Hybrid Evolutionary Multi-Objective Optimization of Machining Parameters

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Abstract

Evolutionary multi-objective optimization (EMO) has received significant attention in recent studies in engineering design and analysis due to their flexibility, wide-spread applicability and ability to find multiple trade-off solutions. Optimal machining parameter determination is an important matter for ensuring an efficient working of a machining process. In this paper, we describe the use of an evolutionary multi-objective optimization (EMO) algorithm and a suitable local search procedure to optimize the machining parameters (cutting speed, feed, and depth of cut) in turning operations. We demonstrate the efficiency of our methodology through two case studies – one having two objectives and the other having three objectives. EMO solutions are modified using a local search procedure to achieve a better convergence property. Here, we also suggest a heuristic-based local search procedure for a computationally faster approach in which the problem-specific heuristics are derived from an *innovization* study performed on the EMO solutions. The methodology adopted in this paper can be used in other machining tasks or in other engineering design activities.

Keywords:
Multi-objective optimization, NSGA-II, ϵ-constraint method, local search, hybrid algorithm, machining parameters, innovative design principles.

1 Introduction

Multi-objective optimization brings the optimization literature close to practice, as in a real-world design or operation, often there exists not one, but a few goals or objectives that must be considered simultaneously. In most interesting scenarios, the objectives are in conflict in nature, meaning that the optimal solution of one objective is *not* the optimal solution of any other objective. Minimization of cost of fabricating an object and the minimization of surface roughness of the main machining process involved in the fabrication process are two objectives which will not have a single optimal solution. In such a scenario, the resulting problem gives rise to a set of trade-off solutions, largely known as the *Pareto-optimal* solutions. The Pareto-optimal set (or front) is likely to include individual optimal solutions and many other compromised solutions which are optimal with respect to certain trade-offs among other objectives.

Thus, the consideration of multiple conflicting objectives in a design or an operation involves two tasks: (i) searching for trade-off Pareto-optimal solutions so that the user can get an idea of the extent and nature of trade-off among the objectives, and (ii) choosing a particular preferred solution from the obtained trade-off set so that the preferred solution can be implemented in practice. The first task is directly related to an optimization task and requires a multi-objective optimization algorithm to search for a set of trade-off solutions. Due to their population approach and wide-spread applicability, evolutionary optimization algorithms are often employed for this purpose [12, 7]. Started in the beginning of nineties, the so-called evolutionary multi-objective optimization (EMO) literature has many useful algorithms for finding a well-distributed set of solutions on or close to the Pareto-optimal front of a multi-objective optimization problem [12, 6]. The second task of choosing a preferred solution requires a more subjective decision-making task which are well-studied by the multi-criterion decision making (MCDM) researchers [25, 34]. Some recent studies have attempted to incorporate an MCDM idea into an EMO and instead of serially applying two methods, an integrated method has been tried [14, 16]. All these studies indicate that the EMO literature has advanced significantly over the past decade and half, and is ready to be applied to various applied problems.

Another advantage of finding a set of trade-off solutions is that these solutions can be analyzed to decipher salient common principles present in them. Since these solutions are optimal (or, often near-optimal), the derived principles are likely to portray useful knowledge about 'what makes a solution optimal' for the problem at hand. This useful post-optimality task is termed as the task of *innovization* elsewhere [15].

In this paper, we apply an EMO methodology on two machining process optimization problems. The primary objective in every machining process is to produce high-quality components with a low cost. Optimization of machining parameters plays a significant role to achieve this goal. Over the last few decades, optimal machining parameters have been extensively studied [21, 20, 29, 24, 30, 17]. In most of these studies, three main machining parameters — speed, feed, and depth of cut — are usually considered. The well-known tool life equation was first proposed by Taylor ([31]), where he found the existence of an optimum or economic cutting speed, which could maximize the material removal rate.

The skilled and familiar shop floor machine operators are the main resource for optimal selection of cutting tools and parameters. But real-life problems face many constraints like low machine tool power, maximum amount of force that can be applied, torque etc. Complex mathematical models have been formulated in the past few years in order to determine the optimal machining parameters. It associates the cutting parameters with the cutting performance [18].

Evolutionary algorithms are recently gaining significant attention from researchers in manufacturing area [1, 26, 3, 2] due to their flexibility, efficiency and robustness in finding global solutions. Many researchers used genetic algorithms to find the machining parameters optimally [23, 9, 17, 10, 18]. This paper uses a multi-objective genetic algorithm (NSGA-II) [13, 12] and verified with single-objective genetic algorithm along with classical optimization methods to obtain optimized machining parameters for turning operations.

Many methods have been used so far for optimization of cutting parameters [24, 30, 36, 17, 10]. Direct search methods were very popular optimization methods as it doesn't require any gradient information. For applying gradient based methods, the objective function must be a continuous function and also need to be twice differentiable. Such conditions do not always satisfy in real-world problems [33]. Quiza et al. [28] found that single-objective approaches have limited value for fixing optimal cutting conditions, because of complex nature of machining processes as we need to optimize various different and conflicting objectives. Wang [32, 37] suggested a neural network based approach to optimize cutting parameters in multi-objective optimization. Amiolemhen et al. [2] proposed a methodology based on genetic algorithms to determine the cutting parameters in multi-pass machining operations by simultaneously considering single-pass finishing and multi-pass roughing operations. Quiza et al. [28] proposed a multi-objective optimization methodology to optimize the cutting parameters in turning processes using genetic algorithms. Another study [22] formulated a multi-objective hard turning process and proposed a methodology to solve the problem using neural network modeling along with the dynamic neighborhood particle swarm optimization and demonstrate the procedure through three case studies. A recent study [35] used multi-objective differential evolution (MODE) and non-dominated sorting genetic algorithm (NSGA-II) to solve optimization of machining parameters in a turning process. Earlier, we proposed a multi-objective optimization based hybrid strategy using genetic algorithms to find optimum machining parameters [11].

In this paper, we highlight the importance of using a local search procedure along with an EMO procedure. In addition to the usual procedure of serially applying a local search from every EMO solution (which can be computationally expensive), we propose an innovization-based local search approach in which the common principles present in EMO solutions are first deciphered and then used as heuristics in the local search operation. Both local search approaches have been demonstrated to find a well-converged set of trade-off solutions, revealing the true nature of the objective interactions and the hidden operating principles related to the machining parameters.

In the remainder of the paper, first we outline the working procedure of our proposed approaches. The next section consists of the problem formulation of the two case studies. Thereafter, we solve each problem and describe the effectiveness of our methodology. An innovization study is then carried out to reveal any useful relationship among the Pareto-optimal solutions. Two different approaches to ensure convergence property of EMO solutions are then illustrated. Finally, the paper ends with conclusions of this study.

2 Multi-Objective Optimization and Analysis

Multi-objective optimization problems involving conflicting objectives give rise to a set of Pareto-optimal (trade-off) solutions. Although at the end of optimization one preferred Pareto-optimal solution must be chosen, the task of finding a set of representative trade-off solutions from the Pareto-optimal front has a number of advantages. First, it provides the decision-makers the idea of the range of optimal solutions. Second, it provides the nature of trade-off present among the solutions. Third and importantly, it has been argued elsewhere [15] that the presence of multiple trade-off optimal solutions allows to decipher hidden knowledge common to the solutions. This post-optimality *innovization* task has been used to unveil useful design principles on a number of engineering design tasks.

Evolutionary multi-objective optimization (EMO) algorithms have been successfully used to find a set of well-converged and well-diversified trade-off solutions due to their (i) population approach, (ii) use of non-domination and (ii) use of a diversity-preserving mechanism. However, since EMO algorithms do not use any mathematical optimality conditions in their operators, the obtained solutions after a finite number of computations are not guaranteed to be optimal, although an asymptotic convergence for EMOs with certain properties have been proven in the past [27]. To enhance the convergence properties of EMO algorithms, often they are applied with a local search procedure. One common approach is to first apply an EMO and then the obtained solutions are modified by using a local search procedure one at a time. This is termed as 'Approach 1' in Figure 1. Although this hybrid procedure is commonly employed, the overall computational effort needed in executing the local search from each EMO solution can be burdensome. Any trick to reduce the local search effort would make the overall procedure computationally tractable.

