Example 4.8 Thermal efficiency of the Otto engine (Continued)

Solution:

$$\text{Basis: model as ideal gas, } Q_H = C_V(T_3 - T_2) \quad (^*\text{ig})$$

$$Q_C = C_V(T_1 - T_4) \quad (^*\text{ig})$$

$$W_{S,net} = Q_H + Q_C = C_V(T_3 - T_2 + T_1 - T_4) \quad (^*\text{ig})$$

$$\eta = C_V(T_3 - T_2 + T_1 - T_4)/[C_V(T_3 - T_2)] + 1 - (T_3 - T_2)/(T_3 - T_2) = 1 + (T_1 - T_4)/(T_3 - T_2) \quad (^*\text{ig})$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{R/C_V}; \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{R/C_V} = \left(\frac{V_2}{V_1}\right)^{R/C_V} \quad (^*\text{ig})$$

$$\Rightarrow T_4 = T_3\, r^{-R/C_V};\ T_1 = T_2\, r^{-R/C_V} \quad (^*\text{ig})$$

$$\eta = 1 - r^{-R/C_V} = 1 - r^{(1-\gamma)} \quad (^*\text{ig})\ 4.8$$

The Diesel Engine

The Diesel Engine is similar to the Otto engine except that the fuel is injected after the compression and the combustion occurs relatively slowly. This necessitates "fuel-injectors," but has the advantage that higher compression ratios can be obtained without concern of pre-ignition (ignition at the wrong time in the cycle). Pre-ignition occurs when the temperature rise due to compression goes past the spontaneous ignition temperature of the fuel, creating an annoying pinging and knocking sound, and reducing the efficiency of the cycle. Pre-ignition does not occur in a diesel engine because there is no fuel during compression. Some spontaneous ignition temperatures are given below.

Spontaneous ignition temperatures (°C) of sample hydrocarbons

Isooctane	447
Benzene	592
Toluene	568
n-Octane	240
n-Decane	232
n-Hexadecane	230
Methanol	470
Ethanol	392

Diesel fuel is much like decane and hexadecane and burns without a spark. Auto fuel is much like isooctane and benzene and, ideally, will not ignite until the spark goes off. But all fuels are mixtures, and if the gasoline contains enough *n*-octane instead of isooctane, then undesirable pre-ignition will occur.

The thermodynamics of the air-standard Diesel engine can be analyzed like the Otto Cycle. But, instead of rapid combustion at constant volume, the Diesel engine has relatively slow combustion at constant pressure. In the air-standard Diesel cycle shown in Figure 4.18, step 1-2 is an adia-

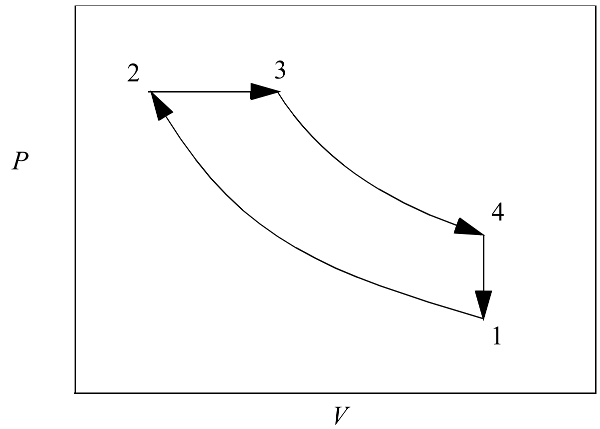

Figure 4.18 *Schematic of air-standard Diesel cycle.*

batic compression, step 2-3 represents the combustion where heat is added, step 3-4 is adiabatic, step 4-1 is an isochoric cooling.

Example 4.9 Thermal efficiency of a Diesel engine

Develop an expression for the thermal efficiency of the air-standard diesel cycle as a function of the compression ratio $rc = V_1/V_2$ and the expansion ratio $re = V_4/V_3$. Assume the working fluid is an ideal gas, and the volume effect of moles of gas generated is small relative to the effect of heating from combustion.

