Individual Penalty Based Constraint handling Using a Hybrid Bi-Objective and Penalty Function Approach

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Abstract

The holy grail of constrained optimization is the development of an efficient, scale invariant and generic constraint handling procedure in single and multi-objective constrained optimization problems. In this paper, an individual penalty parameter based methodology is proposed to solve constrained optimization problems. The individual penalty parameter approach is a hybridization between an evolutionary method, which is responsible for estimation of penalty parameters for each constraint and the initial solution for local search. However the classical penalty function approach is used for its convergence property. The aforesaid method adaptively estimates penalty parameters linked with each constraint and it can handle any number of constraints. The method is tested over multiple runs on six mathematical test problems and a engineering design problem to verify its efficacy. The function evaluations and obtained solutions of the proposed approach is compared with three of our previous results. In addition to that, the results are also verified with some standard methods taken from literature. The results show that our method is very efficient compared to some recently developed methods.

1 Introduction

Evolutionary algorithms (EAs) has been successfully applied in wide range of optimization problems from past two decades. Most real-world optimization problems involve constraints mainly due to the availability of limited resources. However, some practical applications can be modelled as unconstrained optimization to simplify the problem, and there is not much effect of ignoring constraints in optimal solutions. But in general constraints play a vital role during the optimization process. So, it is not wise to ignore the constraints in during the optimization process. A standard constrained optimization problem in literature is defined as follows:

$$\begin{array}{rl}\text{Minimize} & f(\mathbf{x}),\\ \text{Subject to} & g_j(\mathbf{x}) \geq 0, j = 1, .., J,\\ & h_k(\mathbf{x}) = 0, k = 1, .., K,\\ & x_i^l \leq x_i \leq x_i^u, i = 1, .., n.\end{array} \tag{1}$$

In Equation 1, n are number of variables, J and K are number of inequality and equality constraints. The $f(\mathbf{x})$ is the objective function to be optimized, $g_j(\mathbf{x})$ is the j-th inequality constraint, and $h_k(\mathbf{x})$ is the k-th equality constraint. The i-th variable lies in between $[x_i^l, x_i^u]$.

Researchers have proposed various mechanisms to deal with constraints using evolutionary algorithms (EAs), which were initially developed for solving unconstrained problems. Some comprehensive survey on constrained optimization using EAs can be found in ([14, 1, 12]). Coello [2] maintains a constraint handling repository which holds broad spectrum of constraint handling techniques.

The simplest and well established technique in constrained optimization is the penalty function approach ([10, 20, 3, 13, 8]). In this method, objective function is penalized based on the extend of violation of

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constraints. Some of the penalty function methods are parabolic penalty, death penalty, log penalty, static penalty and dynamic penalty etc. Two commonly used penalty function methods are interior penalty and exterior penalty approach. Of these two classes, the exterior methods are considered better than the interior one for the reasons: (i) interior point method starts with a feasible solutions which in most cases is a critical challenge, (ii) interior methods penalize feasible solutions close to constraint boundary and (iii) handling of equality constraints is not possible using interior penalty function methods.

The concept of bi-objective optimization is also used by several researchers for constraint handling using evolutionary algorithms ([19, 9, 16, 23]). A growing number of researchers hybridize classical optimization techniques with EAs where evolutionary techniques are used for the generation of initial guess and classical methods are used for convergence ([15, 9, 21]).

In our earlier works, we proposed a hybrid bi-objective evolutionary combined with classical penalty function based method for constrained optimization problems [7]. In that strategy, the penalty parameter (R) is estimated from the obtained non-dominated solutions between the objective function and overall constraint violation. The estimated penalty parameter and the best solution (in terms overall constraint violation) is used by the classical local search method for exact convergence. The present methodology is an extension of that approach.

In the next section, we give a clear description of individual penalty based constraint handling. Then, we present the proposed algorithm using individual penalty based approach. Consequently, we test the proposed approach in some test problems. The obtained function evaluations (FE) are compared with a penalty parameter less approach [5] and two of our previously developed methods [7, 4]. The results are also compared with three standard constrained optimization methodologies taken from literature [18, 11, 22].

2 Description of Individual Penalty Approach

In this section, we briefly explain the proposed individual penalty based constrained optimization methodology. The conventional way to deal with constraints in penalty function approach, is to assign a penalty to each constraint for a violation. In penalty function approach, number of penalty parameters are taken to be equal to the number of constraints i.e. dimension of penalty parameter (R) is equal to total number of constraints (J). The advantage of individual penalty is that, it will further normalize each constraint, if it is not properly normalized using problem specific normalization.

In our proposed method we treat a single objective constrained optimization as a bi-objective problem. In the bi-objective problem one objective is the overall constrained violation and the other is the original objective function. Based on the bi-objective formulation and individual penalty function method, the following modified formulation is obtained:

$$\begin{array}{ll} \text{minimize} & f_1(\mathbf{x}) = \mathrm{CV}(\mathbf{x}) \\ \text{minimize} & f_2(\mathbf{x}) = f(\mathbf{x}) \end{array} \tag{2}$$

where $\mathrm{CV}(\mathbf{x}) = \sum_{j=1}^{J} R_j \langle g_j(\mathbf{x}) \rangle$ and R_j is the penalty parameter for j^{th} constraint. We use NSGA-II [6] to solve the above multi-objective problem.

At every generation, first rank solutions with zero constraint violation of Equation 2 are considered to calculate all the penalty parameters associated with each constraint. From the obtained non-dominated solutions, we isolate each constraint separately and the original objective function to find the new non-dominated solutions between objective function and individual constraints. Each set of non-dominated solutions between constraint versus original objective function is used to calculated the corresponding penalty parameter of the constraint. The best solution point in the non-dominated front in terms of the constraint is taken as reference point. From the reference point, slope is calculated for every other point of the non-dominated front. From all the calculated slopes, the maximum value is taken as the associated penalty parameter value for that constraint.