Here, we suggest an alternate strategy ('Approach 2' shown in the figure) in which first an innovization study can be performed on the EMO solutions to derive useful relationships among

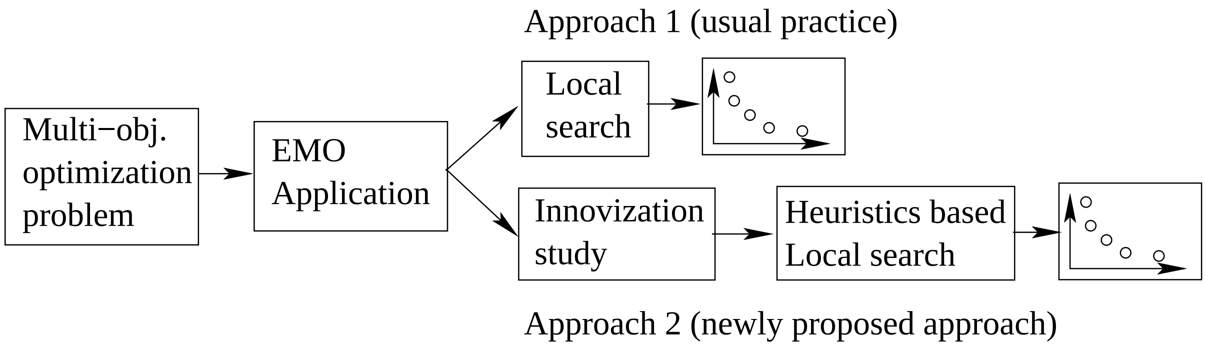

Figure 1: Two approaches of performing a local search to improve an EMO's convergence property are illustrated.

decision variables, objectives and constraint functions that commonly exist on the EMO solutions. The derived relationships may then be used as *heuristics* in the local search operator to reduce the computational burden of the local search procedure. Since the derived (innovized) principles reveal hidden properties present in optimal solutions, such principles should provide useful search rules that may cut down the computational burden of the local search procedure.

In this paper, we apply evolutionary multi-objective optimization methodology to determine optimal machining parameters and discover hidden operating principles on a couple of machining operations. In both problems, the decision variables are cutting speed (v), feed rate (f) and depth of cut (a). Objectives involve more than one conflicting goals of minimizing surface roughness, minimizing operation time, and minimizing machining cost, etc. For the first problem, we employ the well-known ϵ-constraint strategy as the local search operator to improve each EMO solution. However, on the second problem, we demonstrate the use of our proposed innovization-based local search procedure outlined above.

The tasks performed in this paper are outlined below:

1. A trade-off frontier is first obtained using the non-dominated sorting genetic algorithm (NSGA-II) algorithm, which emphasizes elites, non-dominated solutions and less-crowded solutions within the GA population [13].

2. Extreme points of the Pareto-optimal front are found by converting the multi-objective problem into equivalent single-objective optimization problems. Resulting solutions are then compared with the extreme solutions obtained by NSGA-II.

3. Intermediate trade-off solutions obtained using NSGA-II are also verified for their accuracy using the well-known classical multi-criterion optimization methods [25].

4. A post-optimality analysis is then performed with the obtained NSGA-II-cum-local-searched solutions to reveal any common relationship among the trade-off solutions. This so-called *innovization* procedure — revealing innovation through optimization [15] — is expected to unveil salient operating principles for the machining tasks.

5. The innovization task provides a feedback that not all obtained trade-off solutions agree exactly with the deciphered common operating principles. When operating principles are enforced on these solutions, better non-dominated solutions are obtained. This suggests that not only do we obtain salient innovized relationships through a multi-objective study, the innovization study can also provide us with clues for a faster local optimization study.

3 Problem Formulation

The multi-objective optimization problem formulation is adopted from [10, 28]. The decision variables are $\mathbf{x} = (v, f, a)^T$, where v is the cutting speed, f is the feed rate and a is the depth of cut. The objectives are as follows:

- **Operation time:** The operation time is measured as the total time required to manufacture a product $T_p(\mathbf{x})$ and is to be minimized:

$$T_p(\mathbf{x}) = T_s + V(1 + T_c/T(\mathbf{x}))/MRR(\mathbf{x}) + T_i. \tag{1}$$

  It is function of metal removal rate $MRR(\mathbf{x})$ and tool life $T(\mathbf{x})$. The term T_s is the tool-set-up time, T_c is tool-change time, T_i is idle time between two consecutive cuts, and V is volume of material removed. For a particular machining operations, T_s, T_c, T_i, and V are constant so that $T_p(\mathbf{x})$ is function of $MRR(\mathbf{x})$ and $T(\mathbf{x})$ only.

- **Metal removal rate:** $MRR(\mathbf{x})$ can be expressed as the product of cutting speed, feed, and depth of cut and is to be maximized for a better machining operation:

$$MRR(\mathbf{x}) = 1000vfa. \tag{2}$$

- **Tool life:** The tool life $T(\mathbf{x})$ is measured as the average time between the tool changes or tool sharpening, which is to be maximized. The relation between the tool life and the parameters is expressed with the well-known Taylor's formula:

$$T(\mathbf{x}) = K_T/(v^{\alpha_1} f^{\alpha_2} a^{\alpha_3}), \tag{3}$$

  where k_T, α_1, α_2, and α_3, are positive constants.

- **Operation cost:** The operation cost can be expressed as the cost per product, as follows:

$$C_p(\mathbf{x}) = T_p(\mathbf{x})(C_t/T(\mathbf{x}) + C_l + C_o), \tag{4}$$

  where C_t is tool cost, C_l is labor cost, and C_o is the overhead cost. For a specific machining operation, the C_t, C_l, and C_0 are constant.

- **Cutting quality:** The most important criterion for the determination of the surface quality is roughness and is given as follows:

$$R_a(\mathbf{x}) = kv^{\gamma_1} f^{\gamma_2} a^{\gamma_3}, \tag{5}$$

  where γ_1, γ_2, γ_3, and k are constants relevant to a specific tool-workpiece combination.

- **Limitations:** There are several factors that limit the cutting parameters. Permissible range of cutting conditions are as follows:

$$v_{\min} \leq v \leq v_{\max}, \tag{6}$$
$$f_{\min} \leq f \leq f_{\max}, \tag{7}$$
$$a_{\min} \leq a \leq a_{\max}. \tag{8}$$

- **Cutting power P and force F:** Consumption of power can be expressed as a function of the cutting force and cutting speed, as follows:

$$P(\mathbf{x}) = \frac{vF(\mathbf{x})}{6122.45\eta}, \tag{9}$$

where η is the mechanical efficiency of the machine, and F is given by

$$F(\mathbf{x}) = K_F f^{\beta_2} a^{\beta_3}. \tag{10}$$

The limitations of the power and cutting force are given as follows:

$$P(\mathbf{x}) \leq P_{\max}, \tag{11}$$
$$F(\mathbf{x}) \leq F_{\max}. \tag{12}$$

In the following sections, we consider two different turning operations for which the parameters mentioned above are given in the metal-cutting literature and we borrow the optimization problem formulation from those studies and perform a multi-objective study.

4 Case Study I

This problem is borrowed from [28]. A steel bar is being machined on a CNC lathe, using $P20$ carbide tool. The lathe has a 10 kW motor and a transmission efficiency of 75%. A maximum allowable cutting force of $5,000$ N is applied. It is considered that the operation will remove $219,912$ mm^3 of material. The set-up time, tool-change time and the time during which the tool does not cut have been assumed as 0.15, 0.20 and 0.05 min, respectively. The objectives are minimization of operation time (T_p) and used tool life (ξ). The multi-objective optimization problem formulation is given below:

$$\begin{array}{ll} \text{Minimize} & T_p(\mathbf{x}), \\ \text{Minimize} & \xi(\mathbf{x}), \\ \text{subject to} & g_1(\mathbf{x}) \equiv 1 - \frac{P(\mathbf{x})}{\eta P^{\max}} \geq 0, \\ & g_2(\mathbf{x}) \equiv 1 - \frac{F_c(\mathbf{x})}{F_c^{\max}} \geq 0, \\ & g_3(\mathbf{x}) \equiv 1 - \frac{R(\mathbf{x})}{R^{\max}} \geq 0, \\ & x_i^{\min} \leq x_i \leq x_i^{\max}, \end{array} \tag{13}$$

where

$$T_P(\mathbf{x}) = 0.15 + 219912 \left(\frac{1 + \frac{0.20}{T(\mathbf{x})}}{MRR(\mathbf{x})} \right) + 0.05, \tag{14}$$

$$\xi(\mathbf{x}) = \frac{219912}{MRR(\mathbf{x})\, T(\mathbf{x})} \times 100, \tag{15}$$

$$T(\mathbf{x}) = \frac{5.48(10^9)}{v^{3.46} f^{0.696} a^{0.460}}, \tag{16}$$

$$F_c(\mathbf{x}) = \frac{6.56(10^3) f^{0.917} a^{0.1.10}}{v^{0.286}}, \tag{17}$$

$$P(\mathbf{x}) = \frac{v F_c}{60,000}, \tag{18}$$

$$MRR(\mathbf{x}) = 1,000 v f a, \tag{19}$$

$$R(\mathbf{x}) = \frac{125 f^2}{r_n}. \tag{20}$$

The objective $\xi(\mathbf{x})$ is considered as the part of the whole tool life which is consumed in the machining process, hence an operating condition that will minimize this objective will use the machining task optimally. Here, R is the surface roughness and r_n is the nose radius of the tool. The constant parameter values are given below:

$$P^{\max} = 10 \text{ (kW)}, \quad F_c^{\max} = 5{,}000 \text{ N}, \quad R^{\max} = 50\ \mu\text{m},$$
$$r_n = 0.8 \text{ mm}, \qquad \eta = 0.75.$$

