On the Sizing of a Solar Thermal Electricity Plant for Multiple Objectives Using Evolutionary Optimization

Kalyanmoy Deb[a], **Francisco Ruiz**[b], **Mariano Luque**[b], **Rahul Tewari**[c], **J. M. Cabello**[b] **and J. M. Cejudo**[b]

[a] Indian Institute of Technology Kanpur, Kanpur, PIN 208016, India, deb@iitk.ac.in, Also Department of Information and Service Economy, Aalto University, School of Economics, 00100 Helsinki, Finland

[b] University of Málaga, Campus El Ejido s/n, 29071 Málaga, Spain, {rua,mluque,jmcabello,jmcejudo}@uma.es

[c] Deutsche Bank Group, Mumbai, India, rtewari.iitk@gmail.com

KanGAL Report Number 2011008

Abstract

Design, implementation and operation of solar thermal electricity plants are no more an academic task, rather they have become a necessity. In this paper, we work with power industries to formulate a multiobjective optimization model and attempt to solve the resulting problem using classical as well as evolutionary optimization techniques. On a set of four objectives having complex trade-offs, our proposed procedure first finds a set of trade-off solutions showing the entire range of optimal solutions. Thereafter, the evolutionary optimization procedure is combined with a multiple criterion decision making (MCDM) approach to focus on preferred regions of the trade-off frontier. Obtained solutions are compared with a classical generating method. Eventually, a decision maker is involved in the process and a single preferred solution is obtained. Starting with generating a wide spectrum of trade-off solutions to have a global understanding of feasible solutions, then concentrating on specific preferred regions for having a more detailed understanding of preferred solutions, and then zeroing on a single preferred solution with the help of a decision maker demonstrates the use of multiobjective optimization and decision making methodologies in practice. As a by-product, useful properties among decision variables that are common to the obtained solutions are gathered as vital knowledge for the problem. The procedures used in this paper are ready to be used to other similar real-world problem solving tasks.

1 Introduction

Energy is directly related to sustainable human development. Energy consumption affects social aspects (two billion people do not have access to modern energy supplies), and its production damages human health and alters the atmosphere causing the global warming, among other environmental impacts.

All the energy sources came, directly or indirectly, from the sun. Nowadays, there exist many technologies that use this enormous source of energy. Among them, solar thermal electricity is a very promising one that will contribute significantly to increase the electricity generation by renewable sources.

The conventional solar thermal electricity plants convert solar radiation into heat that is transferred to a heat transfer fluid. The transfer fluid generates steam that is expanded in a Rankine cycle. There are two commercially available types of plants for electricity generation in the order of MW: heliostats with a central receiver, and parabolic trough collectors. Both technologies are currently working in Spain in the "Plataforma Solar de Almería". While the central receiver type is working commercially in Spain only since 2007, the second one has been proved since the 80's and it is commercially available since 1984 in the USA. In Mills (2004), a review of the solar thermal electricity technology can be found.

Like in any other thermal machine, the efficiency of the cycle is higher when the temperature and pressure of the steam entering in the turbine increase. The limit to this temperature is given by the characteristics of the heat transfer fluid that is used in the collectors. Usually in parabolic trough plants, synthetic oils are used as a heat transfer medium between collectors and Rankine cycle. The limit to the temperature of the steam is 400^{o}C approximately. Important efforts are being carried out to generate directly the vapor in the collector field. This technology is called "direct steam generation" (DSG) and it is expected to reduce the cost of the solar electricity. The advantages of DSG are:

- To raise the limit to the operation temperature of the cycle, and consequently increase the efficiency.
- To diminish the initial cost of the plant because of the reduction of the number of heat transfer equipments.
- To reduce the auxiliary pumping cost.
- To eliminate the cost of reposition of the synthetic oil.
- To eliminate the risk of contamination due to losses in the oil circuit.

Obviously the main problem of the extension of thermal solar plants is the cost. They require very high investments and the electricity production cost is lower in conventional fossil fuel plants (if no internatilization of the external costs is performed). This situation has been modified in Spain since Royal Decree 661/2007 that guarantees an incentive to solar electricity. A maximum price of 34.3976 c€/kWh can be obtained for the solar electricity sold to the grid. Nowadays there are more than 500 MW under construction in Spain.

This paper analyses the optimal sizing of a DSG solar thermal electricity plant that is promoted by the private firms Endesa (`www.endesa.es`) and Solar Millenium (`www.solarmillenium.com`) in the framework of a collaborative project between German and Spanish enterprises and public research centers. The project is called GDV-500 Plus.

From the mathematical point of view, we want to determine the optimal size of the main components, in order to simultaneously optimize several economical and environmental objectives (profits, total investment costs, internal rate of return and global emissions). These objectives are in conflict to each other, thereby making the resulting multi-objective optimization task interesting. To this end, a multiobjective optimization model has been built. Examples of mathematical programming models for this kind of problems can be found in Doering and Lin (1979), where an integer optimization problem is built to determine the equipment operating configuration of a central energy plant, and in Triki *et al.* (2005), where an optimization model is presented that defines a multi-auction capacity allocation strategy which is optimal with the explicit representation of uncertainty.

There are a number of difficulties in solving the resulting optimization model, which we narrate next. First, the conflicting nature of multiple objectives considered here yields in a set of trade-off solutions. To find a suitable preferred optimal solution, it is, in general, better to first find a set of representative solutions from the entire trade-off frontier and then analyze the solutions to select a preferred solution Coello *et al.* (2002); Deb (2001). The traditional multi-objective optimization methodologies require one to solve parameterized version of the optimization problem repeatedly with different values of the problem parameters. Since this can be a tedious method to solve the parameterized optimization problem many times for different parameter values, in general, recently proposed evolutionary multi-objective optimization (EMO) methodologies, such as NSGA-II Deb *et al.* (2002) or SPEA2 Zitzler *et al.* (2001), are capable of finding a set of well-distributed trade-off solutions in a single execution of the algorithms. However, one of the major difficulties with our particular model is that, due to the complex nature of the system and legal regulations, different objective functions cannot be expressed as *explicit* functions of the decision variables. All objective functions depend on the operation schedule of the plant, which takes the form of a black box that has been modeled as an evaluation subroutine. This subroutine takes into account all the technical and legal requirements, in order to determine the working strategy of the plant and, as a result, the values of the objective functions. The problem is that the function implicitly defined by the subroutine, due to the nature of the process modeled, is not even continuous (hence not differentiable) and it has many local optima. To its worst, no mathematical structure of the objective functions can be exploited to help choose an appropriate optimization algorithm. This has made it impossible to solve the resulting problem using traditional optimization solvers (even able to handle non-convex global optimization problems), and this is another reason why we have chosen to use evolutionary approaches.

The rest of the paper is organized as follows. In Section 2, we describe the operation of the solar plant and the multiobjective model we have built to solve the problem. The use of EMO techniques in decision making is described in Section 3, and EMO is used to approximate the four-objective Pareto-optimal front of the problem in Section 4. In Section 5 we describe the EMO based reference point scheme used to solve the problem, and we present some results. Some conclusions learned from these results are presented in Section 6, while the approach to the final solution is outlined in Section 7. The paper ends with some generic conclusions from the study.

2 Solar Thermal Electricity Plant Model

In this section, we describe the solar thermal electricity plant model.

2.1 The DSG solar plant

Figure 1 shows the elements of the solar plant. In the solar field, solar radiation is converted into heat. The condensate that comes from the block of power (BOP) increases its temperature and pressure and it is again suitable to produce work. When the radiation level is insufficient to produce the required mass flow of steam, a thermal storage and auxiliary power system are disposed in parallel to produce the supplementary energy to the BOP. Thermal storage is designed to collect energy during daylight and dispatch it when necessary. This system increases the number of hours of operation of the plant. The auxiliary system is a gas boiler that is designed to maintain a minimum temperature in the plant in order to reduce start up periods, and to contribute to electricity generation.

Therefore, there are three main quantities to be dimensioned in the optimization process: the solar collector area, the storage capacity and the power of the auxiliary boiler.

2.2 Main assumptions

Due to the complex technical limitations of the plant, and in agreement with the organizations participating in the study, the following assumptions have been made on the component systems of the plant and on the operation strategy.

