Customized Evolutionary Optimization of Compliant Mechanism to Generate Multiple Light Weight Topologies

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Abstract

The present work focuses on evolving the multiple light-in-weight topologies of compliant mechanism tracing user defined path. Therefore in this paper, the bi-objective set is formulated first on the optimization frame-work in which the helper objective of maximum diversity is introduced with the primary objective of minimum weight of elastic structures. Thereafter, the evolutionary algorithm (NSGA-II) is customized to efficiently deal with the constraint bi-objective, non-linear and discrete problem of compliant mechanisms. The existing NSGA-II algorithm is modified with various aspects and schemes such as, domain specific population strategy, domain specific crossover, parallel computing, minimum weight local search method etc. The flexibility of identifying the applied and boundary conditions of elastic structures are also coupled with the customized NSGA-II algorithm to promote non-dominated solutions. Two examples of compliant mechanisms tracing (i) curvilinear path and (ii) straight line path are solved and their light-in-weight topologies are presented.

Key words: Evolutionary Optimization, Compliant Mechanism, Helper Objective, Crossover Operator, Initial Population Strategy

1. Introduction

The present study aims to evolve the multiple light-in-weight topologies of compliant mechanisms. This target needs (i) a formulation that can evolve the multiple topologies which trace-out the prescribed path defined by the user and, (ii) an optimization technique which can efficiently solve the given optimization problem. Therefore in this paper, an equal attention is paid to both, the problem formulation and optimization algorithm. Here, the bi-objective set is formulated on the optimization frame-work in which the helper objective of maximizing the diversity of elastic structures is introduced with the primary objective of minimizing the weight of elastic structures. The functional aspect of these compliant mechanisms to trace the user-defined path is ensured by imposing the constraint on each precision point of user-defined path [1–6]. To efficiently solve the constraint bi-objective, non-linear and discrete problem of compliant mechanisms, the evolutionary algorithm (NSGA-II [7]) is customized with various schemes. From the problem formulation to optimization technique, various aspects and schemes of structural topology optimization are involved in this paper, therefore the associated literature is classified in different categories and reported in the following sections.

1.1. *Definition of Compliant Mechanism*

Compliant mechanisms (CM) are flexible elastic structures which can deform to transmit the force and/or generating some desired path on the application of applied load [8,9]. The two approaches for designing the compliant mechanism are found in literature. In the Ist approach, the designs are inspired by the traditional kinematic synthesis of rigid-body mechanism called *pseudo-rigid-body mechanisms* [10,11]. The IInd approach is a continuum mechanics based approach which generates monolithic elastic structure called *compliant mechanism*. These mechanisms have shown many advantages over pseudo-rigid-body mechanisms as jointless and monolithic structures, involved less friction, wear and noise [12], ease of manufacturing without assembly, light weight devices [13] etc. The applications of compliant mechanism are in the areas of product design, off-shore structures, smart structures, MEMS [9] etc.

1.2. *Single and Multiple Criteria of Designing Compliant Mechanisms*

The different criteria of providing the flexibility and required stiffness to the compliant mechanisms have been incorporated in the past studies. A few common measures which are formulated using either a single- or multi-objective sets are listed as, (i) minimization of weighted linear combination of deformation at the prescribed output port and strain energy [14], (ii) minimization of least-square errors between prescribed and actual values of geometric [15,16] and mechanical advantages [17], (iii) maximization of the ratio of mutual energy to strain energy [18,19], (iv) maximization of mechanical efficiency, geometric and mechanical advantage, and minimization of the maximum compressive load [20], (v) maximization of mutual potential energy [21] (vi) minimization of the weight and the supplied input energy to the structures [1,4–6] etc. With the help of these measures, various aspects of compliant mechanisms are incorporated.

1.3. *Methods to Synthesis Compliant Mechanism*

Several studies based on the continuum mechanics approach have been done by considering the homogenization method [22,23] or material density approach [24] in which the discrete nature of designing problem is converted into the continuous variables problem. It results an easy handling of the optimization problem solving but simultaneously, a threshold value is required for each assigned variable. Sometimes, any arbitrary assignment of the threshold value may lead to non-optimal solution. The classical methods of optimization (sequential linear programming, sequential quadratic programming, methods of moving asymptotes etc.) are used to deal with the continuous variables problem of compliant mechanism but these methods can stuck at some local optimum design while solving the non-linear problems. Therefore to overcome the present issue, another approach is discussed in the next paragraph.