Following are the variable bounds for cutting speed, feed, and depth of cut:

$$v_{\min} = 250 \text{ m/min}, \qquad v_{\max} = 400 \text{ m/min},$$
$$f_{\min} = 0.15 \text{ mm/rev}, \quad f_{\max} = 0.55 \text{ mm/rev},$$
$$a_{\min} = 0.5 \text{ mm}, \qquad a_{\max} = 6 \text{ mm}.$$

4.1 Multiple to Single-Objective Methodologies

Here, we show a chronology of application of different optimization tasks on Case Study I to build in the confidence of the accuracy of obtained solutions. Since a genetic algorithm (GA) is a heuristic based search procedure, an application of a GA or its variant is not always guaranteed to produce the true Pareto-optimal front. In such a scenario, we propose to use a number of independent optimization methods to solve the same problem. If all methods find similar solutions, it is then expected that the obtained solutions are close to the true optimal solutions.

4.2 Bi-Objective Optimization using NSGA-II for Finding Trade-off Solutions

This problem has two objectives and three constraints. Both objective functions depend on the decision variables (cutting speed, feed and depth of cut). The objective functions are non-linear in nature. Here we use NSGA-II [13] to solve this bi-objective optimization problem. The parameter values of NSGA-II used in this study are as follows:

a. Population size = 200

b. Number of generations = 500

c. Probability of crossover = 0.9

d. Probability of mutation = 0.333

e. Distribution index for real variable SBX crossover = 10

f. Distribution index for real variable polynomial mutation = 50

In Figure 2, the obtained trade-off front is shown, where the x-axis represents the operation time and the y-axis represents the consumed tool life.

4.3 Single-Objective Optimization using a Classical Method for finding Extreme Points

To verify the extreme points on the Pareto-optimal frontier, we convert the problem into two single-objective optimization problems. First, we consider $F_1(\mathbf{x})$ as the objective and neglect $F_2(\mathbf{x})$. The `fmincon()` routine of MATLAB is used to solve these single-objective optimization problems. After obtaining the optimal point $(\mathbf{x}^{(1,*)})$ for $F_1(\mathbf{x}^{(1,*)})$, $F_2(\mathbf{x}^{(1,*)})$ value is then computed and the point $(F_1(\mathbf{x}^{(1,*)}, F_2(\mathbf{x}^{(1,*)})$ is plotted in the figure. Similarly, the second objective $F_2(\mathbf{x})$ is optimized alone and at the obtained minimum $\mathbf{x}^{(2,*)}$, the point $(F_1(\mathbf{x}^{(2,*)}, F_2(\mathbf{x}^{(2,*)})$ is computed and marked in the figure. It is clear from the figure that both extreme points obtained from two independent single-objective optimization studies exactly match with the corresponding extreme points of the trade-off frontier obtained by NSGA-II.

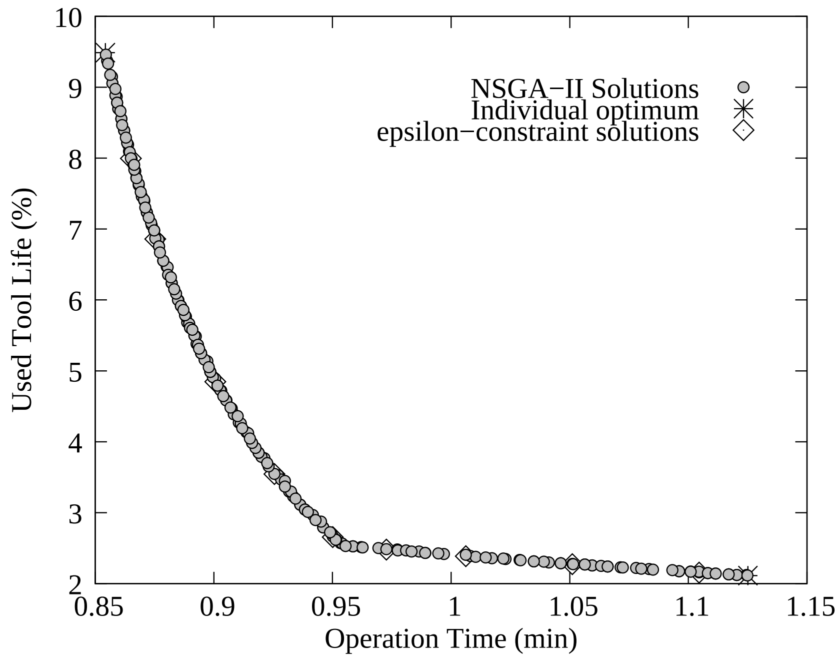

Figure 2: Non-dominated front obtained using NSGA-II, extreme and ϵ-constraint points are shown for Case Study I.

4.4 Single-Objective Optimization using a Classical Method for finding Intermediate Points

Next, to verify some of the intermediate points of the NSGA-II obtained front, the bi-objective problem is converted into suitable single-objective optimization problems using the ϵ-constraint strategy at each NSGA-II point $\bar{\mathbf{x}}$. All previous three constraints are kept the same with the addition of one more constraint from the corresponding objective function:

$$\begin{array}{rl} \text{Minimize} & F_1(\mathbf{x}), \\ \text{subject to} & g_1(\mathbf{x}) \geq 0, \\ & g_2(\mathbf{x}) \geq 0, \\ & g_3(\mathbf{x}) \geq 0, \\ & g_4(\mathbf{x}) \equiv F_2(\mathbf{x}) \leq \epsilon_2 (= F_2(\bar{\mathbf{x}})), \\ & x_i^{\min} \leq x_i \leq x_i^{\max}, \quad \forall i. \end{array} \tag{21}$$

Here, different values of the parameter ϵ_2 is chosen in the range $[F_2(\mathbf{x}^{(2,*)}), F_2(\mathbf{x}^{(1,*)})]$. The bounds on the variables are same as those discussed earlier. MATLAB's `fmincon()` routine is used to solve the resulting optimization problem. However, the NSGA-II solution $(\bar{\mathbf{x}})$ is chosen as the initial point for the `fmincon()` procedure. At the obtained solution $\hat{\mathbf{x}}$ to the above problem, both objective values are computed and the point $(F_1(\hat{\mathbf{x}}), F_2(\hat{\mathbf{x}}))$ is marked on the figure. Figure 2 shows that obtained points from the ϵ-constraint approach fall on the NSGA-II front, thereby establishing the accuracy of the obtained NSGA-II frontier.

Table 1 shows the extreme solutions and some of the intermediate solutions. Interestingly, constraint g_1 representing the power requirement constraint is active for all the points. The table also shows that $P = 7.5$ kW for all the points, which is equal to 75% of allowable power of 10 kW. None of the other two constraints is active at any other point on the trade-off frontier.

Table 1: Extreme and certain intermediate trade-off points for Case Study I.