With respect to the solar collector field, it uses direct solar radiation. The steam mass generated has been considered to depend only on the direct solar radiation received and on the ambient

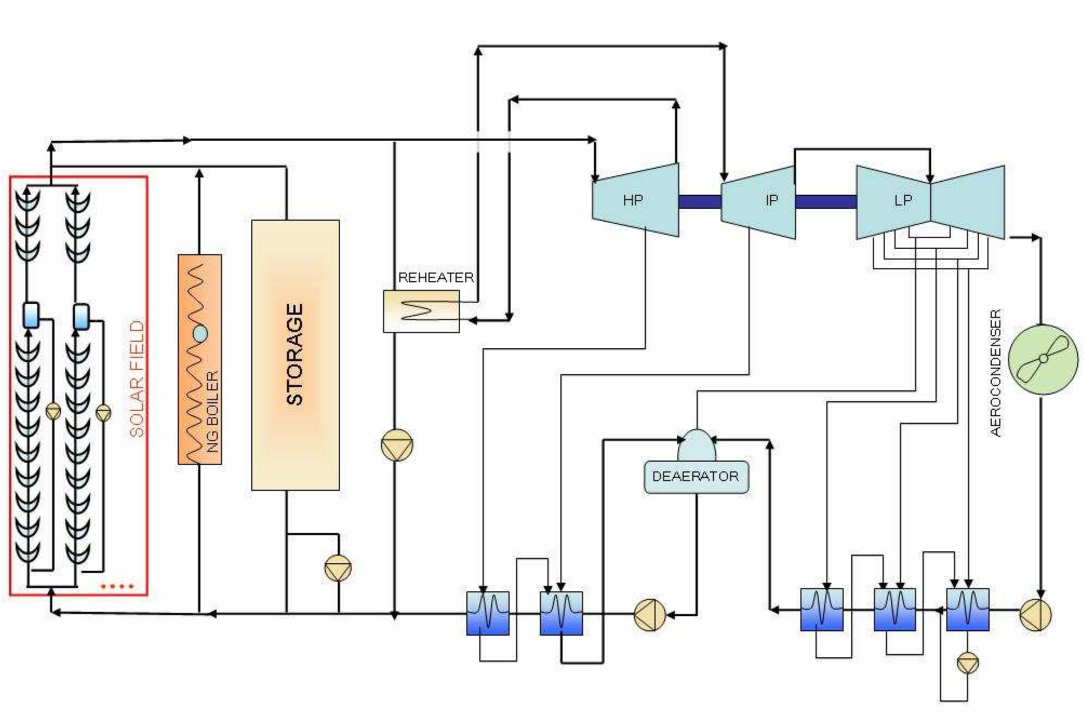

Figure 1: DSG solar plant modeled.

temperature. Therefore, based on a file of expected hourly solar radiation and temperature for the whole year, the steam mass flow produced per square meter at the solar field has been determined, and these data are used to feed the operation subroutine. Due to technical reasons, the maximum size of the solar field has been set to $750,000\,\mathrm{m}^2$.

The capacity of the storage is measured in terms of the number of hours that the tanks can provide the energy necessary to drive the block of power at full capacity. But a tank cannot be arbitrarily large. Therefore, whenever a tank reaches a maximum possible capacity (equivalent to 8 hours of storage), a new tank has to be built. This causes discontinuities in the costs function, given that every 8 hours of storage, the cost is incremented in 15 million € (the fixed cost of building a new tank). On the other hand, in order to account for ambient losses, the energy flow coming from the storage is multiplied by 0.9 if one tank is used, by 0.85 if two tanks are used, by 0.8 if three tanks are used, and so on.

On the other hand, the operation strategy affects the optimal size of the components of the solar plant. In this paper, the operation strategy has been defined in order to reproduce the complexity of the problem. The strategy defined is based on experience of operation of this kind of plants. In this paper, we will assume that a load fraction lower limit L is given by the decision makers. This means that the plant will never produce electricity under the L% of its full capacity. The firm did not want to consider any value of L greater than 75%. The operation for each hour can be summarized as follows

1. Evaluate direct solar radiation and temperature, and calculate the mass steam production with the collector field model.

2. If the mass flow is enough to activate the plant to at least a L% load fraction, the plant is producing electricity just with solar energy. If the mass flow exceeds the necessary amount for a 100% fraction, the remaining energy is stored.

3. In the case that the steam mass production does not reach the minimum value fixed before,

Variable	Description	Unit
A_C	Solar collector field size	m^2
E	Storage capacity	kJ
P_{AUX}	Power of the auxiliary boiler	kW
L	Lower load fraction limit	%

Table 1: Decision variables of the model.

the storage complements the required energy. This is only possible if there is enough energy already stored.

4. When the steam mass cannot be obtained with the solar collector field and the storage, the auxiliary boiler supplements the rest. Due to legal regulations, the overall yearly operation of the auxiliary boiler is limited to 15% of the net electricity production of the plant.

5. The collector field charges the storage system during daylight if L% of the gross power of the plant cannot be obtained with the previously described scheme.

6. If the plant has stopped for eight or more hours, then the next working hour is used just to re-start the system, and thus, the electricity produced is not sold to the grid.

Taking these assumptions into account, the operation model has been built as follows.

2.3 The operation model

As previously mentioned, the decision variables of the model are the sizes of the three main components of the central, as well as the value of the lower load fraction limit, as displayed in Table 1.

As for the constraints, the following simple bounds on the decision variables are considered:

$$\begin{aligned} 0 &\leq A_C \leq 750,000, \\ 0 &\leq E, \\ 0 &\leq P_{AUX}, \\ 0 &\leq L \leq 75. \end{aligned} \tag{1}$$

The rest of the constraints are in fact determined by the operation scheme of the plant, which is in turn influenced by technical and legal regulations. These requirements cannot be expressed as explicit mathematical functions of the decision variables. In order to model the operation scheme for each value of the decision variables, the following subroutine (which contains all the assumptions described in Section 2.2) must be run.

2.4 Operation subroutine

In this section, we will outline the main steps of the operation subroutine, which has been implemented in C++ language, in order to compile it together with the solver. This way, the subroutine is called every time the solver needs function evaluations. In summary, once the values of the decision values are set, the subroutine determines the operation strategy of the plant for each of the 8,760 hours of the year. To this end, the subroutine creates a series of variables that define the operation strategy, as displayed in Table 2. Variable $FUNC_i$ indicates the load fraction at hour i, and thus it can be equal to 0 if the system does not work, or any value between L and 100.

Let us now describe the evaluation subroutine step by step. Let us assume that certain values of the decision variables, A_C, E, P_{AUX} and L are given. Then, we proceed in the following way.

Variable	Description	Unit
E_i	Energy stored after hour i	kJ
$FUNC_i$	Load fraction of hour i	%
$EAUX_i$	Energy generated by the auxiliary system in hour i	kJ
$PERC_i$	Accumulated percentage of energy generated by the auxiliary system until hour i	%

Table 2: Operation strategy related variables (here, $i = 1, \ldots, 8760$)

1. **Initial calculations.** Given the value of E,
   (a) Calculate the number of tanks to be installed, by dividing E by the maximum capacity of a tank.
   (b) Determine the performance of the tanks, which depends on the number of tanks installed, as described in Section 2.2.
   (c) The number of tanks also influences the amount of soil that has to be used for the plant. Namely, for any new tank starting from the third one, a supplementary amount of soil has to be considered.
2. **Operation loop**. The operation strategy has to be determined now, according to points 1–5 of Section 2.2. Namely, for each hour of the year, we determine the load fraction of the plant, in the following way.
   (a) The direct solar radiation and the temperature of hour i are read from the data file, and the steam mass per square meter is calculated accordingly. This value is multiplied by A_C to obtain the total steam mass of the hour.
   (b) If the steam produced at the solar field is enough for a 100% fraction, $FUNC_i$ is given the value 1 (100%), and the remaining energy is added to the previously stored amount, and accounted for in variable E_i. This value can never exceed the total storage capacity given by the decision variable E. The auxiliary system is not used.
   (c) If the steam mass provides a power between L% and 100%, the plant works at the highest possible load fraction (this is the value given to $FUNC_i$), with no aid from the storage or from the auxiliary system.
   (d) If the steam mass generated at the solar field does not suffice for a L% fraction, then several situations can occur:
      i. If there is enough energy stored to reach the L% load fraction, then the necessary amount is taken from the tanks, E_i is actualized accordingly, $FUNC_i$ is set to L, and the auxiliary system is not used.
      ii. If there is not enough energy stored, we need to complement the rest from the auxiliary system. In order to do this, the two following conditions must hold:
         - The installed capacity of the auxiliary system (given by decision variable P_{AUX}) must be enough to produce the required energy.
         - The accumulated (up to hour i) percentage of energy supplied by the auxiliary system cannot exceed the limit (15%).

         If any of these two conditions fail, then the system does not work at hour i. Therefore, the energy produced at the solar field is stored, E_i is actualized accordingly, and $FUNC_i$ is set to 0.