A method of using a binary (0-1) representation of material for the continuum mechanics based approach helps to preserve the discrete nature of the structural and compliant mechanism related problems [25–29]. The binary, material-void design domain results in a discrete, typically non-convex space [30] and allows for a precise, although discretized, topology boundary. But, this representation usually results in 'checker-board' pattern problem and 'floating elements' of material which are disconnected from the main part of structure in the design domain. This approach can easily integrate with any evolutionary algorithm (EA) to evolve the optimum designs and structures because these algorithms can handle the non-linearity and discreteness involved in the topology optimization of structures and compliant mechanisms. Further, the EAs directly deal with the multi-objective problems, so there is no need to convert the multi-objective set into single objective. Generally, the evolutionary algorithms consume much higher time to evolve the optimum set of solutions.

1.4. *Binary Representation of Structures*

In this section of the paper, a few important studies using binary representation of material for the synthesis of compliant mechanisms are discussed which are modeled using either truss/frame ground structures or two-dimensional continuum structures and are optimized using any evolutionary algorithm. In truss/frame ground structures, the presence of a truss/frame element depends on the value of a binary bit. With the additional approaches of flexible building blocks [31], spanning tree theory [32] and load path synthesis [21], the topologies of compliant mechanisms are generated which are well-connected and free from the gray scale and hence, result in the improved designs.

For representing the two-dimensional continuum structure using a Boolean variables, a design domain is discretized into quadrilateral elements and each element of the structure is either represented by material or void depending on the boolean variable value. Using a modified evolutionary structural optimization (ESO) procedure [33], genetic programming [20] and genetic algorithms [1–6,34–41], the compliant mechanisms are designed with different objectives and tasks. Using the morphological technique of representing a structure, various problems of compliant mechanisms and structural optimization are solved in which Bezier curves are used to represent the shape, size and topology of structure [34–37].

1.5. *GA Operators*

In the early studies, the genetic algorithm (GA) was used for structural topology optimization using single point crossover and bit-wise mutation operators [25]. Extending the idea of using GA, the structural optimization problems with different kinds of objective functions and constraints were solved. Later, a two-dimensional crossover operator which divided the design domain into four rectangular sub-domains was used for further improving the GA evolved designs [26–28]. After the feasibility of GA-based optimal designs, the study [29] emphasized on its flexibility and demonstrated its potentialities over the classical optimization methods. A special two-dimensional crossover operator and a mutation operator were introduced to solve the various structural topology optimization problems.

Later, the novel morphological representation scheme using the arrangement of skeleton and flesh to define a structure was used for designing the structural and compliant mechanism problems [34–37]. Different ways of performing the crossover operator were suggested in these studies using morphological representation scheme of strings.

A crossover operator was introduced elsewhere [42] in which the ends of chromosome string was joined to form a circular loop. The recombination operator than divided the loop into two halves and swapped between the two parents. Later, the chromosome repairing scheme further modified the working of operator [43].

A two-dimensional crossover operator which exchanged the row/column of the design domain was introduced for shape optimization [44,45]. The same operator was also successfully used for generating the topologies of path generating complaint mechanisms [1,2,4–6] by the authors of this paper.

1.6. *Studies Based on Path Generating Compliant Mechanism*

The present study concentrates on those compliant mechanisms whose functional aspect is to generate the user-defined path (knows as *path generating compliant mechanisms*). These mechanisms are synthesized on the non-linear FE models to capture the large deformation of elastic structures [46–48]. In the relevant studies, first the path generating compliant mechanisms are designed with single-objective optimization formulation of minimizing the summation of deviation between the precision points of prescribed path and corresponding points on actual path [37]. This optimization problem is solved using the morphological representation scheme with GA. In another attempt, the Euclidean distance between each precision point of prescribed path and corresponding point on actual path is represented as one objective. Therefore, the optimization problem becomes multiple objective and the number of objectives are equal to the number of precision points [39]. The NSGA-II algorithm [7] is used in the study. Finally, the solution which shows the minimum sum of these individual objectives, is chosen. The above two studies are based on Euclidean distance based objective function which can not limit the gap between the prescribed path and actual path traced by the compliant mechanisms. Therefore, it can even perform in the worst case when these paths are apart from each other [39].

Another problem associated with the Euclidean distance based objective function on precision points is that, these precision points are characterized either by a same level of input load or input displacement which can result in an artificial constraint in the design problem. It might also misrepresent the nature of design problem by requiring the shape, size, orientation and position of the prescribed path to be optimized all at once. Therefore, the Fourier shape descriptor-based objective function is introduced in the study [49]. The same objective function is used in study [40] for the synthesis of path generating compliant mechanism using curved frame element and optimized using GA. But, this formulation needs the tuning of multiple user-specified parameters which can effect the optimum solution.