The way in which we handle the constraints come under exterior penalty method. In exterior penalty method, we have to ensure that value of the penalty parameter is not dominating the objective function during initial generations; so at very first generation, we start with $R_j = 1$. The exterior penalty method starts with a sequence of infeasible solutions in the whole optimization process and finally converges to the desired solution at the optimum.

In Figure 1, Pareto-optimal solutions are shown with respect to CV and f as two objective functions. Suppose we have two constraints for this problem. From these non-dominated solutions, we consider first constraint (g_1) and the original objective function to find the non-dominated points between them. Figure 2 shows the non-dominated and dominated points with respect to g_1 versus f. From the non-dominated points, the best point in terms of g_1 is identified. Here it is marked with A (Figure 2). The slope is calculated for every other point from A and the highest slope is fixed as the penalty parameter (R_1), which is also shown in Figure 2. The non-dominated solutions and highest slope for second constraint is also shown in Figure 3. These slopes will be used as penalty parameters in evolutionary as well as local search method to get the constrained minimum.

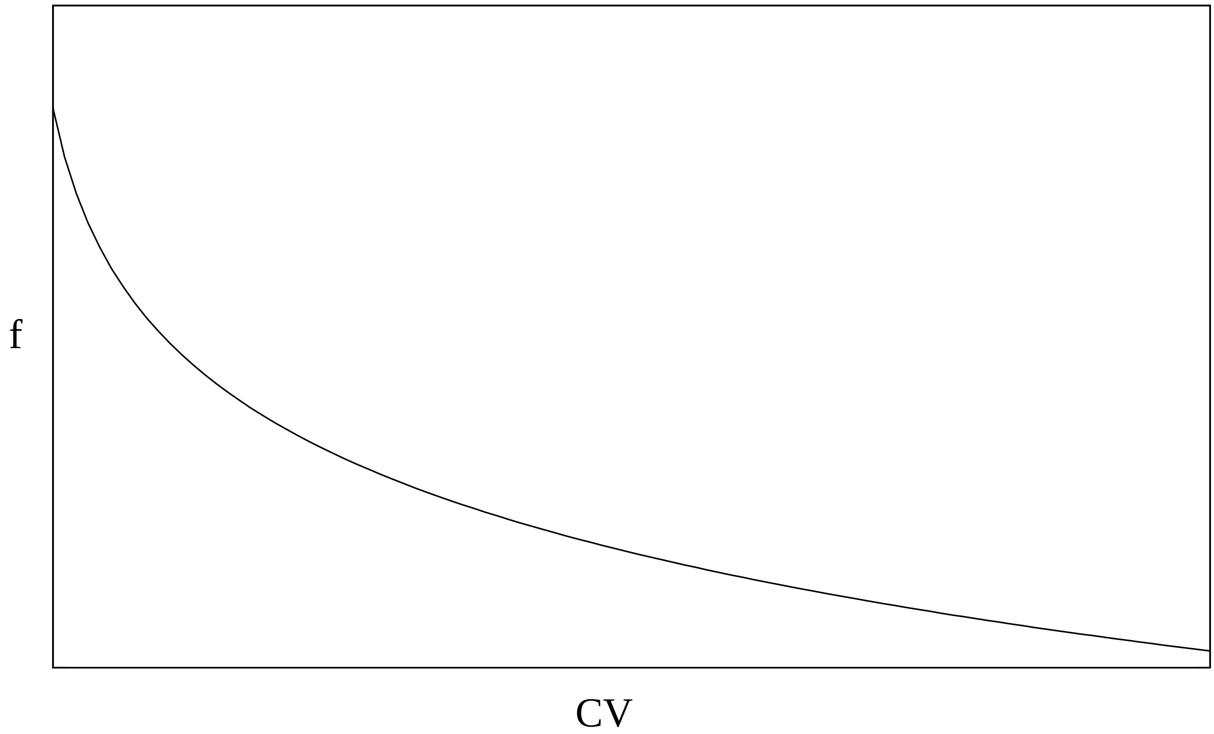

Figure 1: Non-dominated solutions between CV ($=R_1 \times g_1 + R_2 \times g_2$) and f.

3 Proposed Hybrid Approach

Constraint handling using evolutionary algorithms is continuously procuring a lot of popularity for its robustness, ability to solve complex problems, ease of integration with any other algorithm and many more. In our earlier studies, we described the advantage of bi-objective EA and the penalty function approach [7, 4]. In the present study, we are interested in developing an algorithm where each penalty parameter is linked with each constraint. In this section, we will describe our individual penalty based constrained optimization. The individual penalty based constraint handling is a hybridization between a bi-objective EA (using NSGA-II) coupled with a classical penalty function method.

3.1 Proposed Algorithm

The working principle of the individual penalty based constrained optimization methodology is a hybridization between a bi-objective genetic algorithm coupled with a classical penalty function method. The evolutionary part consists of bi-objective method of handling constrained for single objective optimization problem, where the role of evolutionary part is to estimate penalty parameters for each constraints and provide an initial guess to start the classical penalty function method. The steps of the hybrid algorithm is described below.

First, we set the generation counter as $t = 0$ and all penalty parameters $R_j = 1$, where j = 1,2,..number of constraints.