If the two conditions hold, then the storage is emptied ($E_i = 0$), the value of $EAUX_i$ is the energy supplied by the auxiliary system at this hour, and $FUNC_i$ is set to L.

(e) The accumulated hybridization percentage $PERC_i$ is actualized, depending on the values of $EAUX_i$ and $FUNC_i$.

Once these calculations are completed, the subroutine goes back to point a) for the next hour.

The global scheme of the subroutine can be seen in the flowchart displayed in Figure 2.

2.5 The Multi-Objective Model

Originally, the decision makers wanted to maximize the expected yearly return of the plant. Therefore, the first model we solved was the following:

$$\begin{aligned} \text{maximize} \quad & PRO(A_C, E, P_{AUX}, L), \\ \text{subject to} \quad & 0 \leq A_C \leq 750,000, \\ & 0 \leq E, \\ & 0 \leq P_{AUX}, \\ & 0 \leq L \leq 75, \end{aligned} \tag{2}$$

where PRO is the profit function. Broadly speaking, $PRO = I - C$, where I are the expected incomes obtained by selling the electricity, and the costs C include installation, maintenance, fuel, insurance and contingency costs. More precisely, once the operation subroutine has been run, the profits are calculated as follows:

1. The incomes corresponding to the hour i are calculated according to the value of $FUNC_i$ and to the selling price p_i, and taking into account the re-start hours after long stops.
2. The global yearly incomes are calculated as the sum of the 8,760 hourly incomes.
3. The installation costs comprise the costs of the solar panels, the tanks, the auxiliary system (these three depend on the values of the decision variables), the block of power and the soil. All these costs are annualized taking into account given life cycle and discount rate.
4. Annual maintenance costs are assumed to be a fixed percentage of the total installation costs.
5. The fuel cost (for the auxiliary system) has a fixed monthly component and a variable component which depends on the corresponding values of $EAUX_i$.
6. Insurance and contingency costs are also assumed to be fixed percentages of the total installation costs.

In an earlier study Cabello *et al.* (2009), we employed a continuously updated single-objective genetic algorithm (GA) to solve the above optimization problem. The variables are used as real numbers and a real-parameter GA with simulated binary crossover (SBX) operator and polynomial mutation operator Deb and Agrawal (1995); Deb (2001) was used. After every 50 generations, the variable bounds were changed so as to reinitialize the population around $[x_i^* - \sigma_i, x_i^* + \sigma_i]$, where x_i^* is the i-th variable value of the population-best solution after 50 generations and σ_i is the standard deviation of population members for the i-th variable after 50 generations. The GA was terminated when two consecutive updates of bounds did not result in any significant improvement in the best-found solution.

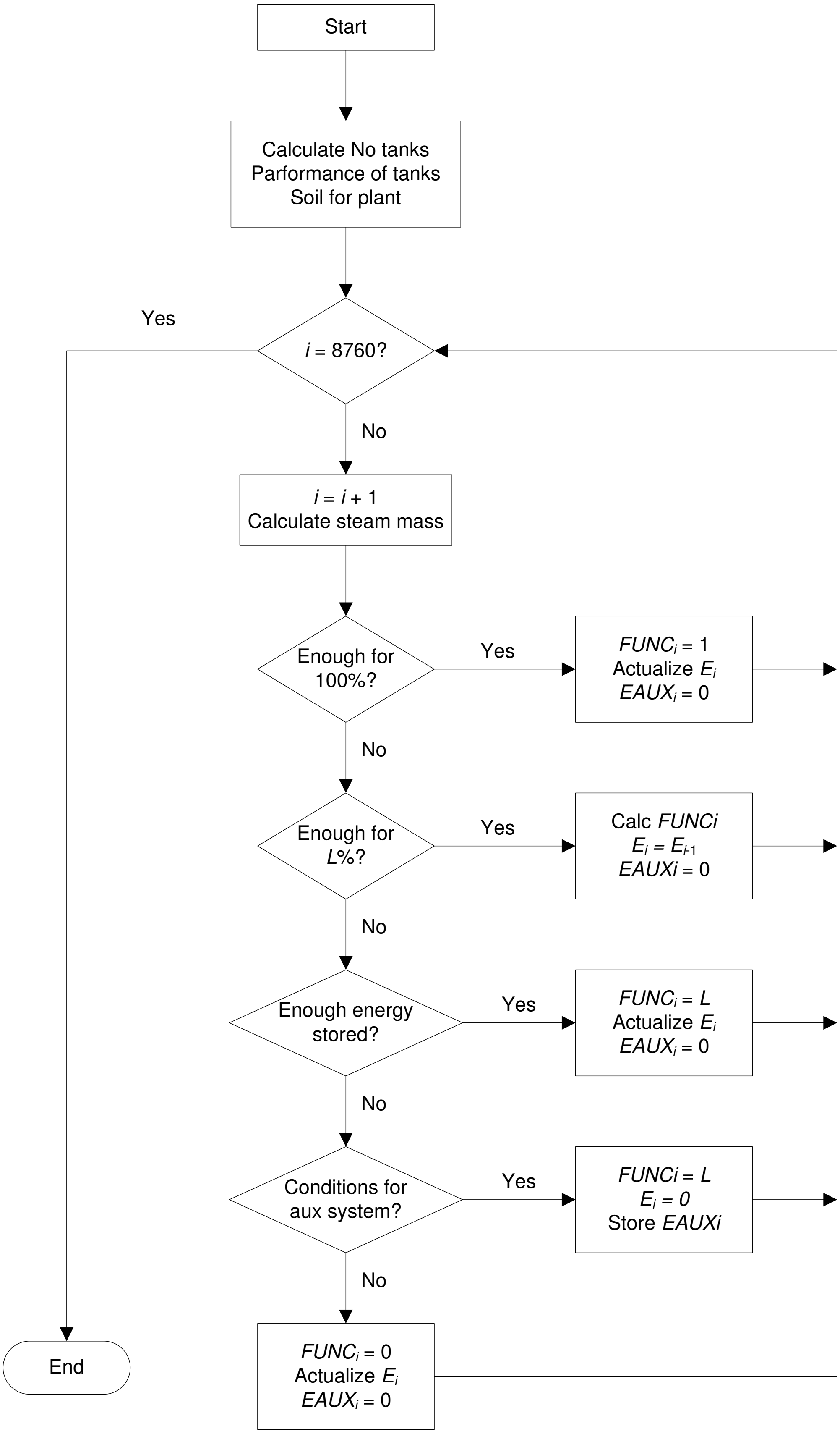

Figure 2: Flowchart of the operation subroutine.

The optimal solution of problem (2), found by the above-mentioned GA, is the following:

$$\begin{array}{ll} A_C = 749,541.16 & E = 6,624,719,730.57 \\ P_{AUX} = 105,021.57 & L = 74.99 \end{array} \quad (3)$$

At this solution, the optimal profit is 27,796,339.23 €. Therefore, both the size of the solar field and the limit L are in practice at their corresponding upper bounds. The value of E corresponds to 15.68 hours stored (that is, nearly 2 full capacity tanks). The optimal value of P_{AUX} is the size of the auxiliary system that allows to produce electricity at a 75% load fraction when no steam (coming from the solar field or from the storage) is available. That is, it is profitable to produce electricity only with the auxiliary system. This is why the legal 15% hybridization limit is reached at this solution. Problems with the optimal profit solution are the following:

- The total investment costs (that is, considering the costs of the solar field, the tanks, the auxiliary system, the block of power and the soil) are really high (406,769,471 €). The firm would like to look for other cheaper options.
- The internal rate of return (IRR) at this solution is 13.32%. Again, we and the firm felt that this IRR can be made better if the costs are decreased.
- The auxiliary system (that works with natural gas) is the only part of the plant that generates direct polluting emissions. At this solution, the auxiliary system is dimensioned and used at its maximum legal capacity, and thus, the pollution can also be decreased if smaller auxiliary systems are considered.