To overcome the above defined problems, the authors of this paper suggest to consider the functional aspect of these compliant mechanisms as a compulsory task. Therefore, the constraint is imposed at each precision point to limit the maximum gap between the prescribed and actual path [1–6]. These constraints are designed on the basis of one user-defined parameter called *percentage of allowable deviation* (η) which can be tuned beforehand [5]. The details of the same formulation are discussed in Section 2.

1.7. *Closure*

The above mentioned studies describe the various properties entangled with the topology optimization of compliant mechanisms. Similarly, a few schemes and aspects are incorporated in the present work. Thus, the paper first describes the constraints and bi-objective optimization formulation in Section 2. The local search based customized NSGA-II algorithm is explained in Section 3. Thereafter, the generated topologies tracing (i) curvilinear path, and (ii) straight line path are presented in Section 4. The study is concluded in Section 5 with prospective future work.

2. Bi-Objective Formulation

In the present work, the bi-objective set is formulated on the optimization frame work. We keep the primary objective as minimization of weight of elastic structures but, the helper objective is designed to deals with the shape, size and topology of evolved structures. Thus, it is evaluated with respect to some reference solution or design. This design can be chosen from the available set of optimum elastic structures based on the previous practice of the designers and decision makers. As the problems solved in this work are different than the problems exist in literature, the topology generated after the single objective optimization of minimum weight of elastic structure is considered as the *reference* design. The estimation of diversity objective is explained in Section 3.2 which indicates that this helper objective of maximum diversity with respect to the *reference* design assists the optimization algorithm (NSGA-II) to maintain the diversity in the GA population [2].

Before going into the details of the formulation, first the design domain of compliant mechanism (50 mm by 50 mm) is explained as shown in Figure 1 which is categorized into three regions of interest.

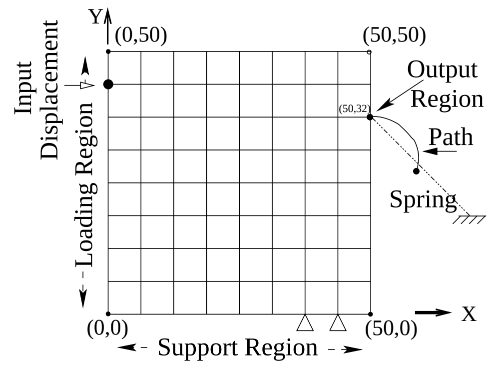


Fig. 1. A design-domain with loading, output and support regions.

The Ist region is called support region where the nodes of an element of the elastic structure are restrained with zero displacement. In the IInd region (loading region i.e. a node of an element), some input displacement boundary condition is applied. The output region is the IIIrd region, that is, a fixed point on the elastic structure which traces out the desired path defined by user.

In this work, the origin of the design domain is fixed on its left hand side and the output region is positioned at the coordinate $(50, 32)$ of the structure. As Figure 1 shows, a spring of constant stiffness ($\kappa = 0.4$ KN/m) is attached at the output point for providing some resistance during the deformation of elastic structure.

As discussed earlier, the essential functional aspect of path generating compliant mechanism is to trace the prescribed path. Thus, the same compulsory task of compliant mechanism is accomplished by imposing the constraints at precision points [1–6]. These hard constraints bound the maximum distance between the prescribed and actual paths for all feasible designs. A hypothetical case is shown is Figure 2 in which a prescribed path and an actual path traced by the elastic structure after FE analysis are drawn. Here, the prescribed path is represented by N precision points. The corresponding points on an actual path traced by the elastic structure is evaluated from geometrical non-linear FE analysis based on equal load steps.

To physically represent the constraints, first an euclidean distance (say d_1) is evaluated by estimating the distance between the current (i) and previous ($i-1$) precision points and get multiplied by a factor η called as *percent of allowable deviation*. Then, another euclidean distance (say d_2) between the current precision point (i) and the correspond-

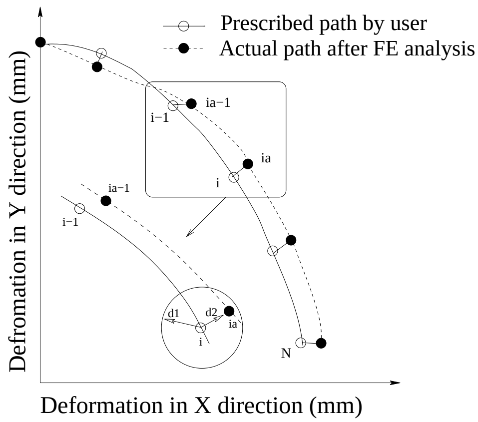


Fig. 2. The prescribed path and an actual path traced by the elastic structure after FE analysis.