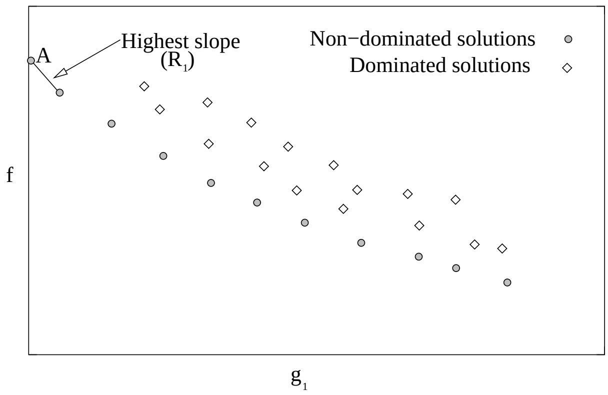

Figure 2: Non-dominated solutions between f and g_1 with highest slope.

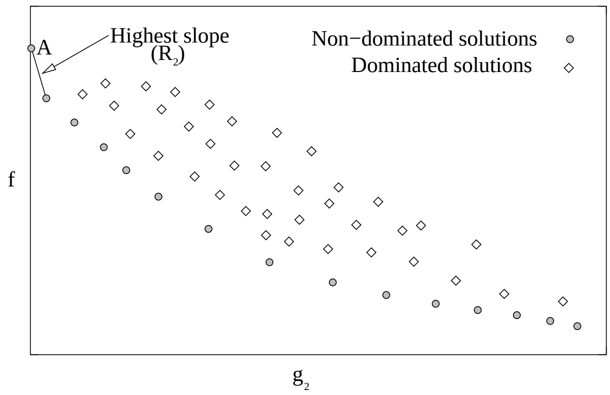

Figure 3: Non-dominated solutions between f and g_2 with highest slope.

Step 1:The evolutionary algorithm (NSGA-II) is an elitist, non dominated sorting and crowding distance based multi-objective genetic algorithm. It is applied to the following bi-objective optimization problem (which is the modified single objective constrained optimization problem). This means that the non-dominated solutions in the objective space are formed using two conflicting objectives (constraint violation versus the original objective function).

The bi-objective problem is defined as follows:

$$\left.\begin{array}{rl} \text{minimize} & \text{CV}(\mathbf{x}), \\ \text{minimize} & f(\mathbf{x}), \\ \text{subject to} & \text{CV}(\mathbf{x}) \leq c, \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}. \end{array}\right\} \tag{3}$$

The definition of overall constraint violation (CV($\mathbf{x}$)) is given in equation 2. In the first generation

$$\text{CV}(\mathbf{x}) = 1 \times \langle g_1(\mathbf{x})\rangle + 1 \times \langle g_2(\mathbf{x})\rangle + ... + 1 \times \langle g_J(\mathbf{x})\rangle.$$

To focus the search near the boundary of feasible region a constraint is added to CV($\mathbf{x}$). We kept $c = 0.2J$ in this study (as is used in our earlier studies [7, 4]).

Step 2:If $t > 0$, first rank solutions are identified for which constraint violation is zero from equation 3.

Step 3:Non-dominated solutions are identified considering each g_j versus f($\mathbf{x}$) using non-dominated sorting of the obtained solutions in Step 2.

Step 4:The penalty parameters are estimated using current non-dominated solutions with respective g_j versus f($\mathbf{x}$), obtained in Step 3. The minimum first objective (in terms of g_j) is identified as reference point. From the reference point, the slope is calculated for every other points in the non-dominated solutions. From these calculated slopes, the highest one is identified as the associated penalty parameter for that constraint.

Step 5:If $t > 0$ and ((t mod τ) = 0), using R_j computed in Step 4, the following local search problem is solved starting with the current minimum-CV solution:

$$\begin{array}{rl} \text{minimize} & P(\mathbf{x}) = f(\mathbf{x}) + \sum_{j=1}^{J} R_j \langle \hat{g}_j(\mathbf{x})\rangle, \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}. \end{array} \tag{4}$$

The solution from the local search is denoted by $\bar{\mathbf{x}}$. The value of τ is fixed to 5 (as is used in our earlier studies [7, 4]).

Step 6:The algorithm is terminated in case $\bar{\mathbf{x}}$ is feasible, and the difference between two consecutive local search solutions is smaller than a small number δ_f. Then $\bar{x}$ is identified as the optimized solution. Here, $\delta_f = 10^{-4}$ is used like earlier studies. Else, t is incremented by one, and we continue with Step 1.

It is noted that due to Step 4, the penalty parameters R_j is not a manual user-tunable parameter as it is determined from the obtained non-dominated solutions between each constraint (g_j) versus the objective function $f(\mathbf{x})$.

For the local search in Step 5 Matlab's `fmincon()` procedure with reasonable parameter settings is used to solve the penalized function.

Lastly, some crucial points are given below:

1. During the optimization process, some constraints may be easier to satisfy, as they will be satisfied from the first generation of NSGA-II. In that case, their R_j values will always be 1, as there will be no change of the objective function with respect to the g_j.
2. After some generations, some g_j may have zero value. These are points satisfying the g_j constraint. In that case, penalty parameter R_j is fixed to 10^6. Most likely this will happen near the optimum. Based on the exterior penalty approach, penalty parameter should start with a small value and gradually increase, thus justifying the use of higher R_j.

The hybridization is a synergistic combination between a bi-objective EA (using NSGA-II) and penalty function based classical approach for faster convergence. The anticipated outcome is the effective solution for the problem at hand with major improvements compared to the several approaches mentioned in literature review.

4 Simulation Results on Standard Test Problems

In order to study the effectiveness of the individual penalty based constraint handling methodology, we first test our proposed methodology in a two variable test problem and then apply to some standard single-objective constrained test problems, having inequality constraints. The details for all test problems is available in [7].

The parameter values for our experiments are given in the following:

Population size = $16n$ (unless stated otherwise),
SBX probability= 0.9,
SBX index = 10,
Polynomial mutation probability = $1/n$, and
Mutation index = 100.