The above-mentioned issues are facts with most real-world system. Usually, no real-world system can be considered effectively from a single perspective. When the profit is maximized, a solution is achieved with a sacrifice from some other important considerations, such as costs, pollution, etc. For these reasons, in this paper, we have decided to carry out a multiobjective study of the plant, considering, apart from the yearly profit, the other three objectives mentioned above: total investment costs (TIC), internal rate of return (IRR), and pollution (POL). Given values of the decision variables, the corresponding values of these three objectives can be easily derived making use of the data of the operation subroutine. Namely, TIC is obtained by adding the costs of the three components, the block of power and the soil; IRR is the annual interest rate that makes financially equivalent the yearly incomes and all the costs (investment and operation costs). Finally, for POL, we have used as an estimate the energy produced in the auxiliary system, that is $\sum_{i=1}^{8760} EAUX_i$. This value has been divided by 3,600 for convenience. Therefore, the resulting four-objective multiobjective problem is the following:

$$\begin{array}{ll} \text{maximize} & PRO(A_C, E, P_{AUX}, L), \\ \text{minimize} & TIC(A_C, E, P_{AUX}, L), \\ \text{maximize} & IRR(A_C, E, P_{AUX}, L), \\ \text{minimize} & POL(A_C, E, P_{AUX}, L), \\ \text{subject to} & 0 \leq A_C \leq 750,000, \\ & 0 \leq E, \\ & 0 \leq P_{AUX}, \\ & 0 \leq L \leq 75. \end{array} \quad (4)$$

Before facing the resolution of the multiobjective problem, we have individually optimized the new three objective functions, in order to investigate the degree of conflict among the different functions. The most interesting findings of these optimizations are the following:

- Obviously, the optimal value of the total investment function (TIC) takes place when there is no activity, that is, all the decision variables are zero, making it a trivial solution. In general, we found out that the cost of the tanks is the greatest component of the overall cost, and thus, the best results for TIC are obtained for low values of E. This also implies that lower values of L are better for the cost, because there is no need to store that much energy.

- When optimizing the IRR, we get the following result:

$$\begin{array}{ll} A_C = 295,437,32 & E = 0 \\ P_{AUX} = 50,385.36 & L = 25 \end{array} \tag{5}$$

  Therefore, the optimal IRR (which equals 16.16%) is obtained when no energy is stored, which, as previously mentioned, reduces costs significantly. This result brings the profits down to 13,734,685.95 € (about 50% less), while the investment costs are down to 145,214,222.30 € – a whopping 64.3% reduction from maximum-profit solution. The pollution also takes a reduced value: 69,697,697.25 €. Nevertheless, after a discussion with the firm, we found that the profit for this solution was too low. We also observed that, if we wish to store energy for higher profits, there are a series of local optima for IRR, with values around 14.84%, which can be obtained for different values of L. In these cases, the profit can be over 23 million Euros.

- Finally, when the pollution function is minimized, the resulting solution causes the pollution to attain its minimal value. At this solution, the auxiliary system is not suggested to be used. This situation can occur at two different situations: for small plants and small values of L, where the auxiliary system is not installed (which corresponds to small profits), or when the storage is big enough so that the auxiliary system is never needed (which usually yields to higher profits, but also higher costs).

These individual objective considerations provided us with extreme trade-off information among the four objectives and matched our expectations. Now, we employ an EMO methodology to find intermediate trade-off solutions that will provide varying degrees of importance to each of the four objectives.

3 Evolutionary Multi-Objective Optimization (EMO) and Decision Making

Multi-objective optimization problems involving conflicting objectives give rise to a set of efficient (or Pareto-optimal) solutions Deb (2001). Each such solution is a potential candidate for implementation, but exhibits a trade-off between the objectives. Many classical methods require a priori preference information and then optimize a preferred single objective version of the problem Miettinen (1999). If different efficient solutions are needed to investigate the effect of different preference values before choosing a final solution, such a classical approach requires to be applied again and again. Studies have shown that such independent optimizations may be computationally expensive in complex problems Shukla and Deb (2005).

However, evolutionary algorithms are ideal for such problem solving tasks. This is because the EA population can be used to store different efficient solutions obtained in a single simulation run. A number of efficient methodologies exist for this purpose Deb (2001). Here we use a commonly-used procedure: NSGA-II Deb *et al.* (2002). A code implementing NSGA-II is available at $http://www.iitk.ac.in/kangal/codes.shtml$ and is employed here.

Popularly used elitist Non-dominated Sorting Genetic Algorithm (NSGA-II) has shown to have a good convergence property to the global Pareto-optimal front as well as to maintain a good diversity of population members on the Pareto-optimal front for two and three objective problems. A detailed description of NSGA-II can be found in Deb *et al.* (2002). In short, NSGA-II is a population based evolutionary optimization procedure which uses a non-domination sorting of population members to emphasize non-dominated solutions systematically and a crowding distance scheme to emphasize isolated population members in every iteration, maintaining a widely distributed set of solutions in the objective space. An elite-preserving procedure also ensures inclusion of previously found better solutions to further iterations for faster convergence. The overall procedure with N population members has a computational complexity of $O(NlogN)$ for two and three objectives problems and has been popularly used in many studies.

Although NSGA-II produces reasonably good results for two and three objective problems, recent studies Deb and Saxena (2006) have discovered that NSGA-II faces difficulty in solving problems with a large number of objectives. Some major difficulties are as follows: (i) an emphasis of population members based on domination principle causes a major fraction of the population members to be non-dominated to each other, thereby stalling the search, (ii) the crowding distance operator used within NSGA-II is not able to maintain a diverse set of solutions in the objective space, (iii) an exponentially large population size is needed to represent a higher-dimensional Pareto-optimal front, thereby demanding a large computational burden. Although these aspects had been one of the recent challenges of EMO research, here we address the difficulty with crowding distance operator and use a clustering strategy Deb *et al.* (2005) to alleviate the diversity maintenance problem of crowding distance operator in a four-objective problem. On the flip side, the clustering operator increases the time complexity slightly, but it has been demonstrated that on problems having more than two objectives the distribution becomes better with the clustering approach Deb *et al.* (2005); Deb (2001).

Following is how the clustering algorithms works. Let us assume that, at a given generation, the number of rank r members is n. If, $k < n$ members with rank r are to be added to the population, the procedure is the following:

1. Initially each point represents a cluster centroid so there are n clusters to start with.
2. Compute the distances between all the centroids and add the two closest clusters together. This reduces the number of clusters by 1.
3. Recompute the centroid of the new cluster formed by adding the two clusters in step 2.
4. If the number of clusters is equal to k then go to step 5 otherwise go to step 2.
5. Choose the point within the cluster which is at minimum distance from the centroid as the representative of the cluster.
6. Add all the k representative members to the parent population and ignore the rest of the members.

4 Four-Objective Front and Verification

At the first step of the solution process, we apply NSGA-II algorithm to the four-objective optimization problem (4) to find a set of four-objective efficient solutions. NSGA-II parameters used in this study are as follows: Population size is 50, SBX recombination probability is 0.9, polynomial mutation probability is 0.25 ($1/n$ where n is the number of variables), and distribution indices for recombination and mutation are 10 and 20, respectively. Figure 3 shows the solutions obtained (in shaded circles) in a three-objective plot with f_1, f_2 and f_4. The points appear to represent a non-convex surface. It is interesting to note how the NSGA-II procedure with the clustering mechanism is able to find and maintain a well-distributed set of efficient solutions. To ensure that obtained four-objective solutions are close to true optimal solutions, next we perform several independent bi-objective optimizations. By taking each pair of objectives (in total $\binom{4}{2}$ or six two-objective problems), we employ the above NSGA-II to find the respective trade-off frontier. Thereafter, the solutions from these fronts and solutions obtained earlier from four individual single-objective optimizations are all plotted together in Figure 3. Except the f_4-f_3 optimization results, all other pair-wise fronts seem to lie on the boundary of the four-objective NSGA-II front. The f_4-f_3 front lies on the intermediate part of the three-dimensional frontier. For a three-objective plot involving f_1, f_2 and f_4, all two-objective optimizations and single-objective optimizations are theoretically supposed to produce boundary solutions. We observe a similar phenomenon in this problem. Interestingly, the single-objective solutions seem to match the extreme points of the four-objective NSGA-II front obtained. The agreement of solutions obtained in different optimizations gives us confidence on the closeness of the four-objective NSGA-II front to its true front.