Solution: This process is a little more complicated than the Otto cycle because heat addition and work occurs during constant pressure combustion. The energy balance for the combustion step is:

$$dU = Q_H + W_{S,combustion} \Rightarrow Q_H = \Delta U - W_{S,combustion} = \Delta U + P_2(V_3 - V_2) = \Delta(U + PV) = \Delta H$$

$$\Rightarrow Q_H = \Delta H = C_P(T_3 - T_2) \qquad \text{(*ig)}$$

For isochoric cooling

$$Q_C = C_V(T_1 - T_4) \qquad \text{(*ig)}$$

For the cycle, the energy balance is $0 = Q_H + Q_C + W_{S,net}$, giving the thermal efficiency

$$\eta = -W_{S,net}/Q_H = (Q_H + Q_C)/Q_H = 1 + \frac{C_V(T_1 - T_4)}{C_P(T_3 - T_2)} = 1 + \frac{1}{\gamma}\left[\frac{T_1}{T_3 - T_2} - \frac{T_4}{T_3 - T_2}\right] \qquad \text{(*ig)}$$

where

$$\frac{T_3 - T_2}{T_1} = \frac{T_3}{T_1} - \frac{T_2}{T_1} = \frac{P_3V_3}{P_1V_1} - rc^{\gamma-1} = rc^{\gamma}/re - rc^{\gamma-1} \qquad \text{(*ig)}$$

where for T_2/T_1 we have used Eqn 2.53, and for T_3/T_1 we have used: the ideal gas law; $P_3 = P_2$; Eqn. 2.55; and $V_1 = V_4$, as

Example 4.9 Thermal efficiency of a Diesel engine (Continued)

$$\frac{T_3}{T_1} = \frac{P_3V_3}{P_1V_1} = \left(\frac{P_2}{P_1}\right)\left(\frac{V_3}{V_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}\left(\frac{V_3}{V_4}\right) = rc^{\gamma}/re \qquad (\text{*ig})$$

For the last term in brackets from the formula for efficiency,

$$\frac{T_3 - T_2}{T_4} = \frac{T_3}{T_4} - \frac{T_2}{T_4} = re^{\gamma-1} - \frac{P_2V_2}{P_4V_4} = re^{\gamma-1} - \frac{P_3V_2}{P_4V_4} = re^{\gamma-1} - re^{\gamma}/rc \qquad (\text{*ig})$$

where for T_3/T_4 we have used Eqn 2.53, and for T_2/T_4 we have used: the ideal gas law; $P_2 = P_3$; Eqn. 2.55; and $V_4 = V_1$, as

$$\frac{T_2}{T_4} = \frac{P_2V_2}{P_4V_4} = \left(\frac{P_3}{P_4}\right)\left(\frac{V_2}{V_4}\right) = \left(\frac{V_4}{V_3}\right)^{\gamma}\left(\frac{V_2}{V_1}\right) = re^{\gamma}/rc \qquad (\text{*ig})$$

Substuting the intermediate results and rearranging the formula for efficiency,

$$\begin{aligned}\eta &= 1 + \frac{1}{\gamma}\left\{\frac{1}{rc^{\gamma}/re - rc^{\gamma-1}} - \frac{1}{re^{\gamma-1} - re^{\gamma}/rc}\right\} \\ &= 1 + \frac{1}{\gamma}\left\{\frac{re}{rc^{\gamma} - re\cdot rc^{\gamma-1}} - \frac{rc}{rc\cdot re^{\gamma-1} - re^{\gamma}}\right\}\end{aligned} \qquad (\text{*ig})\ 4.9$$

4.7 FLUID FLOW

Consider the general flow system of Fig. 4.19*a* in which work and heat are transferred and the fluid undergoes changes in kinetic and potential energy. Recognize that the compressor or pump in the schematic could be replaced with an expander or turbine. Rather than deriving an integral equation between points 1 and 4 in the schematic, let us consider a balance over a differential element at steady-state as shown in Fig. 4.19*b* where the possibility of heat and work transfer are permitted. The steady-state balance for a single stream becomes:[1]

$$0 = \lim_{dL \to 0}\left\{\left[H + \frac{u^2}{2g_c} + \frac{gz}{g_c}\right]^{in} - \left[H + \frac{u^2}{2g_c} + \frac{gz}{g_c}\right]^{out}\right\} + dQ + dW_S$$

For a differential element, $H^{out} = H^{in} + dH$, and

$$\left(\frac{u^2}{2g_c}\right)^{out} = \left(\frac{u^2}{2g_c}\right)^{in} + d\left(\frac{u^2}{2g_c}\right), \quad \left(\frac{gz}{g_c}\right)^{out} = \left(\frac{gz}{g_c}\right)^{in} + d\left(\frac{gz}{g_c}\right)$$

The differential balance becomes

$$0 = -dH - d\left(\frac{u^2}{2g_c}\right) - d\left(\frac{gz}{g_c}\right) + dQ + dW_S$$

1. Note that the velocity in this equation is a mean velocity.