The termination criterion for individual penalty is given in Section 3.1. For all the test problems, we test the algorithm randomly 50 times, each time starting with a different initial population.

4.1 Problem P1

First, we solve the following two-variable problem having two non-linear inequality constraints and non-convex feasible region [8]:

$$\begin{array}{rl} \text{minimize} & f(\mathbf{x}) = (x_1-3)^2 + (x_2-2)^2, \\ \text{subject to} & g_1(\mathbf{x}) \equiv 4.84 - (x_1-0.05)^2 - (x_2-2.5)^2 \geq 0, \\ & g_2(\mathbf{x}) \equiv x_1^2 + (x_2-2.5)^2 - 4.84 \geq 0, \\ & 0 \leq x_1 \leq 6, \quad 0 \leq x_2 \leq 6. \end{array}$$

Based on our single penalty approach [7], the optimum solution was found after 677 function evaluations (600 needed by EMO and 77 by `fmincon()` procedure) and using adaptive normalization [4], the best function evaluation was found after 731 function evaluations (600 needed by EMO and 131 by `fmincon()` procedure). In the present study, the proposed hybrid approach take 691 function evaluations (600 needed by EMO and 91 by `fmincon()` procedure) to solve the problem. Identical optimum was found in all the cases.

Table 1 compare best, median and worst performance values of the individual penalty approach with single penalty approach and adaptive normalization approach with 50 different runs. In all these 50 cases, our present approach finds the true optimal solution. The function evaluations for all three strategies are comparable. From Table 1, we can see, that in terms of range of function evaluations, individual penalty strategy outperforms both the earlier methodologies.

Table 2 shows different penalty parameter values with respect to generations. Initially both penalty parameters are started with 1.0. As constraint g_2 is inactive from initial generation, value of it is fixed to

Table 1: Function evaluations, FE (NSGA-II and local search) and optimal solution, by the Earlier approach and Proposed hybrid approach in 50 runs.

		Best	Median	Worst
Single	FE	677 (600 + 77)	733 (600 + 133)	999 (900 + 99)
Penalty	f	0.627380	0.627379	0.627379
Adaptive	FE	731 (600 + 131)	1297 (900 + 397)	1,623 (1200 + 423)
Normalization	f	0.627378	0.627393	0.627377
Proposed	FE	691 (600 + 91)	765 (600 + 165)	900 (600 + 300)
Approach	f	0.627379	0.627380	0.627397

1 during whole optimization process. Figure 4 shows the performance of the individual penalty approach for a simulation run, out of 50 runs. In the figure X-axis is the generation counter, Y-axis is the change in best objective value and Y2-axis is the adaptive penalty parameter values. The adaptation of R_1 is shown in Figure 4 (also in Table 2). For this typical run, all solutions are feasible right from the initial population. After first local search, the algorithm finds the optimum. But due to our termination criteria (described in Section 3.1), termination takes place after two local searches.

Table 2: Adaptation of penalty parameter values for problem P1.

Gen.	R_1	R_2
0	1.0000	1.0000
5	0.7564	1.0000
10	0.3596	1.0000

After solving the two variable problem successfully, the performance of the proposed methodology is tested on a number of standard test problems, which are available in [7].

4.2 Problem TP3

The TP3 problem has $n = 13$ variables with nine ($J = 9$) inequality constraints. Figure 5 shows, a case out of 50 simulations, where X-axis represents the generation number, Y-axis indicates corresponding best objective function values and Y2-axis is the overall constraint violation. In this simulation (Figure 5), until 6 generations all solutions are infeasible with respect to the original optimization problem. The objective and constraint violation values are drawn using dashed lines till they are infeasible i.e. till 6-th generation.

The feasible solutions for both objective and constraint violation values are shown with continuous line after 6-th generation. The first local search is executed after 5 generations, which helps both objective function and constraint violation to decrease in magnitude. The optima is achieved after 10 generations (second local search). The algorithm is continued up to another local search to meet termination criteria (Section 3.1).

Table 6 compares the function evaluations needed by our proposed approach with penalty parameter less approach [5], our single penalty approach [7] and our adaptive normalization approach [4]. Firstly, instead of comparing obtained solutions with objective function values of these algorithms, here we compare the number of function evaluations needed by each algorithm to achieve an identical accuracy in the final solution. From the obtained solutions and the function evaluations we can see that in all the cases the proposed approach find the optimum solution more accurately than the all other algorithms.

After comparing with our previous approaches, now we compare the solutions with some standard optimization methodologies. In Table 6 we compare the achieved objective function values using individual penalty approach with three other standard methodologies taken from existing literature [22], [11], [18]. In this problem, in terms of best function value, all the four strategies performed equally.

4.3 Problem TP4

The objective function of this problem is linear and the problem has six constraints with eight decision variables. This problem is very difficult to solve due to the asymmetric scaling of constraints. Here, first three constraints are normalized and other three are of different scale. Table 4 depicts that in all cases the