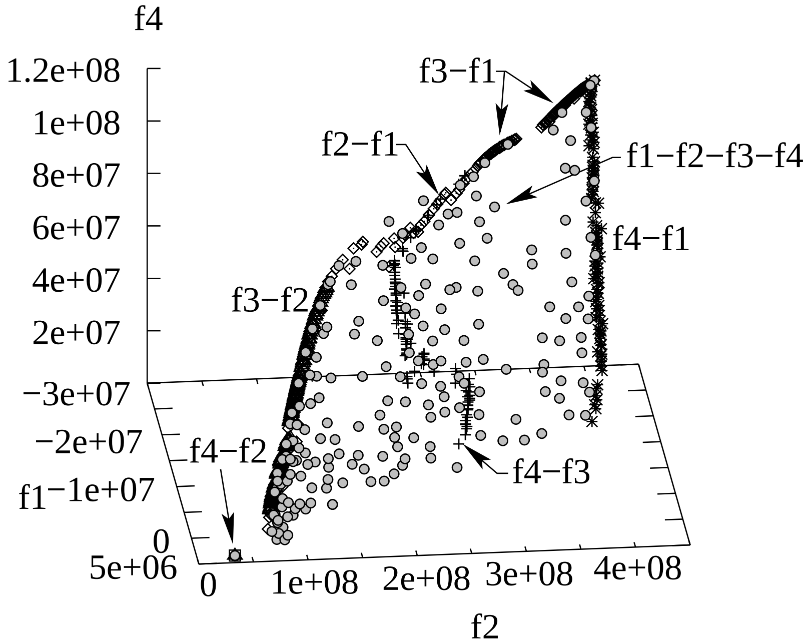

Figure 3: Four-objective NSGA-II points are shown in f_1-f_2-f_4 space.

From the extreme points of the front obtained, we construct the ideal and nadir point as $\mathbf{z}^* = (27,796,339.23;\ 35,000,000.00;\ 0.161627;\ 0.00)^T$ and $\mathbf{z}^{\rm nad} = (-3,620,836.00;\ 406,769,470.88;\ 0.003522;\ 113,2$
Note that the first and third objectives were maximized and the second and fourth objectives were minimized in this problem. It is interesting to note also that the efficient solutions possess a wide variation in all objective values, making the problem an interesting one to study.

Next, we consider the four-objective NSGA-II solutions and investigate pair-wise trade-offs among the objectives to get a better insight to the problem. Figure 4 shows the plot of f_1 and f_2. Since f_1 is maximized and f_2 is minimized, the solutions seem to produce a good trade-off between these two objectives. Next, we plot the four-objective NSGA-II solutions on a f_1-f_3 space in Figure 5. It is interesting to note that since both objectives are maximized, there is not much of a trade-off between these two objectives for a profit smaller than around $5(10^6)$. Thus, for smaller profit margin (or for an internal return rate smaller than 0.1), we conclude that an increase in internal return rate will cause into a monotonic increase in profit in running the solar plant. However, a trade-off scenario occurs for a larger profit solution or for a larger internal return rate situation. The plot in f_1-f_4 space (Figure 6) shows a wide-ranging trade-off among the objectives.

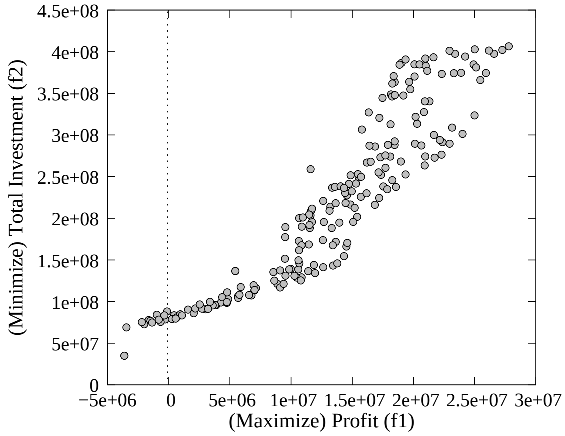

Figure 4: Four-objective NSGA-II points are shown in f_1-f_2 space.

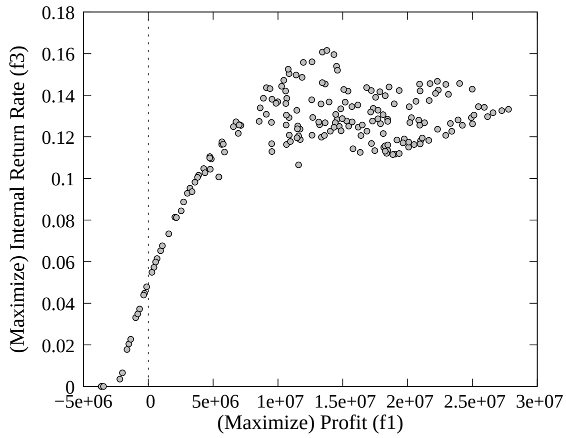

Figure 5: Four-objective NSGA-II points are shown in f_1-f_3 space.

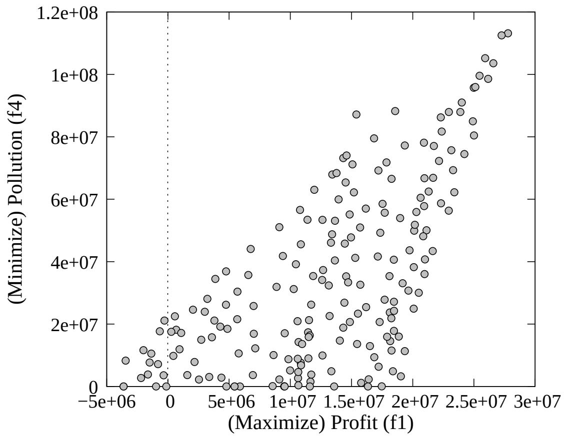

Figure 6: Four-objective NSGA-II points are shown in f_1-f_4 space.

These investigations provide interesting insight to the solar power plant problem. We shall discuss further about useful insights in Section 6. However, for now we shall discuss an important aspect of multiobjective optimization – selection of a specific solution based on some preference information.

5 The EMO based reference point scheme

In the above four-objective problem, although we have managed to find a good set of trade-off solutions using a clustered NSGA-II procedure which was supported by lower-dimensional optimization results, there are certain difficulties associated with optimization and decision making for a four or more objective problem. First, a four-dimensional front requires a large number of solutions to be found. Second, even if a distribution of points is obtained by a suitable algorithm, there may not be enough points present in the vicinity of the desired part of the efficient frontier.

Third, a visualization of the efficient front becomes a difficulty with more than three objectives. In order to overcome these problems, one approach is to employ a multiple criteria decision making (MCDM) technique within an EMO to identify preferred solutions directly instead of first finding a representative set from the entire trade-off frontier and then make a decision of a preferred solution. This will avoid finding the entire frontier and help the decision-maker to find more solutions directly in the region of interest to him or her. Therefore, at sight of the results obtained in the previous section, it was decided to solve the multiobjective problem using a reference point scheme, suggested elsewhere (Wierzbicki, 1980). In this scheme, the decision makers are required to supply values they consider desirable for each objective function (reference levels), and an efficient solution is determined by minimizing a so-called achievement scalarizing function $s()$. The most basic example of such a function for minimization of p objective functions is:

$$s(\mathbf{f}(\mathbf{x}), \boldsymbol{\mu}, \mathbf{z}) = \max_{i=1,\ldots,p} \{\mu_i(f_i(\mathbf{x}) - z_i)\}, \tag{6}$$

where $\mathbf{x}$ is the vector of decision variables, $\mathbf{f} = (f_1, \ldots, f_p)$ are the objective functions, $\mathbf{z} = (z_1, \ldots, z_p)$ are the reference levels, and $\boldsymbol{\mu}$ is a vector of weights which can have just a instrumental (normalizing) role, or a preferential interpretation regarding the importance of achieving each reference level (see Luque *et al.*, 2007; Ruiz *et al.*, 2008, for further details). This achievement function is minimized over the feasible set to obtain an efficient solution.

The evolutionary algorithms used have enabled us to handle the problem, due to the complex nature of the functions involved in it. In this section, we will first describe the reference point based EMO procedure used, and then discuss two different approaches adopted here. In the first approach, we use an EMO procedure to find a set of preferred solutions (instead of the entire Pareto-optimal front) dictated by the reference point concept. In the second approach, an achievement scalarizing function is built from supplied reference levels and solved using a single-objective evolutionary algorithm (an EMO is employed here). These two approaches are compared at the end of this section.

5.1 EMO Aided with Decision-Making

As mentioned before, the use of clustering approach within the NSGA-II solves one of the problems involved in solving many objective problems. The issue of using domination principle and visualization difficulties are still bothersome to a practitioner. One remedy in such cases is to focus on a particular region of the Pareto-optimal front, rather than finding the entire Pareto-optimal front. In this section, we attempt to achieve this task by using the reference point concept.