F_1 (min)	F_2 (%)	v (m/min)	f (mm/rev)	a (mm)	T (min)	MRR (mm^3/min)	F_c (N)	P (kW)	R (μm)
0.854	9.488	400.00	0.550	1.573	6.7	346148.0	1125.0	7.5	47.3
0.863	8.214	380.015	0.549	1.629	7.9	339870.1	1184.1	7.5	47.1
0.881	6.354	346.836	0.550	1.727	10.5	329223.4	1297.4	7.5	47.2
0.894	5.312	325.399	0.550	1.799	12.9	321929.8	1382.8	7.5	47.3
0.911	4.270	301.078	0.550	1.892	16.4	313271.3	1494.4	7.5	47.3
0.933	3.229	272.595	0.550	2.019	22.5	302504.5	1650.6	7.5	47.2
0.959	2.525	250.000	0.536	2.178	29.8	291818.9	1797.0	7.5	44.9
1.029	2.333	250.186	0.310	3.444	35.3	266812.7	1798.5	7.5	15.0
1.083	2.204	250.137	0.211	4.748	39.9	250184.7	1798.5	7.5	6.9
1.125	2.113	250.000	0.159	6.000	43.6	238800.0	1799.9	7.5	4.0

Thus, we conclude the following ideal and nadir point for the problem in Case Study I, showing the range of trade-off values for each objective:

$$\begin{array}{ll} \text{Ideal point:} & (0.8543, 2.1126)\ (\text{min}, \%) \\ \text{Nadir point:} & (1.1251, 9.4880)\ (\text{min}, \%) \end{array}$$

The hypervolume [19] for the original NSGA-II front is calculated to be 1.669133 from the reference point $(1.126, 9.489)^T$ (chosen as a slightly worse point than the nadir point). The hypervolume for the hybrid NSGA-II front is found to be 1.671832 from the same reference point. This indicates an improvement of the hypervolume of 0.16% by the local search procedure.

4.5 Innovative Principles from Optimization: (*Innovization*)

Having found the extreme points and the nature of the trade-off objective information, we first modify all NSGA-II solutions using the ϵ-constraint strategy as a local search operator. The `fmincon()` routine is used for the purpose and the corresponding NSGA-II solution is used as the initial point. The corresponding trade-off front is shown in Figure 3. This front is smoother than the original NSGA-II front, shown in Figure 2.

We now perform a post-optimality analysis of the obtained points to gather useful knowledge about operating principles of the particular machining process. The task is known as *innovization* [15] in which relationships among decision variables and objectives that are common to obtained trade-off solutions are unveiled. Here we employ the manual version of the innovization task by investigating each decision variable at a time and observing if their variation with respect to one of the objective function value follows any pattern. Recent studies towards an automated innovization task [5, 4] using a data-mining technique are promising and can also be applied to this problem. However, for our manual innovization purpose, we sort the points according to one of the objectives and plot each variable at a time.

Figure 4 shows the variation of speed with an increase in operation time (T_p) as they exist among the obtained trade-off solutions. The figure suggests that for a quick operation time requirement, the turning speed must be large. This is an intuitive fact, but what our study additionally reveal is the exact relationship between these two quantities (v is in m/min and T_p is in min):

$$v = \begin{cases} 203.2T_p^{-4.233}, & \text{for } T_p \le 0.9545, \\ 250, & \text{for } T_p > 0.9545. \end{cases} \tag{22}$$

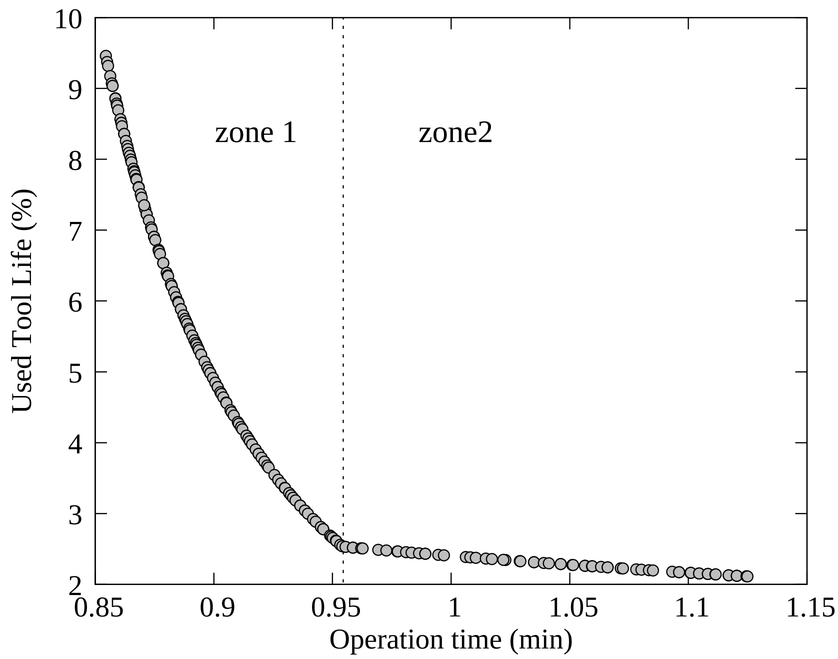

Figure 3: Non-dominated front using hybrid NSGA-II and the ϵ-constraint approach used as a local search for Case Study I.

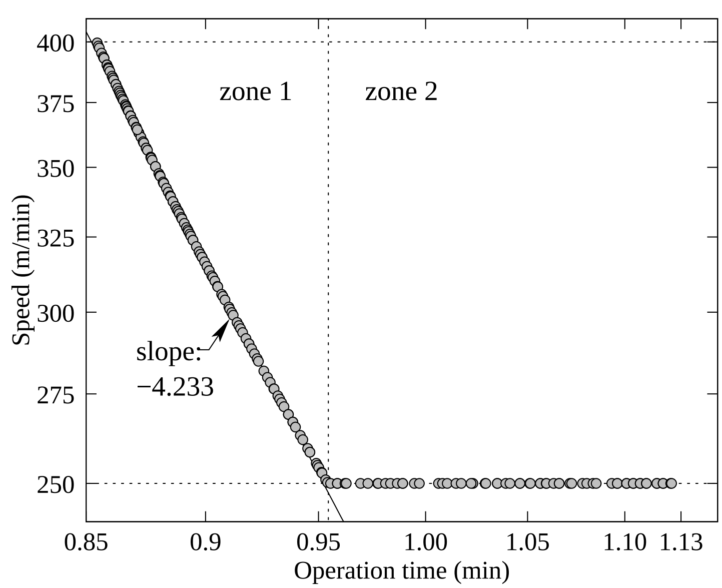

Figure 4: Variation of speed with operation time as it appears on the hybrid NSGA-II solutions for Case Study I.

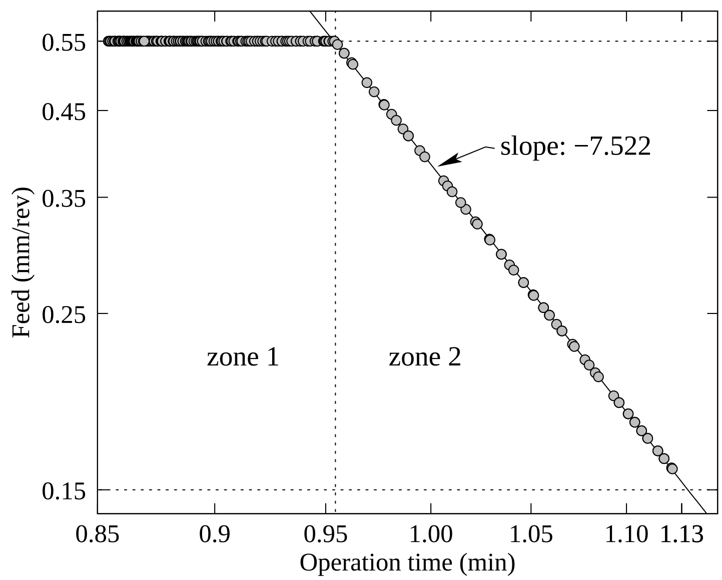

Figure 5: Variation of feed with operation time as it appears on the hybrid NSGA-II solutions for Case Study I.

The speed requirement reduces with T_p and at $T_p = 0.9545$ min, the speed reaches the allowable lower bound and thereafter it needs to be kept fixed at this allowable lower value. This divides the trade-off front into two zones based on this critical value of T_p.

Next, we do a similar study for feed and operation time. Figure 5 shows the variation of feed with T_p for all obtained NSGA-II-cum-local-search solutions. Interestingly, for small values of T_p, feed must be set at its allowable upper bound of 0.55 mm/rev, however in zone 2, the feed (in mm/rev) must be reduced polynomially as follows:

$$f = \begin{cases} 0.55, & \text{for } T_p \leq 0.9545, \\ 0.385T_p^{-7.522}, & \text{for } T_p > 0.9545. \end{cases} \tag{23}$$

Since the surface roughness is not considered in this study as an objective, the bi-objective optimization of the operation time and the used tool life results in higher values of feed as an optimal choice. For our next problem, we include the surface roughness as an additional objective.