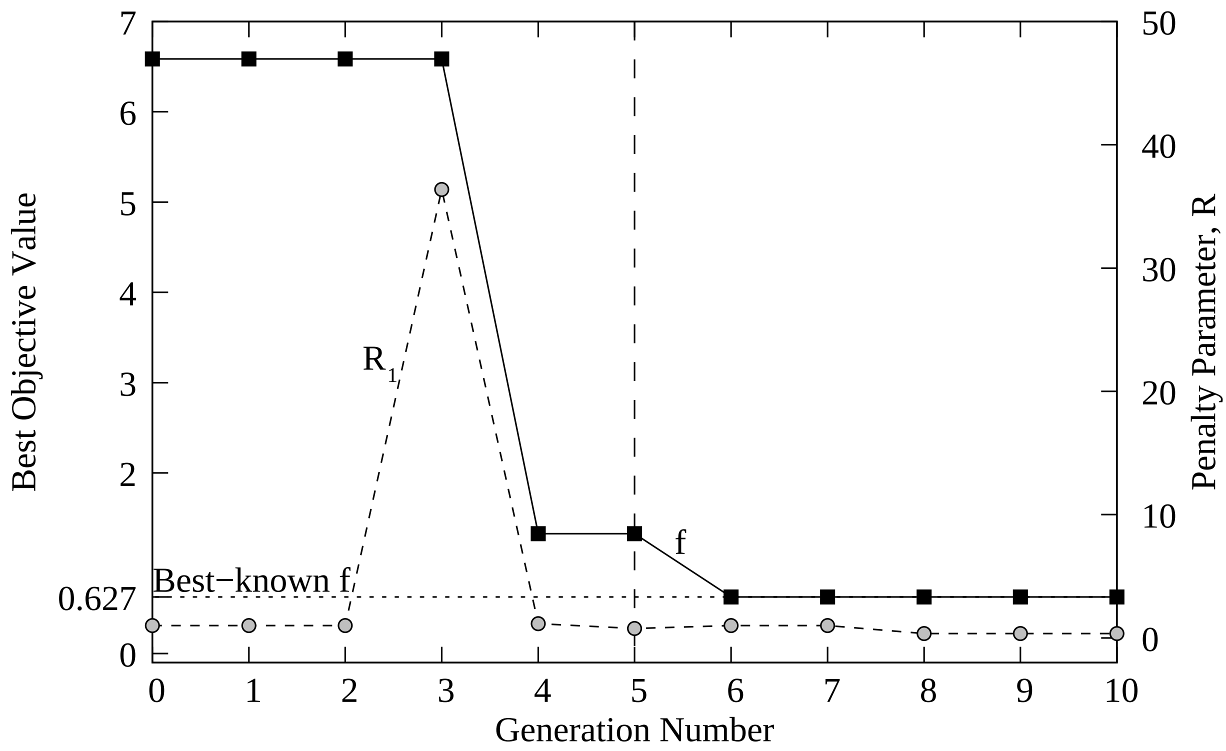

Figure 4: Objective value reduces with generation for problem P1.

proposed algorithm finds the optimum satisfactorily and the results are better in comparison to our previous results even though in terms of function evaluation adaptive normalization [4] out performs the individual penalty approach. Table 6 shows that the present method outperforms all three methods taken from literature [22, 11, 18].

4.4 Problem TP5

Figure 6 shows best objective value from the population members for a particular run out of 50 runs with different initial populations. Proposed method achieves all feasible solutions successfully from initial population members.

Table 3 presents the adaptation of all four penalty parameters. As the algorithm is not able to find adequate points, the penalty parameters are 1.0 until fifth generation. Even after 5-th generations, values of R_2 and R_3 are fixed till the termination of the algorithm. This table tell us that constraints g_1 and g_4 are active at the known minimum point. Figure 6 also indicates the adaptation of penalty parameters R_1 and R_3 with respect to generations. Table 4 shows that in terms of best known optimum and function evaluations proposed method is better than the previous results. If we compare in terms of best known objective value the proposed method performed very efficiently compared to the existing ones [22, 11, 18] shown in Table 6.

Table 3: Adaptation of penalty parameter values for problem TP5.

Gen.	R_1	R_2	R_3	R_4
0	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	1.0000
10	0.9660	1.0000	1.0000	1.5204
15	1.6408	1.0000	1.0000	4.5468

Table 4: Comparison of function evaluations (FE) needed by the proposed approach and the existing earlier approach [7]. Function evaluations by NSGA-II and local search have been shown separately. The better one in terms of FE is shown in bold.

