ECE 305 Prob. Set 1 [redacted]

R.2.2 ⇒ Under what conditions can the dot product of 2 vectors be negative?

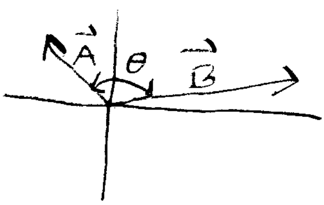
The definition of the dot product $\vec{A}\cdot\vec{B} =$ (where C is a scalar) is:

$$\vec{A}\cdot\vec{B} = C = |A|\cdot|B|\cos\theta$$

Since $|A|$ & $|B| \geq 0$, C can be negative if $\cos\theta < 0$, or if

$$\boxed{\pi/2 < \theta < \pi}$$

Remember, theta is the smaller angle between the 2 vectors, so $0 \leq \theta \leq \pi$ for the definition of the dot product.



R 2.3 Write Down The Results of A·B & A
if $A \parallel B$ & $A \perp B$.

a) For $A \parallel B$: $A \cdot B = |A||B| \cos\theta$ and in
case, $\theta = 0$ so $A \cdot B = |A||B|$

$A \times B = |A||B| \sin\theta \cdot \hat{n}$ and $\theta =$

and since $\sin\theta = 0$, $A \times B = 0$.
The cross product of 2 vectors always
gives a vector perpendicular to the
or The cross product of 2 parallel ve
is 0.

b) For $A \perp B$: $A \cdot B = |A||B| \cos\theta$ and here
$\theta = \pi/2 = 90°$, so $A \cdot B = 0$. The Dot
of 2 vectors gives the projection
one vector on to another. For per
vectors, neither vector has any
in the direction of the other.

$A \times B = \hat{n} \cdot |A| \cdot |B| \cdot \sin\theta$ and $\theta = \pi/2$

$A \times B = \hat{n}\, |A| \cdot |B|$

#3 R 2.4 Which vector operation Below does NOT Make Sense?

a.) $(A \cdot B) \times C$ => Makes No Sense! The result $\vec{A} \cdot \vec{B}$ is a Scalar, and You can't cross a Scalar $(\vec{A} \cdot \vec{B})$ with a vector $(\vec{C})$. (Pg. 16)

b.) $A(B \cdot C)$ => Does Make Sense! $\vec{B} \cdot \vec{C}$ is a s
So $A(B \cdot C)$ is just $\vec{A}$ scaled
The result $(B \cdot C)$. (Pg. 14)

c.) $\vec{A} \times \vec{B} \times \vec{C}$ => No parentheses! which operation Do perform First?

d.) A / B => No Sense at all! There is no Definition For The Division of 2 vectors (Pg. 19)

e.) A / a_A => No Sense! A is a vector; a_A is a unit vector in The A direction. Just like The last problem, you c Divide 2 vectors

$(A \times B) \cdot C \Rightarrow$ Does Make Sense! The result of $\vec{A} \times \vec{B}$ is a vector, which is simply dotted to $\vec{B}$.

R 2.5 Is $(A \cdot B) C = A (B \cdot C)$? No!

$(\vec{A} \cdot \vec{B})$ is a scalar result, and it changes the length of vector $\vec{C}$ in the operation $(A \cdot B) \vec{C}$

$(\vec{B} \cdot \vec{C})$ is also scalar, and changes the length of vector $\vec{A}$.

As long as vectors $\vec{A}$ & $\vec{C}$ are not equal, the result $(\vec{A} \cdot \vec{B}) \vec{C} \neq \vec{A} (\vec{B} \cdot \vec{C})$

P2.1 Given 3 vectors,

$$\vec{A} = \hat{x} + 2\hat{y} - 3\hat{z}$$
$$\vec{B} = -4\hat{y} + \hat{z}$$
$$\vec{C} = 5\hat{x} - 2\hat{z}$$

Find:

$a_A \Rightarrow$ a unit vector in the $\vec{A}$ direction.

$\vec{a}_A = \dfrac{\vec{A}}{|\vec{A}|} \Rightarrow$ This is NOT an illegal

'Vector Division'! We simply scale all co

of $\vec{A}$ by $\dfrac{1}{|\vec{A}|}$, where $|\vec{A}|$ is the

Magnitude of $\vec{A}$. So, $|\vec{A}| = \sqrt{(1)^2 + (2)^2 -}$

$= \sqrt{14}$

So, $\boxed{\vec{a}_A = \frac{1}{\sqrt{14}}(\hat{x} + 2\hat{y} - 3\hat{z})}$

$|\vec{A} - \vec{B}| = |\hat{x}(1-0) + \hat{y}(2-(-4)) + \hat{z}(-3-1)| = |\hat{x} + 6$

$= \sqrt{1^2 + 6^2 + (-4)^2} = \boxed{\sqrt{53}}$

$A \cdot B = (\hat{x} + 2\hat{y} - 3\hat{z}) \cdot (0\hat{x} - 4\hat{y} + \hat{z}) = 2(-4) + (-3$