Figure 6 shows the variation of depth of cut versus operation time as they exist among the obtained NSGA-II-cum-local-search solutions. The variation of depth of cut is not fixed for any part of the trade-off frontier, but with an increase in operation time the depth of cut needs to be monotonically increased. Again, we observe two different patterns in the variation of depth of cut (in mm):

$$a = \begin{cases} 2.443T_p^{2.751}, & \text{for } T_p \leq 0.9545, \\ 2.874T_p^{6.271}, & \text{for } T_p > 0.9545. \end{cases} \tag{24}$$

The same problem was also attempted by using the micro-GA approach [8, 28]. The trade-off results reported in these studies is similar to that reported here. But additionally we report here the key operating principles (equations 22 to 24) that are common to the trade-off solutions - a matter that provides useful insights in operating the process:

1. All trade-off solutions utilizes the maximum allowable power capacity of 7.5 kW.

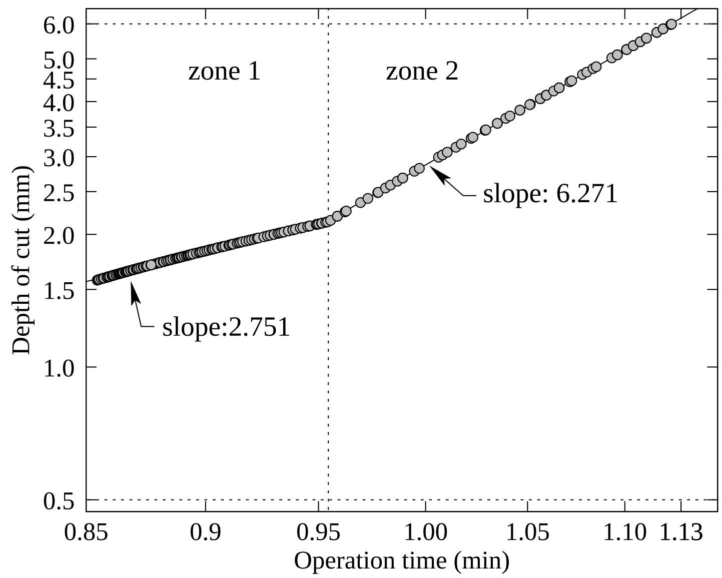

Figure 6: Variation of depth of cut with operation time as it appears on the hybrid NSGA-II solutions for Case Study I.

2. There are two distinct operating principles common to the trade-off solutions: (i) T_p less than or equal to 0.9545 min (zone 1) and (ii) T_p more than 0.9545 min (zone 2). This critical transition operation time depends on the chosen lower and upper values of the decision variables.

3. For zone 1, feed must be fixed at its allowable upper value, speed must be reduced and depth of cut must be increased with an increase in operation time.

4. For zone 2, speed must be set its allowable lower value, feed and depth of cut must be increased with an increased requirement in operation time.

5. A depth of cut less than around 1.5 mm is not an optimal setting at all.

6. The used tool life and operation time has a more meaningful trade-off for zone 1 solutions and an operating condition lying in zone 1 is recommended for this machining operation. In zone 2, however, a large sacrifice in operation time is needed to make a small gain in used tool life and this this zone may not be a practical choice for a machining operation.

5 Case Study II

The machining operation considered here is a turning process of machining a cast steel blank using NC lathe with a HSS cutting tool. The task here is to find the optimum cutting conditions involving the same three decision variables: speed (v), feed (f), and depth of cut (a). The objectives are operation time, cost per product, and surface roughness, which all must be minimized for a better machining operation. The multi-objective formulation is outlined elsewhere [10] and is given below:

$$\begin{array}{ll} \text{Minimize} & F_1(\mathbf{x}) = T_P(\mathbf{x}), \\ \text{Minimize} & F_2(\mathbf{x}) = C_P(\mathbf{x}), \\ \text{Minimize} & F_3(\mathbf{x}) = R_a(\mathbf{x}), \\ \text{subject to} & g_1(\mathbf{x}) \equiv 1 - P(\mathbf{x})/P_{\max} \geq 0, \\ & g_2(\mathbf{x}) \equiv 1 - F(\mathbf{x})/F_{\max} \geq 0, \\ & x_i^{\min} \leq x_i \leq x_i^{\max}, \quad i = 1,2,3, \end{array} \tag{25}$$

where

$$\begin{aligned} T_P(\mathbf{x}) &= 0.12 + 2313.76\left(\frac{1+\frac{0.26}{T(\mathbf{x})}}{MRR(\mathbf{x})}\right) + 0.04, \\ C_P(\mathbf{x}) &= \left(\frac{13.55}{T(\mathbf{x})} + 0.39\right) T_p(\mathbf{x}), \\ R_a(\mathbf{x}) &= 1.001(v^{0.0088} f^{0.3232} a^{0.3144}), \\ T(\mathbf{x}) &= 1575134.21(v^{-1.70} f^{-1.55} a^{-1.22}), \\ MRR(\mathbf{x}) &= 1000vfa, \\ F(\mathbf{x}) &= 1.38 f^{1.18} a^{1.26}, \\ P(\mathbf{x}) &= 0.000626 v f^{0.24} a^{0.11}. \end{aligned}$$

The variable bounds and constant parameter values are as follows:

$$\begin{array}{lll} v_{\min} = 70 \text{ m/min}, & v_{\max} = 90 \text{ m/min}, & f_{\min} = 0.1 \text{ mm/rev}, \\ f_{\max} = 2 \text{ mm/rev}, & a_{\min} = 0.1 \text{ mm}, & a_{\max} = 5 \text{ mm} \\ F_{\max} = 230 \text{ N}, & P_{\max} = 5 \text{ kW}. & \end{array}$$

5.1 Methodology

For the above three-objective optimization problem. we use an identical NSGA-II parameter settings as used in the previous case study. Figure 7 shows the Pareto-optimal front obtained using NSGA-II alone. Interestingly, despite having three objectives, the obtained front is a curve, indicating that some kind of redundancy in the objectives. The projections of the front on three basic planes indicate the trade-offs between the pairs of objectives. It is clear that objectives F_1 (operation time) and F_2 (cost per production) are along most part of the front correlated to each other, meaning that these two objectives do not possess too much of a trade-off between them. The projection on F_1-F_3 plane indicates that these two objectives possess a perfect trade-off between them.

Although the trade-off front seems to be smooth, a local search operation from each point may be an overall computationally expensive proposition. Here, we demonstrate the use of 'Approach 2' introduced in Figure 1. We first perform an innovization study on the obtained NSGA-II solutions and then perform a heuristic-based search from each NSGA-II solution, which is expected to be a computationally less demanding task.

5.2 Innovization Task on Case Study II

Like before, we perform a manual innovization study with three variables as they are observed to vary with respect to one of the objectives (third objective is chosen here). Figure 8 shows the variation of speed with surface roughness. Interestingly, in almost all obtained trade-off solutions, speed is required to be kept fixed to its allowable upper value. However, unlike in the previous case study, here the NSGA-II solutions show some noise in its convergence to any fixed operating