Problem	Penalty Parameter Less Approach [5]			Single Penalty Approach [7]		
	Best	Median	Worst	Best	Median	Worst
TP3 (FE)	65,000	65,000	65,000	2,427	4,676	13,762
NSGA-II+Local				2,000+427	3,000+1,676	11,000+2,762
(f^*)	$\mathbf{-15}$	$\mathbf{-15}$	-13	$\mathbf{-15}$	$\mathbf{-15}$	-12
TP4 (FE)	320,080	320,080	320,080	31,367	54,946	100,420
NSGA-II+Local				14,400+16,967	24,600+30,346	45,600+54,820
(f^*)	7,060.221	7,220.026	10,230.834	7,078.625	7,049.943	7,940.678
TP5 (FE)	350,070	350,070	350,070	6,332	15,538	38,942
NSGA-II+Local				3,920+2,412	9,520+6,018	25,200+13,742
(f^*)	680.634	680.642	680.651	**680.630**	680.634	680.876
TP6 (FE)	250,000	250,000	250,000	1,120	2,016	6,880
NSGA-II+Local				800+320	1,200+816	3,600 + 3,280
(f^*)	$-30,665.537$	$-30,665.535$	$-29,846.654$	$\mathbf{-30,665.539}$	$\mathbf{-30,665.539}$	$-30,649,552$
TP8 (FE)	350,000	350,000	350,000	4,880	23,071	83,059
NSGA-II+Local				3,200+1,680	8,000+5,071	44,800+38,259
(f^*)	24.372	24.409	25.075	**24.308**	25.651	31.254
Weld (FE)	320,000	320,000	320,000	**1,067**	**2,261**	9,115
NSGA-II+Local				800+267	600+661	6,400 + 2,715
(f^*)	2.38119	2.38263	2.38355	**2.38113**	2.38216	2.46835
Problem	Adaptive Normalization Approach [7]			Individual Penalty Approach		
	Best	Median	Worst	Best	Median	Worst
TP3 (FE)	$\mathbf{2,333}$	$\mathbf{2,856}$	$\mathbf{11,843}$	2,593	4,504	27,998
NSGA-II+Local	2,000+ 333	2,000 + 856	8,000 + 3,843	2,000 + 593	3,000 + 1,504	22,000 + 5,998
(f^*)	-12	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-14.999}$	$\mathbf{-15.0}$	$\mathbf{-14.999}$
TP4 (FE)	$\mathbf{2,705}$	$\mathbf{27,235}$	$\mathbf{1,07,886}$	4,456	32,239	1,75,530
NSGA-II+Local	1,200 + 1,505	7,200 + 20,035	45,600 + 62286	4,000 + 456	24,000 + 8,239	1,32,000 + 43,530
(f^*)	7,049.588	7,059.576	7,065.348	$7,049.249220$	**7049.248993**	**7049.248654**
TP5 (FE)	1,961	11,729	42,617	**1,797**	**2,246**	**9,087**
NSGA-II+Local	1,120 + 841	7,280 + 4,449	27,440 + 15,177	1,120 + 677	1,120 + 1,126	6,720 + 2,367
(f^*)	680.635	680.631	680.646	**680.630058**	**680.630059**	**680.630058**
TP6 (FE)	1,123	4,183	13,631	**602**	**12,857**	**25,120**
NSGA-II+Local	800 + 323	2,400 +1,783	8,400 + 5,231	400 + 202	10,000 + 2,857	21,200 + 3,920
(f^*)	$\mathbf{-30,665.539}$	$\mathbf{-30,665.539}$	$\mathbf{-30,665.539}$	$\mathbf{-30665.538}$	$\mathbf{-30665.539}$	$\mathbf{-30665.538}$
TP8 (FE)	7,780	68,977	3,54,934	**3,273**	**13,246**	**47,732**
NSGA-II+Local	5,600 + 2,180	41,600 + 27,377	1,600+1673	8,000+5,246	35,200+12,532	2,20,800 + 1,34,134
(f^*)	24.565	**24.306**	**24.306**	**24.306**	**24.306**	24.308
Weld (FE)	1,607	5,761	13,636	1,159	2,426	**3,606**
NSGA-II+Local	1,200 + 407	4,000 + 1,761	10,000 + 3,636	(800 + 359)	(1,200+1,226)	(2,400+1,226)
(f^*)	2.42551	2.38159	**2.38113**	**2.38116**	2.38117	2.38123

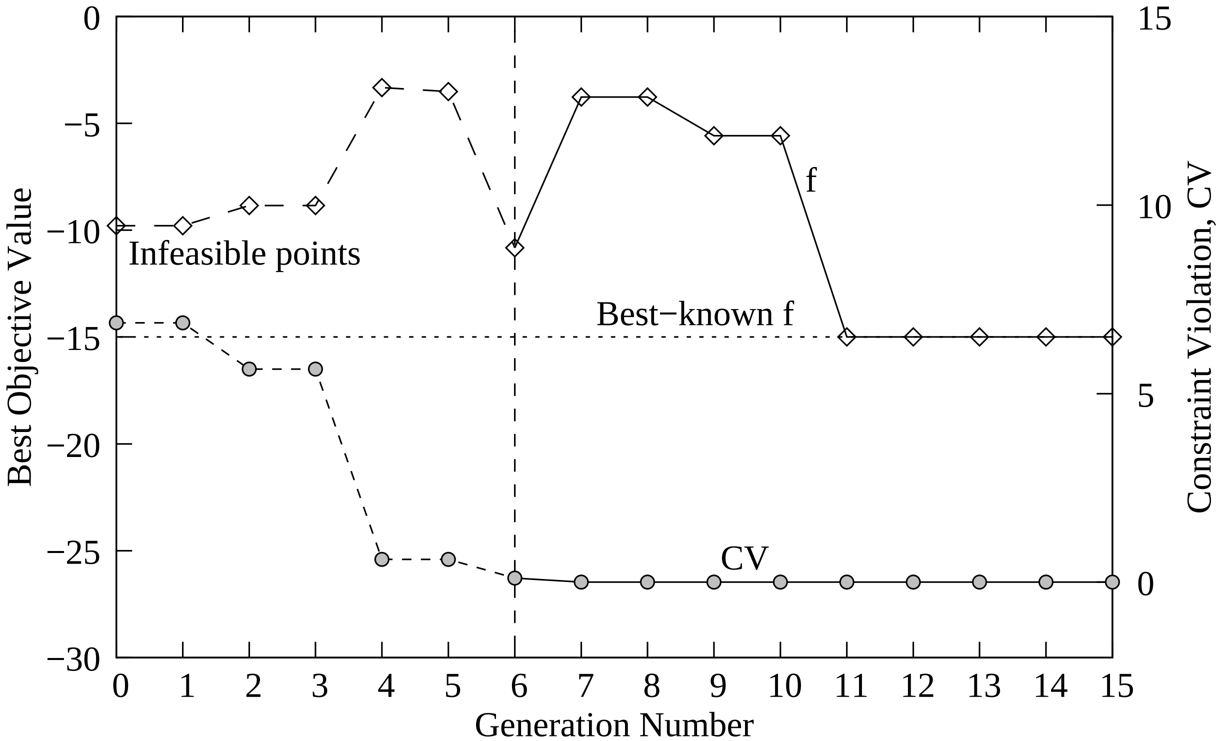

Figure 5: Objective value reduces with generation for problem TP3.

4.5 Problem TP6

Six constraints are constraining the optima for this problem. But out of them, only g_1 and g_6 are active at the constrained minimum point. Table 5 shows the adaptation of penalty parameters for a specific run out of all 50 runs. Initially the algorithm starts working with $R_j = 1$ as described in the algorithm. Constraints (g_2, g_3, g_4 and g_5) are satisfied from the initial population, as penalty parameter values for these constraints are always 1. The minimum number of function evaluations needed for our approach is only 602 (tabulated in Table 4). In comparison to all methods, our proposed method is the best in terms of best function evaluations with desired accuracy. From Table 5, we can conclude that only constraint g_1 and g_6 are active for this problem. From Table 4 it is clear that all the four methods performed equally well for this problem.