ing point (i_a) of actual path is calculated. Based on these calculations, a constraint is imposed at each precision point which ensures that $d_2 \leq d_1$. A pictorial significance is shown in Figure 2 in which, if a circle of radius d_1 at the current precision point (i) is drawn, then the corresponding point (i_a) of actual path must lie within or on the circle to satisfy the constraint on each precision point. The mathematical representation of constraints at each N precision points is given in Equation 1. Any elastic structure which satisfies these constraints can guarantee to accomplish the task of tracing the path based on user-defined allowable deviation (η).

The single- and bi-objective sets are given in Equation 1 in which both the problems are subjected to same set of constraints.

Single-objective optimization:

Minimize: Weight of structure

Bi-objective optimization:

Minimize: Weight of structure (primary obj.),

Maximize: Diversity of structure (helper obj.),

Both problems are subjected to:

$$1 - \frac{\sqrt{(x_{ia} - x_i)^2 + (y_{ia} - y_i)^2}}{\eta \times \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2}} \geq 0, \quad i = 1, 2, ..., N$$

$$\sigma_{flexural} - \sigma \geq 0, \tag{1}$$

where $\eta = 15\%$ is the permissible deviation (kept fixed in this paper, (refer study [5] for more details), and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the structure, respectively.

3. Customized Evolutionary Algorithm

The evolutionary algorithms are widely used to solve the real-world optimization problems. Many times, these algorithms are modified according the nature of problems so that they can evolve improved and refined solutions. In the present work, a popularly used elitist non-dominated sorting genetic algorithm (known as NSGA-II, [7]) is customized to efficiently deal with the constraint multi-objective structural topology optimization problem. The NSGA-II algorithm is used here as global search and optimizer which has shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front for two objective problems. A detailed description of NSGA-II can be found elsewhere [7]. In short, NSGA-II is population based evolutionary optimization procedure which uses mathematical partial-ordering principle to emphasize non-dominated population members and a crowding distance scheme to emphasize isolated population members in every iteration. An elite-preserving procedure also ensures inclusion of previously found better solutions to further iterations. The overall procedure with N population members has a computational complexity of O(NlogN) for two and three objectives problems and has been popularly used in many studies. The NSGA-II is also adopted by a few commercial softwares (such as iSIGHT and modeFRONTIER). A code implementing NSGA-II is available at *http://www.iitk.ac.in/kangal/codes.shtml* website.

A local search method is coupled with the customized NSGA-II algorithm to further refine the non-dominated solution's based topologies of compliant mechanisms. The flow chart of local search based customized NSGA-II algorithm is shown in Figure 3. It shows various schemes like initial population, structure representation and repairing techniques, two-dimensional crossover operator, mutation operator, parallel computing etc. which are expected to efficiently solve the topology optimization problems of structures and compliant mechanisms.

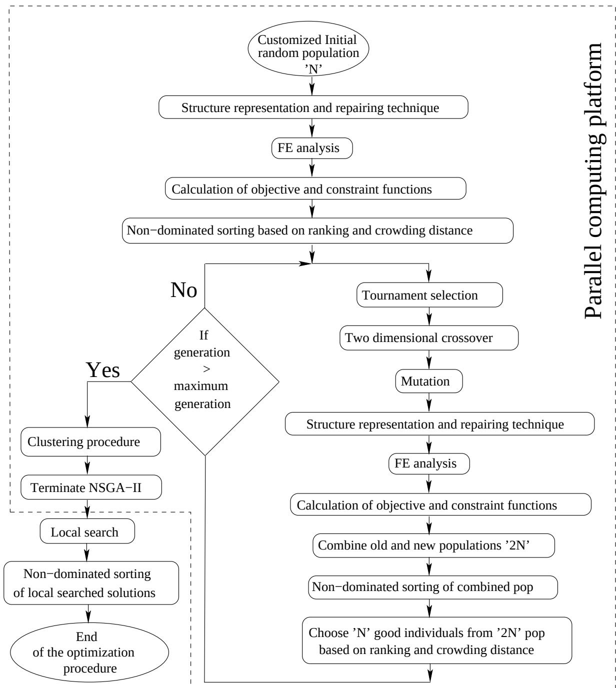


Fig. 3. A flow chart of customized NSGA-II algorithm

3.1. *GA Parameters*

A population of 240, crossover probability of 0.95 and mutation probability of (1/string length) are assigned and the NSGA-II algorithm is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 637 bits is used as shown in Figure 4. This string is made