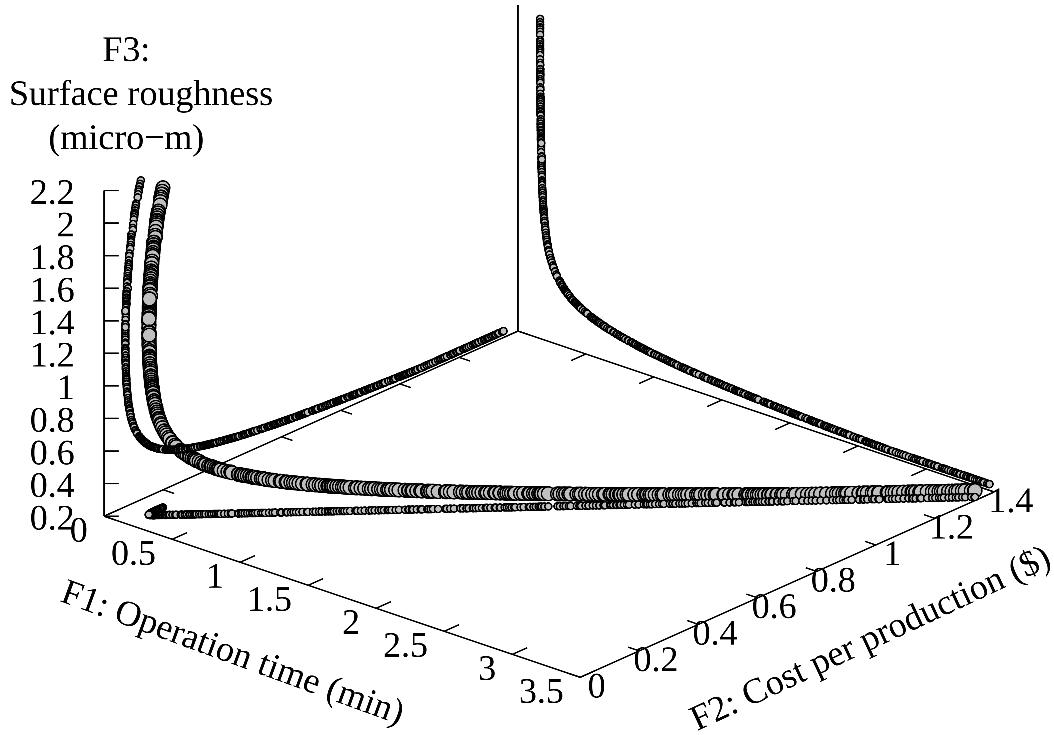

Figure 7: Obtained NSGA-II trade-off front for Case Study II.

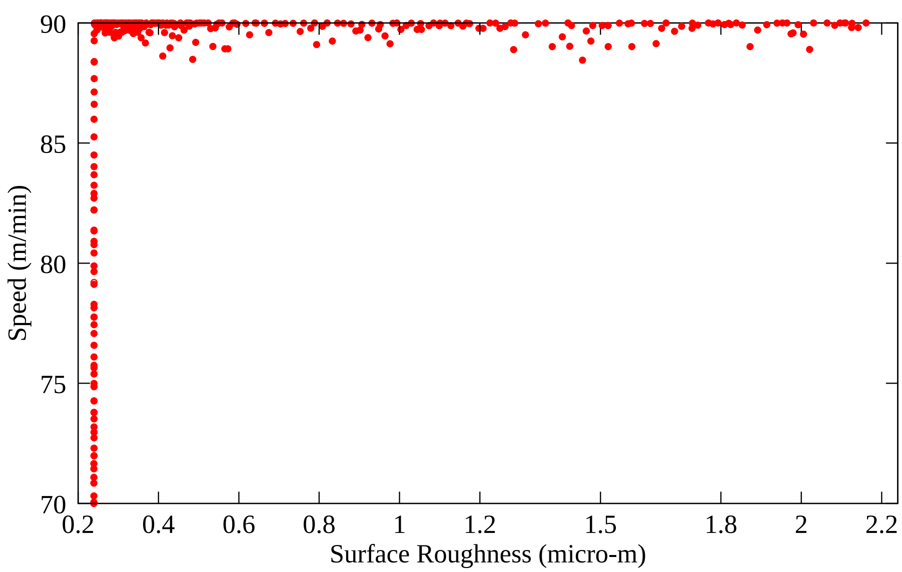

Figure 8: Speed as it changes with surface roughness on Case Study II.

principle. This can indicate one of the two facts: (i) NSGA-II solutions have failed to converge on to the true Pareto-optimal set, or (ii) the problem does not possess this operating principle as a true common principle of the Pareto-optimal front. One way to resolve this issue is to employ a local search procedure and check if the NSGA-II solutions are indeed optimal solutions. Since the local search procedure is a computationally expensive proposition, particularly when it has to be applied many times, the accuracy of an innovization study of the obtained NSGA-II solutions can indicate whether a subsequent local search is necessary for the current problem or not. We continue with a manual innovization study of the other two variables, before we perform the local search procedure.

Figure 9 shows the variation of feed with surface roughness. We observe two different principles

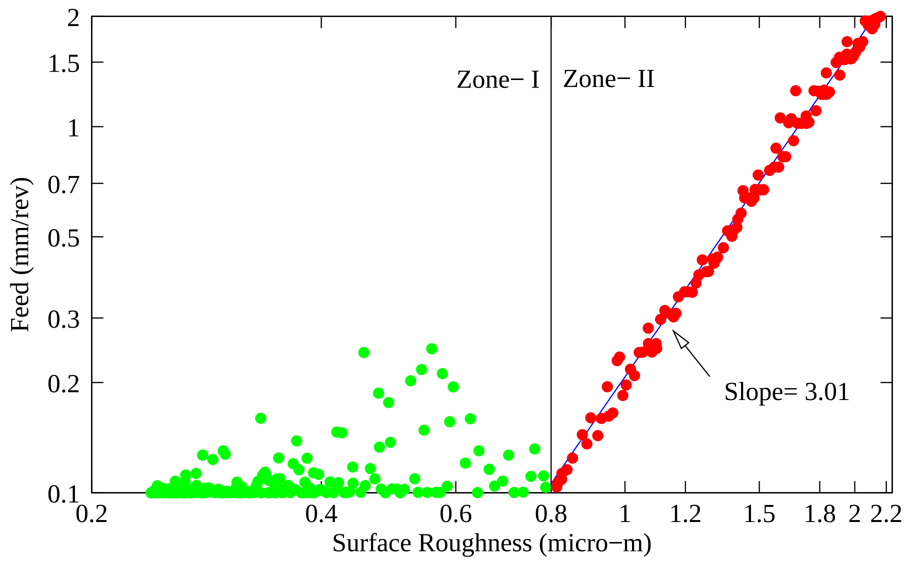

Figure 9: Feed as it changes with surface roughness on Case Study II.

of variation. For surface roughness somewhat less than a critical value $R_a^{\rm cr}$ (around 0.8 μm), the feed seems to be kept fixed at its allowable lower value (0.1 mm/rev) and for surface roughness value more than $R_a^{\rm cr}$, the feed needs to be increased polynomially with increased requirement of surface roughness. A fitted relationship turns out to be as follows: $f = 0.208R_a^{3.01}$ mm/rev (R_a is in μm). Intuitively, an increase or decrease in feed causes an increase in surface roughness, but the fact that for up to a certain surface roughness no increase or decrease in feed is necessary is somewhat not intuitive and comes as an optimal operating principle for this particular machining operation. Obviously, for surface roughness below $R_a^{\rm cr}$, both speed and feed are more of less constant, thus a change in surface roughness must be caused by a change in the third variable (depth of cut), which we investigate next.

Figure 10 shows a somewhat opposite trend. Up to a surface roughness value of around $R_a^{\rm cr}$, the depth of cut varies significantly with surface roughness and beyond $R_a^{\rm cr}$ the depth of cut must be kept almost constant to its allowable upper value. The variation of depth of cut with a fitted curve is found to be as follows: $a = 6.75R_a^{2.95}$ (mm).

The above principles suggest that the critical surface roughness value $R_a^{\rm cr}$ occurs for $v = 90$ m/min, $f = 0.1$ mm/rev and $a = 5$ mm. A computation reveals $R_a^{\rm cr} = 0.82$ μm. With all the above analysis, the operating principles are now quite clear:

1. Keep the turning speed constant to its allowable upper bound (90 m/min) to ensure that the

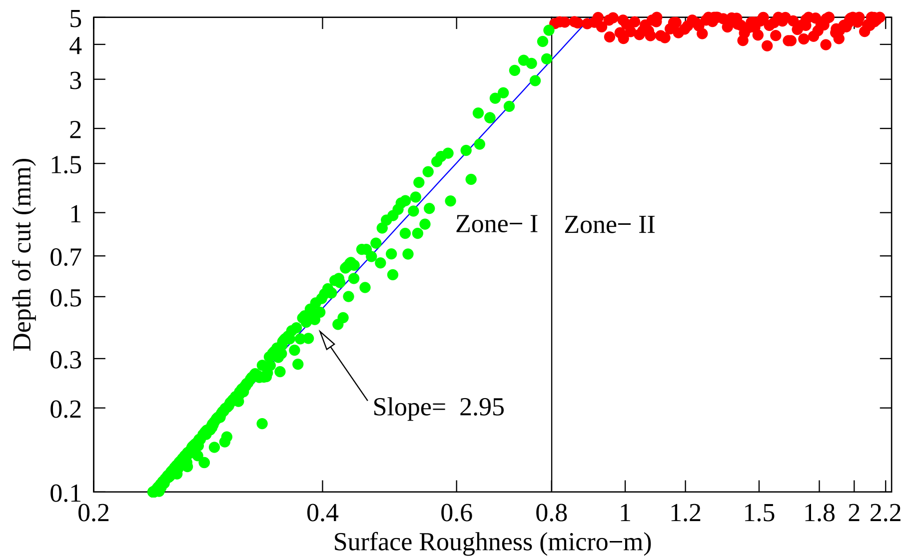

Figure 10: Variation of depth of cut with surface roughness as they appear on the NSGA-II solutions for Case Study II.

resulting operation is optimal. This becomes a necessary condition for optimality and we can use $v = 90$ as the first heuristics for the local search procedure for every EMO solution.