Table 5: Adaptation of penalty parameter values for problem TP6

Gen.	R_1	R_2	R_3	R_4	R_5	R_6
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	24688.3323	1.0000	1.0000	1.0000	1.0000	12870.0566
10	1000000.0000	1.0000	1.0000	1.0000	1.0000	10176.2642
15	17606.0127	1.0000	1.0000	1.0000	1.0000	9178.0350
20	1000000.0000	1.0000	1.0000	1.0000	1.0000	8970.5268
25	16601.2252	1.0000	1.0000	1.0000	1.0000	8654.2382
30	1000000.0000	1.0000	1.0000	1.0000	1.0000	6004.0605
35	11517.3527	1.0000	1.0000	1.0000	1.0000	6004.0605
40	1000000.0000	1.0000	1.0000	1.0000	1.0000	6076.3472
45	11517.3527	1.0000	1.0000	1.0000	1.0000	52321.0725
50	1000000.0000	1.0000	1.0000	1.0000	1.0000	18916.8389

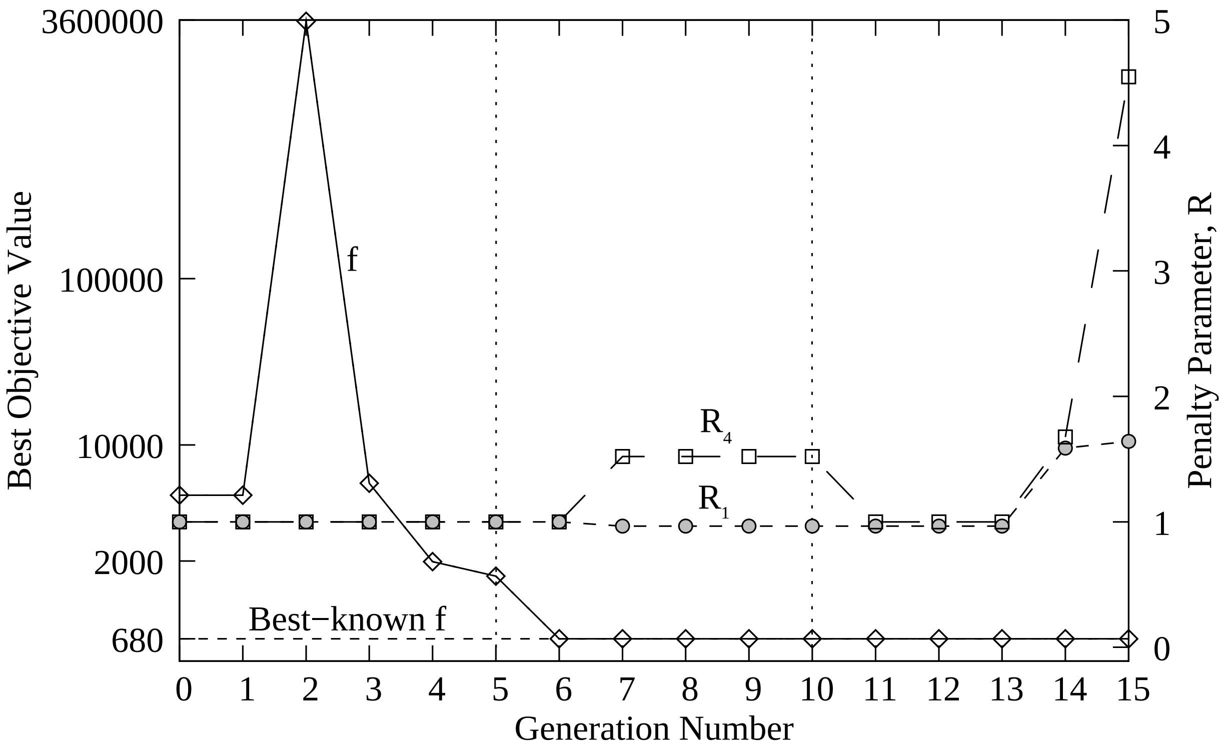

Figure 6: Objective value reduces with generation for problem TP5.

4.6 Problem TP8

The adaptation of objective function value with respect to generation counter is shown in Figure 7. The algorithm is not able to find a feasible solution up to eighth generations.

Even though, the optimum is achieved after 25 generations, the algorithm has to wait for another 5 generations to meet our termination criteria. Our algorithm performed very efficiently in this problem. Table 4 shows that in all cases the proposed method performs outstandingly compared our all three strategies taken. Table 6 clearly shows the robustness of the individual penalty approach.

4.7 Problem Weld

The welded beam design problem consists of five nonlinear and non-convex inequality constraints [17]. The objective of the design is to minimize the fabrication cost. It has four design variables (thickness of the beam b, width of the beam t, length of weld l, and weld thickness h). In Table 4 we can see that the solutions obtained using the proposed individual penalty approach is more robust in terms of function evaluations and solution accuracy compared to our previously developed methods. The proposed individual penalty approach very successfully solve the engineering optimization problem, even though in terms of best and median function evaluations single penalty [7] out performs the present strategy.

5 Conclusions

A hybrid individual penalty parameter based constraint handling approach is proposed in this present study. The present method combines an evolutionary algorithm with a classical penalty function approach as a complementary combination. The penalty parameters for each constraint and the initial solution for classical penalty function approach is evaluated using the evolutionary algorithm and the classical approach is used for its convergence property.