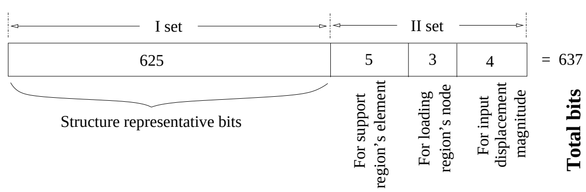


Fig. 4. A binary string comprises of two sets.

of two sets in which the Ist set of 625 bits represents the shape of structure (described in Section 3.2) whereas, the decoded value of IInd set identifies the support and loading positions in their respective regions (refer Figure 1), and magnitude of input displacement. For the same, the remaining 12 bits of IInd set are further divided into three sets of five, three and four bits as shown in Figure 4. The decoded value of first five bits indicates the location of an element from the origin where the elastic structure is to be supported. The decoded value of subsequent three bits helps in determining the loading position, that is, a node where the input load is applied. The decoded value of last four bits are used to evaluate the magnitude of input displacement which can vary from 1 mm to 16 mm at step of 1 mm. The above mentioned flexibility is implemented to come-up with the optimum combinations of support and loading positions, and input displacement magnitude to promote the non-dominated solutions through the NSGA-II run. A detailed significance of additional bits will be discussed later along with the results presented in the study.

3.2. *Structure Representation Scheme*

Before starting the details of initial population strategy, the structure representation scheme is described here. A binary string of 625 bits (refer Figure 4) is used to represent the shape of structure. First, a binary string is copied to two dimensional representation as shown in Figure 5. Thereafter, the

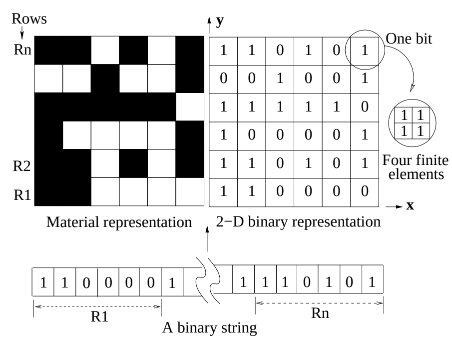


Fig. 5. A representation of structure using binary string and material.

material-void representation of each grid is chosen based on the binary bit value, for example, bit value 1 signifies that material is present whereas, 0 represents the void. This scheme divides a design domain of structure into 25 × 25 (= 625) grids in x and y directions, respectively.

The helper objective of diversity is evaluated in this paper by finding the dissimilarity in the bit value at each gene position of the binary strings of the *reference* design and the elastic structure evolved from the GA population.

3.3. *Domain Specific Population Strategy*

In the present work, a strategy of generating an initial random population using a (0-1) binary representation of elastic continuum structures is used in which the design domain's three regions of interest are connected through the intermediate points [4,6]. A pictorial view is drawn in Figure 6 to show

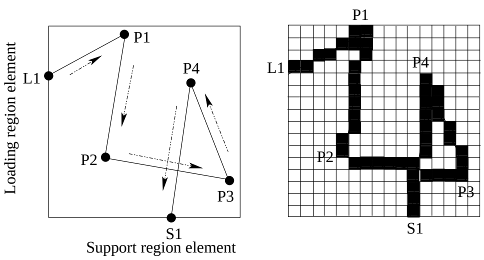


Fig. 6. Connectivity between support and loading regions.

a connectivity between the support and loading region's elements. Here, the positions of support and loading region's elements are calculated after decoding the II nd set of binary string (refer Figure 4) whereas, the location of output region's element is fixed through-out the study.

For connecting the support and loading regions, a random integer number between 1 to 5 is generated to decide the number of intermediate points through which these two regions get connected. Depending upon the number of intermediate points, the coordinates of each intermediate point is randomly generated within the design domain. The points P1, P2, P3 and P4 in Figure 6 show the location of intermediate points and, the support (S1) and loading (L1) region's elements are connected through these points by straight lines. Thereafter, a material is assigned to those elements where these straight lines pass. A material connectivity of the above mentioned regions is also shown in Figure 6. Similarly, a set of piece-wise linear line segments between the support and output regions and another set between the loading and output regions are explained. Therefore depending on the randomly generated intermediate points, an initial population for the NSGA-II algorithm is developed.

A repairing technique is also employed in which if two elements generate a point connection, then the given procedure puts one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity.