2. For a smaller surface roughness requirement up to a value of around $R_a^{\rm cr} = 0.82$ μm, keep the feed constant at its allowable lower bound (0.1 mm/rev), but vary the depth of cut. An increase in depth of cut will result in an increase in surface roughness. Thus, we can use $f = 0.1$ for solutions having $R_a \leq 0.82$ μm as the second heuristics for the local search.

3. For surface roughness higher than around 0.82 μm, the depth of cut must be kept fixed at its allowable upper bound (5 mm), but the feed needs to be monotonously increased from its lower bound to its upper bound. Thus, we can use $a = 5$ for solutions having $R_a > 0.82$ μm as the third heuristics for the local search procedure.

4. For achieving a small surface roughness (that is, having a good surface finish), fixing speed at its allowable upper bound and feed at its allowable lower bound are optimal operating principles. The smallest allowable depth of cut will ensure the minimum surface roughness (best surface finish), but at the expense of operation time and cost per production.

We now apply the above three heuristics in the local search procedure to improve the EMO solutions for getting closer to the true Pareto-optimal front of the problem.

5.3 Heuristics-based Local Search Results

Figures 8, 9 and 10 indicate that NSGA-II solutions did not quite converge to provide an accurate operating principles, but there are some clear indications as outlined in the above enumerated items. These indications can be used as *heuristics* in a local search procedure to improve the convergence of NSGA-II solutions.

In this study, we use the reference-point based approach as the local search procedure [34]. It minimizes the so-called achievement scalarizing function (ASF), which is given below:

$$\begin{array}{ll} \text{Minimize} & \max_{i=1}^{3} \left(\frac{F_i(\mathbf{x})-z_i}{F_i^{\max}-F_i^{\min}} \right) + \rho \sum_{i=1}^{3} \left(\frac{F_i(\mathbf{x})-z_i}{F_i^{\max}-F_i^{\min}} \right), \\ \text{subject to} & g_j(\mathbf{x}) \geq 0, \quad j=1,2, \\ & x_i^{\min} \leq x_i \leq x_i^{\max}, \quad \forall i. \end{array} \tag{26}$$

Here the vector $\mathbf{z} = (F_1(\bar{\mathbf{x}}), F_2(\bar{\mathbf{x}}), F_3(\bar{\mathbf{x}}))^T$ is the objective vector of the NSGA-II solution $\bar{\mathbf{x}}$ which is to be improved by the local search procedure. A small value of ρ (around 10^{-4}) is used to avoid the algorithm to converge to a weak Pareto-optimal point. This problem involves all n decision variables, but minimizes a non-differentiable objective function. All constraints and variable bounds of the original problem are also included in the above problem.

Based on the innovization study performed in the previous subsection, we can set the following condition: $v = 90$ m/min, thereby reducing the number of variables from three to two. Also for all NSGA-II solutions having F_3 less than around 0.82 μm, we set $f = 0.1$ mm/rev, thereby reducing one more variable. Thus, such NSGA-II points ($\bar{\mathbf{x}}$) are local searched for only one variable (that is, depth of cut). The above ASF problem will use $\mathbf{x} = (90, 0.1, a)^T$ and the resulting optimization problem will be a much simpler problem to solve than the original three-variable ASF problem.

Similarly, for all NSGA-II solutions having F_3 values greater than around 0.82 μm, we shall set $a = 5$ mm and use the feed as the only decision variable. Thus, the ASF problem given in equation 26 will use $\mathbf{x} = (90, f, 5)^T$ and the resulting problem is a single-variable problem which will be comparatively easier to solve than the original three-variable problem.

Putting in another way, we can add the above heuristics as additional constraints to the above ASF optimization problem:

$$\begin{aligned} v &= 90, \\ f &= 0.1, \quad \text{for } F_3(\bar{\mathbf{x}}) \leq 0.82, \\ a &= 5, \quad \text{for } F_3(\bar{\mathbf{x}}) > 0.82. \end{aligned}$$

Since the EMO solution $\bar{\mathbf{x}}$ to be local-searched is known before-hand, one of the two conditions from second and third constraints above can be suitably chosen.

After all EMO solutions are modified with the above heuristics-based local search procedure, the resulting solutions are collected together. Figures 11 and 12 show the variations of feed and depth of cut with surface roughness objective for all modified solutions. Compared to Figures 9 and 10, it is clear that the fluctuations in the solutions around the stipulated operating principles is now absent and more reliable principles are obtained. When a polynomial curve is fit around these points, the transition surface roughness (R_a) value is found to be 0.82 μm. The corresponding principles are shown below:

$$v = 90 \text{ m/min}, \tag{27}$$

$$f = \left\{ \begin{array}{ll} 0.1, & \text{if } R_a \leq 0.82 \\ 0.184 R_a^{3.09}, & \text{otherwise} \end{array} \right. \text{mm/rev}, \tag{28}$$

$$a = \left\{ \begin{array}{ll} 9.37 R_a^{3.18}, & \text{if } R_a \leq 0.82 \\ 5.0, & \text{otherwise} \end{array} \right. \text{mm}. \tag{29}$$

Figure 13 shows the three-objective trade-off frontier obtained after the local search procedure and the projections of it on three basic planes. Two zones with $R_a \leq 0.82$ μm and beyond are also marked in the figure. It is clear that surface roughness and operation time objectives are completely conflicting to each other, whereas for the zone 1 surface roughness is conflicting to cost per production as well. Interestingly, operation time and cost per production are mostly non-conflicting to each other, as indicated before.

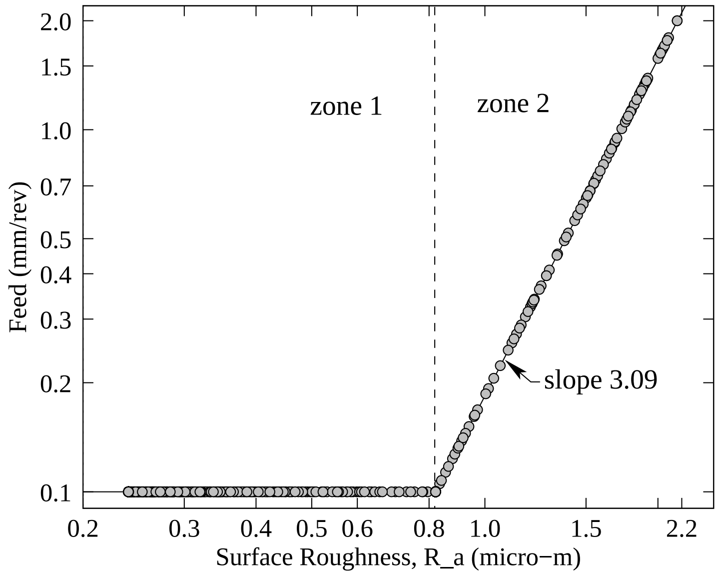

Figure 11: Feed as it varies with surface roughness on hybrid NSGA-II solutions with heuristics-based local search procedure for Case Study II.

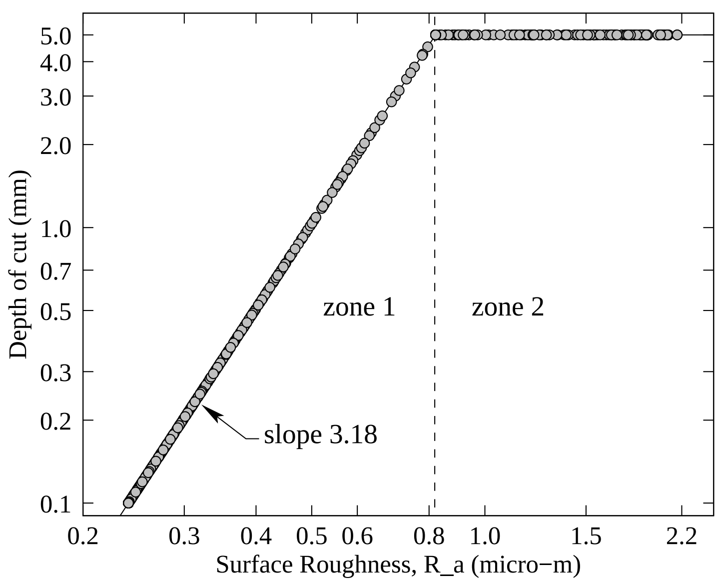

Figure 12: Depth of cut as it varies with surface roughness on hybrid NSGA-II solutions with heuristics-based local search procedure for Case Study II.