In many objective problem-solving tasks, EMO methodologies can be put to benefit in finding a preferred and smaller set of Pareto-optimal solutions, instead of the entire frontier. This approach has a practical viewpoint and allows a decision-maker to concentrate only to those regions on the Pareto-optimal frontier which are of interest to her/him. EMO methodologies may provide an advantage over their classical counterparts for another pragmatic reason. Some classical interactive multi-criterion optimization methods demand the decision-makers to suggest a reference direction or reference points or other clues Miettinen (1999) which result in a preferred set of solutions on the Pareto-optimal front. In these classical approaches, based on such clues, a single-objective optimization problem is usually formed and a single solution is found. A single solution (although optimal corresponding to the given clue) does not provide a good idea of the properties of solutions near the desired region of the front. By providing a clue, the decision-maker is not usually looking for a single solution, rather she/he is interested in knowing the properties of solutions which correspond to the optimum and near-optimum solutions respecting the clue. This is because while providing the clue in terms of weight vectors or reference directions or reference points, the decision-maker has simply provided a higher-level information about her/his choice. Ideally, by providing a number of such clues, the decision-maker in the beginning is interested in choosing a region of her/his interest. We here argue that instead of finding a single solution near the region of interest, if a number of solutions in the region of interest are found, the decision-maker will be able to make a better and more reliable decision. Moreover, if multiple such regions of interest can be found simultaneously, decision-makers can make a more effective and parallel search towards finding an ultimate preferred solution.

In this paper, we use the concept of reference point methodology in an EMO and attempt to find a set of preferred Pareto-optimal solutions near the regions of interest to a decision-maker. We use the reference point based NSGA-II suggested in Deb *et al.* (2006) for this purpose.

The main ideas behind choosing the preferred set of solutions in the reference point based NSGA-II are as follows:

1. Solutions *closer* to the reference points (in the objective space) are to be emphasized more.

2. Solutions within an ϵ-neighborhood to a near-reference-point solution are de-emphasized in order to maintain a diverse set of solutions near each reference point.

The following update to the original NSGA-II niching strategy is performed to incorporate the above two ideas:

1. For each reference point, the normalized Euclidean distance of each solution of the front is calculated and the solutions are sorted in ascending order of distance. This way, the solution closest to the reference point is assigned a rank of one.

2. After such computations are performed for all reference points, the minimum of the assigned ranks is designated as the preference distance to a solution. This way, solutions closest to all reference points are assigned the smallest preference distance of one. The solutions having next-to-smallest Euclidean distance to all reference points are assigned the next-to-smallest preference distance of two, and so on. Thereafter, solutions with a smaller preference distance are preferred in the tournament selection and in forming the new population from the combined population of parents and offspring.

3. To control the extent of obtained solutions, an ϵ-clearing idea is used in the niching operator. First, a random solution is picked from the non-dominated set. Thereafter, all solutions having a sum of normalized difference in objective values of ϵ or less from the chosen solution are assigned an artificial large preference distance to discourage them to remain in the race. This way, only one solution within an ϵ-neighborhood is emphasized. Then, another solution from the non-dominated set (and is not already considered earlier) is picked and the above procedure is performed.

The above procedure provides an equal emphasis of solutions closest to each reference point, thereby allowing multiple regions of interest to be found simultaneously in a single simulation run. Moreover, the use of the ϵ-based selection strategy ensures a spread of solutions near the preferred Pareto-optimal regions.

5.2 Reference Point Based NSGA-II

The previous results have allowed us to identify two different zones in the efficient set. One zone consists of solutions having a large profit, but having a large investment cost, large pollution and small IRR. Another zone consists of solutions having a relatively small profit, but low investment cost, low pollution and high IRR. Considering that we are interested in capturing these two sets of solutions, we use two reference points in our study – one from each of the two zones described above, as follows: $(\mathbf{z}^{(1)} = (25(10^6); 300(10^6); 0.13; 50(10^6))^T$ and

$\mathbf{z}^{(2)} = (5(10^6); 150(10^6); 0.13; 20(10^6))^T)$. Next, we employ the reference-point based NSGA-II approach Deb *et al.* (2006) to obtain solutions simultaneously for both reference points[1]. The solutions obtained are represented by shaded circles in Figure 7, and the reference points are marked using a diamond. For an adequate visualization, we also show the four-objective NSGA-II front on the three-objective space in the figure. It is interesting to note that the reference-point based NSGA-II is able to find two distinct preferred regions near the specified aspiration points ($\mathbf{z}^{(1)}$ and $\mathbf{z}^{(2)}$). Since such an optimization task does not spend its search on the entire Pareto-optimal front, instead concentrates near the reference points, this approach is much more computationally efficient than the original NSGA-II procedure.

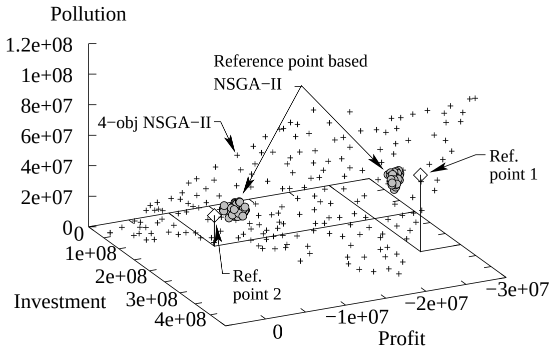

Figure 7: Efficient points obtained with the reference point based NSGA-II approach are shown in f_1-f_2-f_4 space.

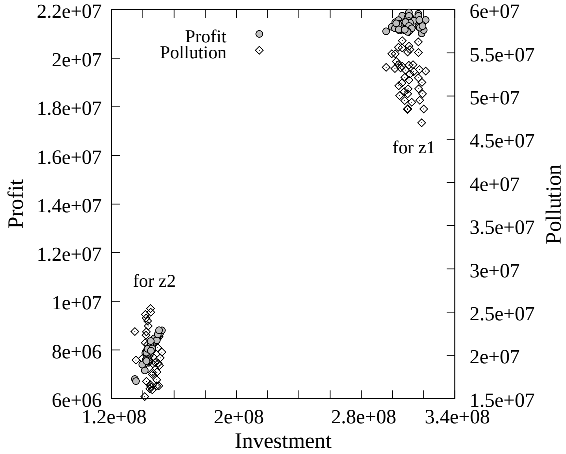

Figure 8: Pair-wise interactions of the objectives are shown.

In Figure 8 we show the reference point based NSGA-II results between two pairs of objectives. It is clear that both profit (f_1) and pollution (f_4) objectives are similarly related with investment made. For a higher total investment, better profit is possible, but at the expense of more pollution. It is clear that the efficient solutions near the first reference point cause more profit arising from a large investment, but expected pollution level is also high. On the other hand, the second reference point causes a smaller profit, but demanding a smaller investment and resulting in also a smaller pollution level. The trade-off between these objectives may eventually help the DM to choose one of the two preferred regions and eventually a single preferred solution from the corresponding set.

5.3 Comparison with Generative Reference Point Based Approach

In the previous section, we showed how a reference point based NSGA-II is able to find near Pareto-optimal points close to the desired reference points. Since a decision-maker may not be able to deal with too many obtained solutions, we need to choose a handful (say, less than 10 points) of well-distributed set of points from the obtained NSGA-II set. Here, we rerun the reference point based NSGA-II till the following termination criterion is obtained: the KKT error (an error in satisfying KKT optimality conditions, often used in terminating an optimization run) is less than 0.001. KKT error is determined by computing the gradient of the objectives numerically. After the optimization task is performed, each solution is close to a respective theoretical optimal solution. Then, we use the k-mean clustering methodology Zitzler and Thiele (1999) and identify

[1]It is important to highlight here is that reference point based NSGA-II employed here is capable of considering two or more reference points simultaneously. This is another advantage of using an EMO approach with a MCDM concept.

nine well-distributed points from the set. These points are identified from our simulation and are shown in Figure 9. This process takes a total of 37,000 solution evaluations.