$\theta_{AB} \Rightarrow \quad A \cdot B = |A||B| \cos\theta_{AB}$

previous prob

$$\cos\theta_{AB} = \frac{A \cdot B}{|A||B|} = \frac{-11}{\sqrt{14}\sqrt{17}} = -.713$$

$$\theta_{AB} = \cos^{-1}(-.713) = \boxed{135.48° = 2.365}$$

The component of $\vec{A}$ in the direction of $\vec{C}$

$$A \cdot C = |A| \cdot |C| \cdot \cos\theta_{AC} = |C| \cdot \mathrm{Proj}_A C = |A| \, \mathrm{Proj}_C A$$

$$\text{So, } \mathrm{Proj}_A C = \frac{A \cdot C}{|C|} = \boxed{\frac{11}{\sqrt{29}}}$$

$A \times C \Rightarrow$ set up a matrix & find the determin

$$\begin{array}{c} \\ A \rightarrow \\ C \rightarrow \end{array} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(-4) - \hat{y}(-2+15) + \hat{z}$$

$$\boxed{= -4\hat{x} - 13\hat{y} - 10\hat{z}}$$

$\cdot(B \times C)$

$$\times C = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(8) - \hat{y}(-5) + \hat{z}(20) = 8\hat{x} + 5\hat{y} +$$

$$(B \times C) = (\hat{x} + 2\hat{y} - 3\hat{z})(8\hat{x} + 5\hat{y} + 20\hat{z}) = 8 + 10 - 60$$

(g CONT'd)

$(\vec{A}\times\vec{B})\cdot\vec{C}$ $\quad \vec{A}\times\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix} = \hat{x}(14) - \hat{y}(1) + \hat{z}($

$= -10\hat{x} - \hat{y} - 4\hat{z}$

$(\vec{A}\times\vec{B})\cdot\vec{C} = (-10\hat{x} - \hat{y} - 4\hat{z})\cdot(5\hat{x} + 0\hat{y} - 2\hat{z})$

$= -50 + 8 = \boxed{-42}$

$(\vec{A}\times\vec{B})\cdot\vec{C} = \vec{A}\cdot(\vec{B}\times\vec{C})$, The expected Res

h.) $(\vec{A}\times\vec{B})\times\vec{C} \Rightarrow A\times B = -10\hat{x} - \hat{y} - 4\hat{z}$ (From Part

$(\vec{A}\times\vec{B})\times\vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -10 & -1 & -4 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(2) - \hat{y}(40) + \hat{z}($

$= 2\hat{x} - 40\hat{y} + 5\hat{z}$

$\vec{A}\times(\vec{B}\times\vec{C})$ $\quad \vec{B}\times\vec{C} = 8\hat{x} + 5\hat{y} + 20\hat{z}$ (part g)

$A\times(\vec{B}\times\vec{C}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 8 & 5 & 20 \end{vmatrix} = \hat{x}(55) - \hat{y}(4$

$= 55\hat{x}$

The expected result: $(A\times B)\times C \neq A\times(B\times C)$

#6 P.2.2 Given $\vec{A} = \hat{x} - 2\hat{y} + 3\hat{z}$
$\vec{B} = \hat{x} + \hat{y} - 2\hat{z}$

Find a unit-vector $\vec{C}$ that is $\perp$ to $\vec{A}$ & $\vec{B}$

To find a vector $\perp$ to $\vec{A}$ & $\vec{B}$, we need to take $\vec{A} \times \vec{B}$:

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -2 & 3 \\ 1 & 1 & -2 \end{vmatrix} = \hat{x}(4-3) - \hat{y}(-2-3) + \hat{z}(1+2)$$

$$= \hat{x} + 5\hat{y} + 3\hat{z}$$

$$\vec{C} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \Rightarrow \sqrt{1+25+9} = \sqrt{35}$$

$$\boxed{\vec{C} = \frac{1}{\sqrt{35}}(\hat{x} + 5\hat{y} + 3\hat{z})}$$

#7 P2.3 Two vector fields $\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$ & $\hat{x}B_x + \hat{y}B_y + \hat{z}B_z$ are parallel eve

What is the relationship btwn. th components?

If 2 vector fields are parallel,

$\vec{A} \times \vec{B} = 0$ so, (next page)

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) - \hat{y}(A_x B_z - \\ + \hat{z}(A_x B_y - A_y B_x)$$

The components of $\vec{A} \times \vec{B}$ must each b
0, so

$$A_y B_z - A_z B_y = 0 \qquad A_y B_z = A_z B_y \quad (1)$$

$$A_x B_z - A_z B_x = 0 \qquad A_x B_z = A_z B_x \quad (2)$$

$$A_x B_y - A_y B_x = 0 \qquad A_x B_y = A_y B_x \quad (3)$$

$$\frac{A_y}{B_y} = \frac{A_z}{B_z} \quad \text{From 1}$$

$$\frac{A_x}{B_x} = \frac{A_z}{B_z} \quad \text{From 2}$$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} \quad \text{From 3}$$

$$\therefore \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z}$$ which means

That for parallel vectors $\vec{A}$ & $\vec{B}$,

$\vec{A} = K \cdot \vec{B}$ where K is a scalar.