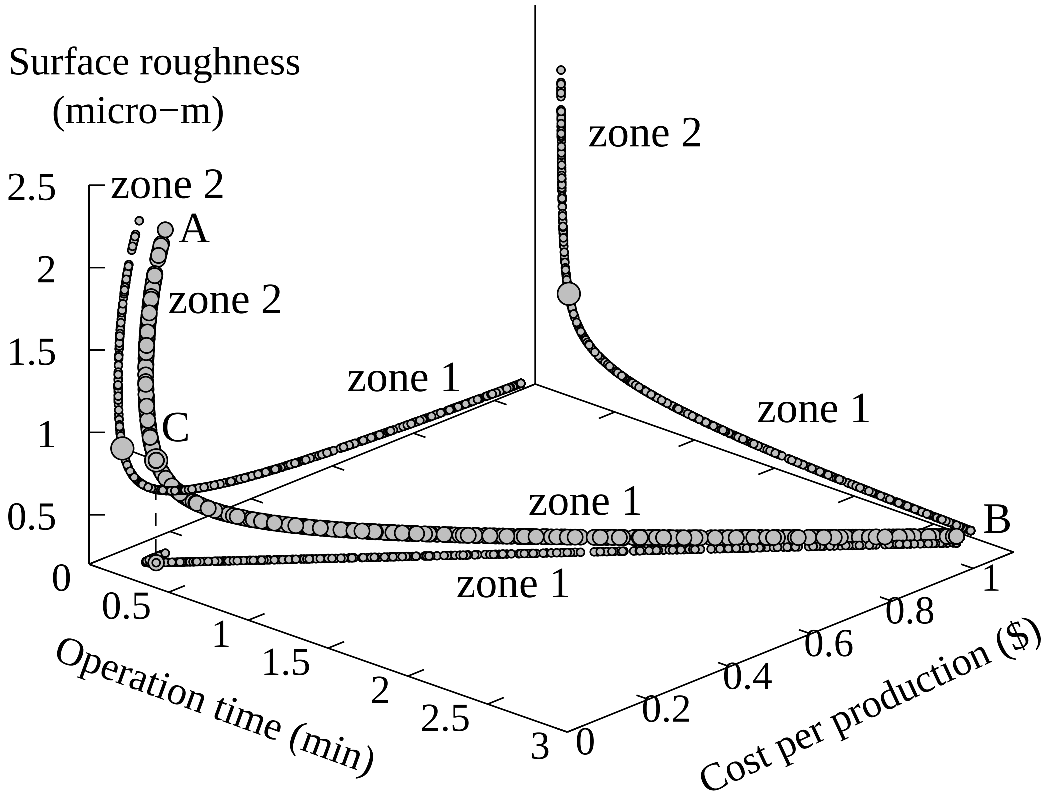

Figure 13: Three-objective trade-off frontier and its projections on basic planes obtained with the hybrid NSGA-II with heuristics-based local search procedure for Case Study II.

Table 2: Two extreme solutions and certain intermediate solutions of Case Study II.

T_p (min)	C_p (\$)	R_a (μm)	v (m/min)	f (mm/rev)	a (mm)	MRR (mm^3/min)	T (min)	F (N)	P (kW)
2.731	1.065	0.240	90.000	0.100	0.100	900.0	441677.5	0.005	0.025
2.304	0.899	0.254	90.000	0.100	0.120	1079.3	353857.5	0.006	0.026
1.715	0.669	0.281	90.000	0.100	0.165	1487.9	239193.6	0.009	0.027
1.194	0.466	0.319	90.000	0.100	0.249	2236.9	145452.1	0.016	0.028
0.601	0.235	0.417	90.000	0.100	0.582	5241.7	51466.3	0.046	0.031
0.295	0.115	0.605	90.000	0.100	1.901	17104.7	12158.5	0.205	0.035
0.211	0.083	0.821	90.000	0.100	5.000	45000.0	3735.6	0.693	0.039
0.181	0.073	1.098	90.000	0.246	5.000	110735.8	925.1	2.005	0.048
0.171	0.073	1.338	90.000	0.454	5.000	204246.0	358.2	4.128	0.056
0.166	0.081	1.627	90.000	0.831	5.000	373833.0	140.3	8.425	0.064
0.163	0.125	2.161	90.000	2.000	5.000	900000.0	36.0	23.757	0.079

Two extreme solutions and certain intermediate trade-off points are shown in Table 2. It is interesting to note that both $P(\mathbf{x})$ and $F(\mathbf{x})$ values are too small compared to their allowable values of 5 kW and 230 N. Thus, both constraints are inactive at the entire trade-off frontier. However, as discussed above the trade-off solutions make some of the variable bounds active, thereby causing these points to become optimal.

The first solution (A) in the table corresponds to the minimization of F_1 and the last solution (B) in the table corresponds to the minimization of F_3. However, a minimization of F_2 does not produce an extreme point on the three-dimensional trade-off frontier in this problem. The solution (C) is $\mathbf{x}^{(2,*)} = (90, 0.3342, 5)^T$ with an objective vector $(0.1754, 0.0725, 1.2121)^T$. Solutions A, B, and C are shown in Figure 13. Thus, the ideal and nadir point of this problem is as follows:

$$\begin{array}{lll} \text{Ideal point:} & (0.1626, 0.0725, 0.2399) & (\text{min, \$, } \mu\text{m}) \\ \text{Nadir point:} & (2.7308, 1.0651, 2.1611) & (\text{min, \$, } \mu\text{m}) \end{array}$$

A hypervolume computation of the NSGA-II front executed from the reference point $(2.731, 1.066, 2.162)^T$ (chosen as a slightly worse point than the above-computed nadir point) is found to be 4.483961, whereas the same for the hybrid NSGA-II front executed from the identical reference point is found to be 4.485401. Thus, in this problem, the hybrid NSGA-II approach is about 0.032% better, indicating that, although marginally, heuristics-based local search is able to improve the trade-off front. But importantly, the above study depicts the following fact. The use of NSGA-II on the three-objective problem can lead to an approximate set of trade-off solutions. However, a subsequent use of an innovization study on NSGA-II solutions may reveal valuable heuristics for a local search operation. A heuristic-based local search procedure may then find a set of well-converged solutions, which may be difficult to obtain using the original NSGA-II procedure.

6 Conclusions

In this paper, we have dealt with a couple of machining processes and demonstrated the use of hybrid multi-objective optimization algorithms (evolutionary and classical) in finding a set of well-converged trade-off compromised solutions. Due to their ability to find a set of trade-off solutions in a single simulation run, an evolutionary multi-objective optimization (EMO) algorithm, particularly the well-known NSGA-II procedure, has been shown to be appropriate. The NSGA-II obtained solutions have been verified by finding the extreme solutions and certain intermediate solutions by converting the multi-objective problem into a suitable single-objective optimization problem.

An EMO algorithm uses heuristics, instead of mathematical optimality conditions, in its search process. Hence, an EMO algorithm need not always converge to the true Pareto-optimal set. In this paper, we have discussed two different strategies to obtain a converged set of solutions. First, the NSGA-II solutions can be improved by using a local search procedure directly. A typical multi-criterion decision-making (MCDM) method can be used for this purpose. Here, we have demonstrated the use of the ϵ-constraint approach for the first problem, in which the ϵ-limit of the constraint comes from the obtained NSGA-II solution.

To reduce the computational effort of the overall local search procedure, for the first time, we have proposed an innovization-based strategy for the local search operation. An innovization task, proposed elsewhere [15], can reveal important properties common to the trade-off solutions. For problems where a clear indication of such properties emerge from the innovization study, the properties can be enforced as heuristics to the local search procedure. For the second case study, this strategy has shown to solve the problem more accurately and efficiently than the original NSGA-II procedure.

Importantly, this study has revealed interesting and useful operating principles that can remain as valuable knowledge to the operators. The study can be repeated for similar other machining tasks.

Acknowledgments

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