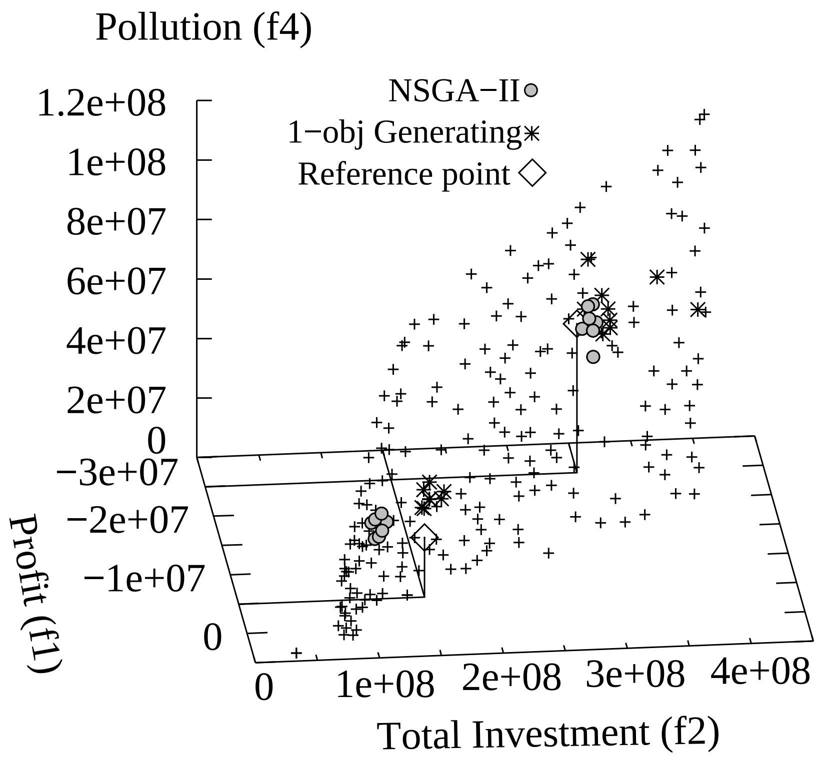

Figure 9: Comparison of solutions obtained by the reference point based NSGA-II and by a generating method using a single-objective GA solving the ASF problem with nine different weights one at a time.

The nine solutions obtained for each reference point is now compared with nine solutions to be obtained by a generating method. Since the achievement scalarizing function requires a weight vector ($\boldsymbol{\mu}$), we use nine different weight vectors and solve the corresponding achievement scalarizing function (6). For this purpose, we use three different values for the weight w_i for the i-th objective – low (0.2), medium (0.5) and high (0.8). Thereafter, we use the following L_9 fractional factorial design idea to create nine, four-objective weight vectors (shown in Table 3). These nine weight vectors are used one at a time from each of the two reference points and the ASF problem is solved using the single-objective RGA Cabello *et al.* (2009). The following termination condition is used. If the normalized change in the achievement function value in the last 20 generations is less than 0.0001 and the KKT error is less than 0.001, then the algorithm is terminated. The results are shown in Figure 9. Although the solutions for the first reference point are close to those found by the reference point based NSGA-II, the solutions for the second reference point are somewhat different. Most of the solutions obtained by the generative single-objective RGA runs are dominated by the reference point based NSGA-II solutions. This can be explained as follows. All NSGA-II solutions have $E = 0$. If E is changed to $E = 0.000001$ in these solutions, we end up getting solutions that are very close to those obtained by the generative single-objective RGA Modified RGA method. That is, if E is exactly made equal to 0, it improves the result. In this sense, reference point based NSGA-II is able to find better solutions (with $E = 0$) compared to the generative single-objective RGA.

The total solution evaluations (including the ones needed to compute the KKT error) in the

Table 3: Nine weight vectors using L_9 fractional factorial design concept.

	f_1	f_2	f_3	f_4
$\mathbf{w}^{(1)}$	low	low	low	low
$\mathbf{w}^{(2)}$	low	medium	medium	medium
$\mathbf{w}^{(3)}$	low	high	high	high
$\mathbf{w}^{(4)}$	medium	low	medium	high
$\mathbf{w}^{(5)}$	medium	medium	high	low
$\mathbf{w}^{(6)}$	medium	high	low	medium
$\mathbf{w}^{(7)}$	high	low	high	medium
$\mathbf{w}^{(8)}$	high	medium	low	high
$\mathbf{w}^{(9)}$	high	high	medium	low

two approaches are as follows:

1. Single-objective RGA: $335,592$
2. Reference point based NSGA-II: $69,660$

Hence, we clearly see that besides the advantage in solution accuracy, the number of solution evaluations required by the multi-objective NSGA-II approach are far less than those required by the generative single-objective RGA approach.

Although, both methodologies produce high quality solutions close to the true Pareto-optimal solutions, the parallel processing of solutions by the reference point based NSGA-II seems to find solutions much faster than the usual generating method. Based on these results, we recommend the use of reference point based NSGA-II for similar tasks.

6 Innovization: Unveiling important properties

Previous studies have indicated the importance of a post-optimality task by which important properties of optimal efficient solutions can be deciphered to obtain a better knowledge of high-performing solutions in a problem. In this case, we investigate pair-wise interactions of objectives with decision variables for the solutions obtained by the reference point based NSGA-II and unveil some interesting properties:

1. As shown in Figure 10, for both reference points, the variable A_C and objective f_2 (total investment) follows a linear relationship. Interestingly, at both cases, the slope of the linear relationship is identical to 0.00275. To lower the total investment cost for the plant, it is better to reduce A_C linearly. Besides, it was also found that E is the variable with the greatest impact on the investment cost.

2. However, the variable P_{AUX} is more or less constant among the solutions of the same preferred region (see Figure 10). Since it is profitable to use the auxiliary system, the variable P_{AUX} is adjusted to a particular value in order to produce as much power as possible through auxiliary means. For the two given reference points, it seems that the corresponding solutions can be characterized separately from their P_{AUX} values. Also, for a smaller investment cost, a smaller value of P_{AUX} is desired and vice versa.

3. Figure 11 shows the variation of decision variables E and L for different f_2 values. It seems that the preferred region near $\mathbf{z}^{(1)}$ is constant at the upper boundary of L (= 75 chosen

here) and at a reasonably high value of E. Thus, a high profit (resulting from a high total investment cost) requires that the variable L be set at its highest possible value and E is set to a large value. It is now clear that a recipe to arrive at large profit solutions is to keep P_{AUX}, L and E values constant at specific large values. The first two variables take their allowed largest values and the variable E gets set to around $3.36(10^9)$. The only way the solutions make a significant trade-off among the objectives is by using different A_C values – linearly requiring a larger A_C value for a larger profit.

Although a smaller E and L must be chosen for smaller profit solutions, interestingly unlike in large profit solutions here solutions may take different E and L values.

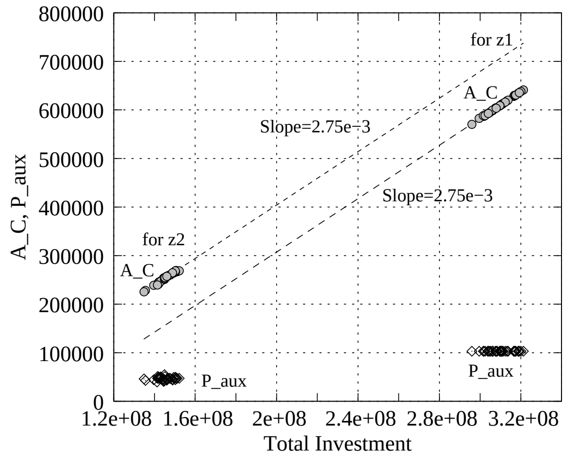

Figure 10: Variable A_C vary linearly with f_2, while P_{AUX} remains more or less constant near the Pareto-optimal region for the chosen reference points.

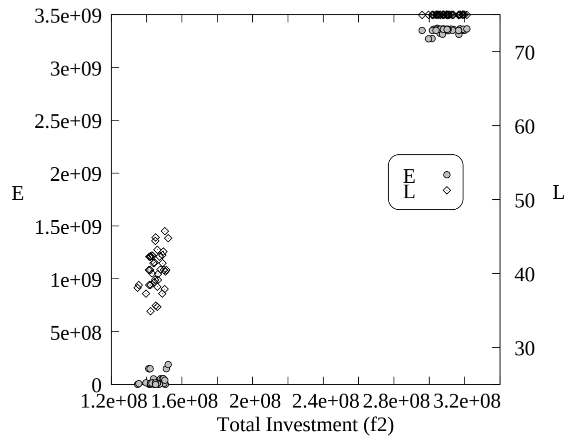

Figure 11: Although E and L are constant for reference point $\mathbf{z}^{(1)}$, they vary for $\mathbf{z}^{(2)}$, but distinct E and L values for the two reference points are evident.

The above observations are useful for the solar plant operator. Importantly, the study provides very pragmatic information of how to design and operate such a plant for an optimal use. It is unclear how such useful information can be obtained by other computational means.

7 Choosing a Single Preferred Solution

It is now clear how the reference point NSGA-II can be used to narrow down the search of solutions near preferred Pareto-optimal regions interesting to a decision-maker, and also to derive interesting properties about the structure of the efficient set of the problem. But, although the choice is narrowed down, we are not still in a position to identify a single preferred solution for implementation and we are still occupied with a number of solutions. It is our surmise that with a few representative solutions being found near two regions of interest, the decision-maker will first be able to favor one of the two regions by exploring the trade-offs between the regions. Once a region is identified, the decision maker can repeat the above procedure by specifying another reference point in that region. Since the new reference point is closer to the Pareto-optimal front than the previously-supplied reference points (without the knowledge of the Pareto-optimal front), the new reference point based NSGA-II run will be able to find solutions closer to each other, thereby allowing the decision maker to compare solutions with finer differences in them than before. Repeating this procedure a few times will enable the decision maker to choose a single preferred solution.

