $z_2/z_1$ for the two unknown media, and
b) Find the ratio of sound speeds $c_2/c_1$ for the two unknown media.

Normal incidence ($\theta = 0^\circ$):

$$Z = \frac{Z_2}{Z_1}, \quad C = \frac{C_2}{C_1},$$

$$T_{PI} = 0.845 = \frac{4Z}{(Z + 1)^2} \quad (\theta = 0^\circ)$$

$$Z^2 + 2Z + 1 - \frac{4Z}{0.845} = 0$$

$$Z^2 - 2.7337Z + 1 = 0$$

$$Z = 2.2987 + Z = .4350$$

Angle of Intromission $\theta = 68.8^\circ$, $Z = 2.3$

$$\sin^2(68.8^\circ) = \frac{2.3^2 - 1}{2.3^2 - c^2} = \frac{4.29}{5.29 - c^2} = 0.8692$$

$$c^2 = 0.3546, \quad \frac{c}{Z} = 1.5$$

see p.158: no critical angle (see figure), $c_1 < c_2$ and $c_2 < c_1$.