3.4. *Finite Element Analysis*

After the custom initialization, structure representation and repairing techniques, the elastic structure is analyzed for stress and deformation by FE analysis. In this study, one grid of a structure (as described in Section 3.2) is further discretized into four finite elements with same binary variable value as shown in Figure 5. Therefore in the present process, the structure is discretized with 4×625 (= 2500) 4-node rectangular finite elements and analyzed through a non-linear large deformation FE analysis using *ANSYS* package. But, the GA operations are performed on the same structure represented by 625 bits.

3.5. *GA Operators*

Crossover is an important GA operator which is responsible for the search aspect of the algorithm. It creates new solutions which differ from the parent solutions. In this paper, a domain-specific crossover operator is used which works on exchanging the sub-domains between the two parent solutions [3]. For this crossover, initially, the three random points (P1, P2 and P3) are generated on their respective sides of the given design domain as shown in Figure 7. Thereafter, all these points are joined

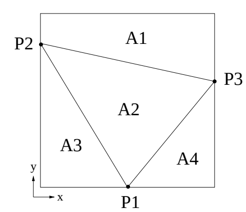


Fig. 7. Sub-divided domain for crossover.

through straight lines which results in dividing the design domain into four sub-domains (A1, A2, A3 and A4). For each sub-domain, a coin is flipped to decide whether to exchange the corresponding sub-domain between the two parents. Therefore, an area of the exchanged sub-domain between the two parents varies and depends on the randomly generated points which incorporates the stochasticity in the operator. For the crossover of 12 bits of IInd set, a standard single point crossover is used in the present study.

Mutation operator is another GA operator which generates new solutions in the population but usually it is done with a low probability. Here, it is done with a probability of (1/string length) on each bit of a string of Ist set to change from a void to a filled or from a filled to a void grid. For mutating the remaining 12 bits of IInd set, first the decoded values of support and loading regions, and magnitude of input displacement are evaluated and then, these values are perturbed within the range of $\{-2, 2\}$ at their original values. Here, it is ensured that the perturbed values of above three applied and boundary conditions do not fall outside their respective bounds. This mutation operator helps to get the nearest integer value at the original. After perturbation, these mutated values are again coded into the binary string of 12 bits.

3.6. *Parallel Computing*

A distributed computing platform is used in the present study to reduce the computational time of designing and synthesis of compliant mechanisms. In this parallelization process, the root processor first initializes a random population. Then, it divides the entire population into different sub-populations in proportion to the number of processors available. After this, each sub-population is sent to different slave processors. These slave processors further evaluate the objective functions and constraints values, and send them to the root processor. Thereafter, root processor performs the GA operators, like selection, crossover and mutation operators, non-dominated front ranking etc. on the population and replaces it with good individuals. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through *ANSYS* FE package which consumes the maximum time of the optimization procedure ([1–5]). A MPI based Linux cluster with 24 processors is used in the present study. A detailed specification and configuration of the Linux cluster are given at *http://www.iitk.ac.in/kangal/facilities.shtml* website.

3.7. *Clustering Procedure*

For an adequate convergence near to the global 'Pareto-optimal' front, the evolutionary algorithms (EA) need a fairly large number of population members and generations depending upon the problem complexity. Thus, the number of feasible solutions after the EA run are usually close to the population size. It is not advisable to represent so many solutions to the end user for a subsequent decision-making task. Therefore, the clustering procedure is employed in the study in which the neighboring solutions are grouped together and solutions from each group representing that region of the non-dominated front are chosen as representative solutions [50]. Figure 8 shows the procedure pictorially. After clustering, the parallel NSGA-II algorithm is terminated. Thereafter, the local search method (described in next section) is employed on each representative NSGA-II solutions based designs to refine them.

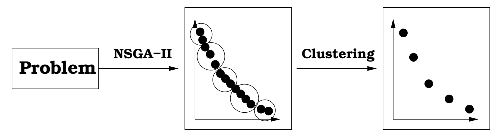


Fig. 8. A clustering Procedure.

3.8. *Local Search Method*

This is an important section of the given optimization procedure because the local search method is executed in such a way to generate minimum weight solutions. The details are as follows. The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is given in Equation 2.

$$F(x) = \frac{\overline{w}_1(f^x_{p_{max}} - f^x_p)}{f^x_{p_{max}} - f^x_{p_{min}}} + \frac{\overline{w}_2(f^x_{h_{max}} - f^x_h)}{f^x_{h_{max}} - f^x_{h_{min}}}, \qquad (2)$$

where, f^x_p and f^x_h are primary and helper objectives, $f^x_{p_{min}}$ & $f^x_{p_{max}}$ and $f^x_{h_{min}}$ & $f^x_{h_{max}}$ are minimum and maximum values of primary and helper objectives in the population respectively, and $\overline{w}_1$ & $\overline{w}_2$ are the corresponding weight to the objective functions. In Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space [51]. In the present work, an emphasis is given to generate the minimum weight topologies therefore, the weight vector assigned to primary and helper objectives is $\overline{w}_1 = 1$ and $\overline{w}_2 = 0$.