As an example, a test run was carried out with real decision makers. At sight of the previous results, they decided to search in the area of the first reference point (higher profits). After a few iterations of the above-mentioned approach, using the reference point (19,000,000; 255,000,000; 0.13; 48,500,000), the decision makers chose the following solution among the set obtained by the reference point based NSGA-II procedure:

$$\begin{aligned} A_C &= 490,312 \qquad E = 2,695,122,967 \\ P_{AUX} &= 90,261 \qquad L = 63.52, \end{aligned} \tag{7}$$

and the corresponding values of the objective functions are:

$$\begin{aligned} PRO &= 18,596,015 \qquad TIC = 259,779,783 \\ IRR &= 13.67\% \qquad POL = 49,956,127. \end{aligned} \tag{8}$$

As can be seen, the solar field is far from the maximum allowed size, and the deposits have been designed for 6.39 hours capacity (less than one maximum size tank). The auxiliary system is dimensioned so that the electricity can be produced (at 63.52% load) when there is no energy coming from the solar field or the tank. This is why the hybridization percentage reaches the allowed maximum (15%) at the solution. The profit is slightly over 18.5 million €, but the total investment cost is significantly lower than the cost for optimal profit. Besides, both the IRR and the pollution take better values at this solution.

8 Conclusions

In this paper, we have attempted to solve a four-objective optimization model of a solar thermal power plan operation problem. Due to the non-availability of exact mathematical structure of the objective functions, this problem is usually difficult to solve in practice. Moreover, the inherent non-linearity and discontinuity of the objectives and complex trade-off behavior of objective interactions have motivated us to use an evolutionary computation procedure.

First, the original four-objective problem has been solved to find a representative set of trade-off solutions over the entire Pareto-optimal front using a clustered NSGA-II procedure. Since in such practical (and non-mathematical) problems, the true Pareto-optimal front cannot be determined exactly, several lower-objective version of the problem have been solved and results are compared with the four-objective results to build confidence on the accuracy of the proposed algorithms. Results have shown a complex trade-off pattern among the objectives.

Second, it has been argued that in dealing with four or more objectives, it is computationally a better approach to find preferred solutions on some parts of the Pareto-optimal front, rather than attempting to find points on the entire front. We have used a reference point based MCDM approach with the clustered NSGA-II approach for this purpose. Two approaches have been considered and compared to each other. In the first approach, the NSGA-II procedure is employed to find solutions on two areas of the Pareto-optimal front, dictated by the supplied reference points. In the second approach, a single-objective achievement scalarizing function has been formulated with each of the two reference points and is solved using an evolutionary algorithm. A comparison has revealed that the first approach is computationally quicker and can produce more accurate solutions. Hence, we recommend the former approach for other similar studies.

Third, we have performed a manual Innovization study Deb and Srinivasan (2006) for the NSGA-II solutions to reveal any hidden yet useful problem properties that are common to them. Since in this paper we have dealt with a real-world optimization problem, such a study has revealed some vital problem knowledge that were not known before. Such information common to optimal solutions are expected to be beneficial to plant operators and designers at large.

Finally, to demonstrate how the methods of a multiobjective optimization procedure and decision making can be used to consider trade-off solutions and eventually lead to a single preferred solution, we have involved a real decision maker in the process and shown the steps and results obtained. The obtained solution makes a good trade-off among the objectives and remain as a good candidate solution for implementation.

Multiobjective optimization methodologies using evolutionary and classical means are well-studied and many different hybrid approaches are available to be applied in practice. This study is a testimony of how such methodologies can be exploited to understand the intricacies of the problem, generate useful trade-off solutions, and finally choose a single preferred solution. The methodologies demonstrated here not only allow practitioners to achieve the above, but also enable them to unveil important problem knowledge that will provide an additional edge and control in solving the problem in its totality. This study should be motivating to both academician and practitioners to achieve similar studies on other more complex real-world problems.

Acknowledgments

K. Deb and R. Tewari acknowledge the financial support provided by Academy of Finland under project no. 133387. This research has been mainly supported by Endesa Generación, S.L., through the research contract "Solar Vapor 500". J.M. Cabello, M. Luque and F. Ruiz also would like to acknowledge the support from the Regional Government of Andalucía (group PAI SEJ-445, and project P09-FQM-05001) and to the Government of Spain (project MTM2010-14992).

References

Cabello, J. M., Cejudo, J. M., Luque, M., Ruiz, F., Deb, K., and Tewari, R. (2009). Optimization of the sizing of a solar thermal electricity plant: mathematical programming versus genetic algorithms. In *Proceedings of the Eleventh conference on Congress on Evolutionary Computation*, CEC'09, pages 1193–1200, Piscataway, NJ, USA. IEEE Press.

Coello, C. A. C., VanVeldhuizen, D. A., and Lamont, G. (2002). *Evolutionary Algorithms for Solving Multi-Objective Problems*. Boston, MA: Kluwer.

Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. Wiley, Chichester, UK.

Deb, K. and Agrawal, R. B. (1995). Simulated binary crossover for continuous search space. *Complex Systems*, **9**(2), 115–148.

Deb, K. and Saxena, D. (2006). Searching for Pareto-optimal solutions through dimensionality reduction for certain large-dimensional multi-objective optimization problems. In *Proceedings of the World Congress on Computational Intelligence (WCCI-2006)*, pages 3352–3360.

Deb, K. and Srinivasan, A. (2006). Innovization: Innovating design principles through optimization. In *Proceedings of the Genetic and Evolutionary Computation Conference (GECCO-2006)*, pages 1629–1636, New York: ACM.

Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T. (2002). A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, **6**(2), 182–197.

Deb, K., Thiele, L., Laumanns, M., and Zitzler, E. (2005). Scalable test problems for evolutionary multi-objective optimization. In A. Abraham, L. Jain, and R. Goldberg, editors, *Evolutionary Multiobjective Optimization*, pages 105–145. London: Springer-Verlag.

Deb, K., Sundar, J., Uday, N., and Chaudhuri, S. (2006). Reference point based multi-objective optimization using evolutionary algorithms. *International Journal of Computational Intelligence Research (IJCIR)*, **2**(6), 273–286.

Doering, R. D. and Lin, B. W. (1979). Optimum operation of a total energy plant. *Computers and Operations Research*, **6**(1), 33–38.

Luque, M., Miettinen, K., Eskelinen, P., and Ruiz, F. (2007). Incorporating preference information in interactive reference point methods for multiobjective optmization. *Omega*, **37**(2), 450–462.

Miettinen, K. (1999). *Nonlinear Multiobjective Optimization.* Kluwer, Boston.

Mills, D. (2004). Advances in solar thermal electricity technology. *Solar Energy*, **76**, 19–31.

Ruiz, F., Luque, M., and Cabello, J. M. (2008). A classification of the weighting schemes in reference point procedures for multiobjective programming. *Journal of the Operational Research Society*. To appear.

Shukla, P. and Deb, K. (2005). Comparing classical generating methods with an evolutionary multi-objective optimization method. In *Proceedings of the Third International Conference on Evolutionary Multi-Criterion Optimization (EMO-2005)*, pages 311–325. Lecture Notes on Computer Science 3410.

Triki, C., Beraldi, P., and Gross, G. (2005). Optimal capacity allocation in multi-auction electricity markets under uncertainty. *Computers and Operations Research*, **32**(2), 201–217.

Wierzbicki, A. P. (1980). The use of reference objectives in multiobjective optimization. In G. Fandel and T. Gal, editors, *Lecture Notes in Economics and Mathematical Systems,* vol. 177, pages 468–486, Berlin. Springer-Verlag.

Zitzler, E. and Thiele, L. (1999). Multiobjective evolutionary algorithms: A comparative case study and the strength pareto approach. *IEEE Transactions on Evolutionary Computation*, **3**(4), 257–271.

Zitzler, E., Laumanns, M., and Thiele, L. (2001). SPEA2: Improving the strength pareto evolutionary algorithm for multiobjective optimization. In K. C. Giannakoglou, D. T. Tsahalis, J. Périaux, K. D. Papailiou, and T. Fogarty, editors, *Evolutionary Methods for Design Optimization and Control with Applications to Industrial Problems*, pages 95–100, Athens, Greece. International Center for Numerical Methods in Engineering (CIMNE).