The traditional way to deal with the constraints in penalty function approach is to use a different penalty parameter associate with each constraint. This allows enough degrees of freedom to take care of normalization,

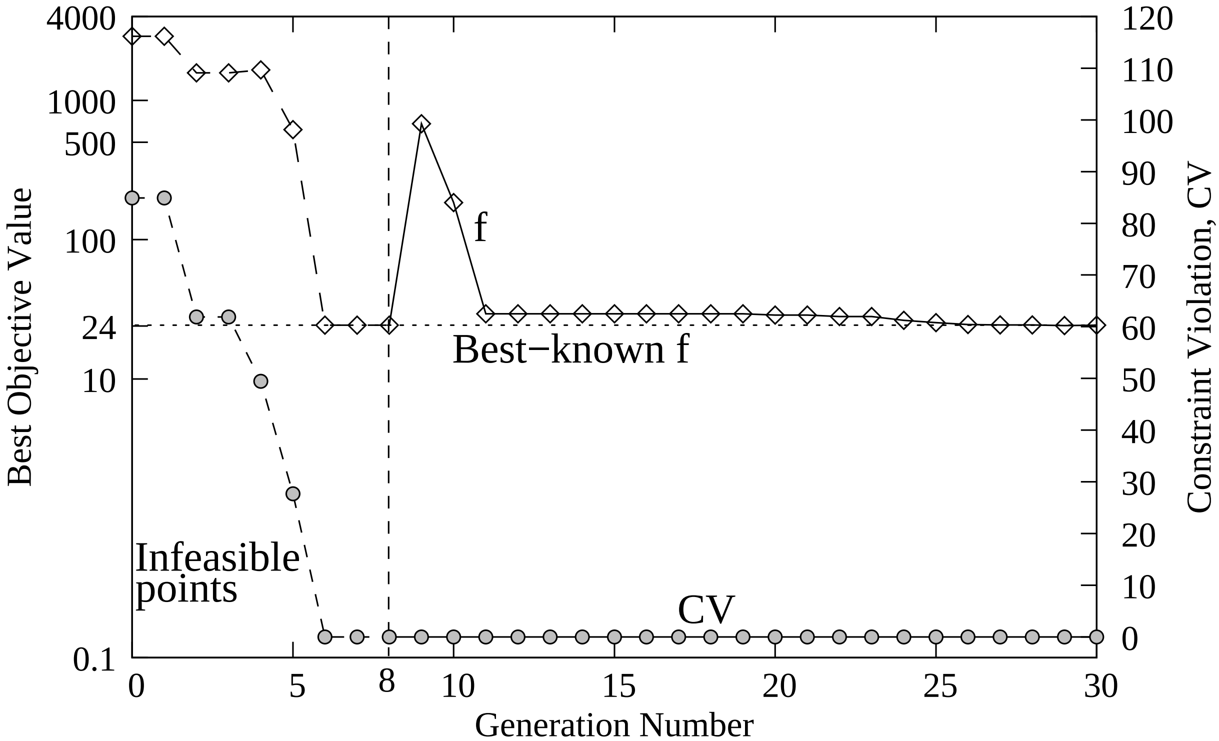

Figure 7: Objective value reduces with generation for problem TP8.

if constraints are of different scale. But in many cases this method is computationally expensive. The complexity of the method increases with the increase in number of constraints. This is one of the reasons why penalty function lost its popularity despite its simplicity. Our adaptive way of estimating individual penalty for each constraint is free from these above mentioned drawbacks and can handle any number of constraints efficiently.

We first test our approach on a model two variable constrained optimization problem and then extend it to various standard test problems from literature. To ensure robustness the simulation run was repeated 50 times and results were found similar in terms of function evaluations as well as solution accuracy. The function evaluations are compared with a penalty parameter less approach and two of our earlier studies and some recently developed methods on constraint handling using evolutionary algorithms. Our proposed approach outperforms existing methods in some of the problems significantly in terms of function evaluations and solution accuracy. In a nut shell, the proposed individual penalty based approach is computationally cheaper in comparison with the other methods.

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Table 6: Comparison of function values accomplished by the proposed approach and three existing constraint handling approach [22], [11],[18]. The best ones in terms of global optimum or in terms of best performance among the four are shown in bold. In some of the problems, the proposed hybrid approach outperforms the existing three approaches.

Problem	Optima	Proposed Hybrid Approach			Adaptive Trade-off Model Evolution Strategy (ATMES) [22]		
	(f^*)	Best	Mean	Worst	Best	Mean	Worst
TP3	-15	$\mathbf{-15}$	$\mathbf{-14.999}$	$\mathbf{-14.999}$	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-15}$
TP4	7,049.248	$\mathbf{7,049.248}$	$\mathbf{7,049.248}$	$\mathbf{7,049.249}$	7,052.253	7,250.437	7,560.224
TP5	680.630	**680.630**	**680.630**	**680.630**	**680.630**	680.639	680.673
TP6	-30665.538	$\mathbf{-30665.538}$	$\mathbf{-30665.538}$	$\mathbf{-30665.538}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$
TP8	24.306	**24.306**	**24.306**	**24.306**	**24.306**	24.316	24.359
		Simple Multi-membered Evolution Strategy (SMES) [11]			Stochastic Ranking (SR) [18]		
		Best	Mean	Worst	Best	Mean	Worst
TP3	-15	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-15}$	$\mathbf{-15}$
TP4	7,049.280	7,051.903	7,253.047	7,638.366	7,054.316	7,559.192	8,835.655
TP5	680.630	680.632	680.643	680.719	**680.630**	680.656	680.763
TP6	-30665.539	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$	$\mathbf{-30665.539}$
TP8	24.306	24.327	24.475	24.843	**24.307**	24.374	24.642

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