In the local search method, first the weighted sum of scaled fitness of a selected representative solution is evaluated as given in Equation 2. Thereafter, one bit of representative solution is mutated at a time and the design is extracted from the new string. This new string's based structure is analyzed by FE package and then, the objective and constraint functions are evaluated. If the new design does not satisfy any constraint, then the change in new string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string values. In case of mutating '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that of old string, or else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old string's weighted sum value, then it is accepted or else the change is discarded. In case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the grids having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the change is accepted. This process is repeated till all bits are mutated once. If there is no change in the value of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As discussed in Section 3.4, one binary bit represents four elements for FE analysis. Therefore, a binary string of 625 bits represents a structure which is discretized with 2500 finite elements. In case of local search, the previous binary strings (625 bit) of representative solutions are reconstructed into the new binary strings of 2500 grids. These grids represent the same structure of 2500 elements and the local search search is performed on these 2500 grids.

4. Evolved Light Weight Compliant Mechanism Topologies

In this section, the evolved topologies of compliant mechanisms are presented which are generated after solving the constraint bi-objective optimization problem of Equation 1 using customized NSGA-II algorithm. Two examples of compliant mechanisms tracing (i) curvilinear path and (ii) straight line path are solved and their associated topologies are presented.

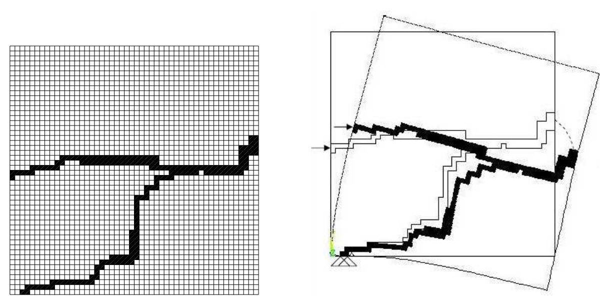
A few parameters are kept constant during the whole study such as, a material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for synthesis of compliant mechanism. The direction of input displacement is fixed along x direction. Here, the prescribed path is represented by five precision points and the trajectory traced by output region's node of the elastic structure is evaluated through a geometric nonlinear FE analysis using $ANSYS$ package. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. After the termination of NSGA-II algorithm, maximum six representative solutions are chosen from the non-dominated set of NSGA-II solutions with the help of clustering procedure.

4.1. *Compliant Mechanisms Tracing Curvilinear Path*

In this example of compliant mechanism, the single and bi-objective optimization problems are solved. The evolved topology from the single - objective study is used as *reference* design to evaluate the helper objective of bi-objective optimization.

4.1.1. *Reference Design Tracing Curvilinear Path*

The single-objective study of minimum weight subjected to the precision point based constraints evolves the *reference* designs of 0.545 gm which is shown in Figure 9. The design consists of open loops



(a) Undeformed topology (b) Final deformed topology

Fig. 9. The undeformed and final deformed *reference* design.

of material joining three regions of interest (support, loading and output regions). This *reference* design is supported at an element which is positioned at 2 mm away from origin. The input load of 5 mm is applied at a node which is located at 24 mm away from origin. The above mentioned applied and boundary conditions are identified by the customized evolutionary algorithm using the IInd set of binary string and are tabulated in Table 1.

4.1.2. *Multiple Light-in-Weight Topologies*

In the last section, the *reference* design was evolved which is now used to determine the diversity of structures of bi-objective optimization problem. The representative NSGA-II solutions (a to f) and the refined solutions (1 to 6) after minimum weight local search are shown in Figure 10. In this figure,

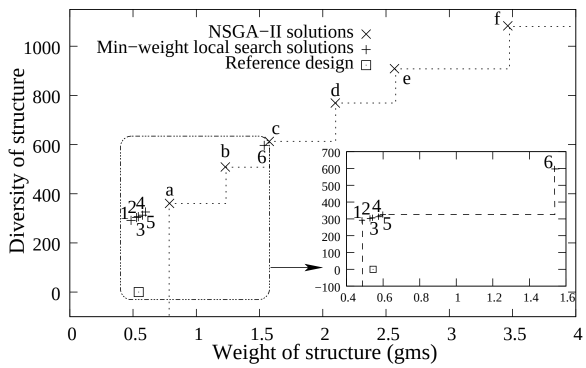
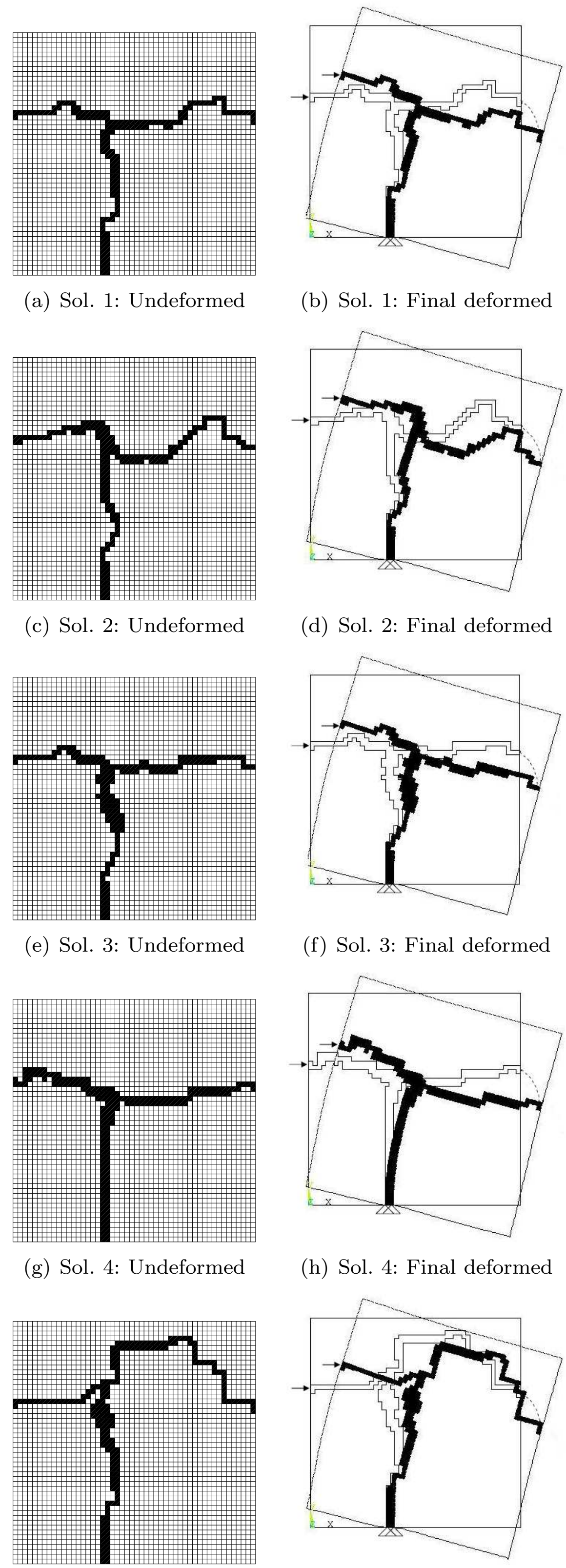


Fig. 10. NSGA-II and Local search solutions.

all the minimum weight local search solutions are clustered near to the *reference* design except solution 6 in primary objective. This solution is bulky too and hence, it is discarded. The position of *reference* design is also shown in Figure 10. Here, the diversity of *reference* design is evaluated as zero because it is calculated with respect to its own design. The Figure 10 indicates that the solutions 1, 2 and 3 are evolved as lighter in weight designs in comparison to the *reference* design. Thus, these solutions dominate the *reference* design obtained from the single-objective study. It reveals a fact that the bi-objective set proposed in the study not only evolves the multiple topologies but also, assists the optimization procedure to generate minimum weight topologies. On the other hand, a single objective study only deals with the optimization of one objective and may result in a premature suboptimal solution for the given example of compliant mechanisms tracing curvilinear path [5,6].

In the Figure 11, the undeformed and final deformed topologies (1 to 5) are presented. Among the non-dominated set of these solutions, the solution 1 is evolved as a minimum weight structure (refer Figure 11(a)) but at the same time, it shows the minimum diversity with respect to the *reference* design (refer Figure 10). On the other end, the topology of



(a) Sol. 1: Undeformed (b) Sol. 1: Final deformed

(c) Sol. 2: Undeformed (d) Sol. 2: Final deformed

(e) Sol. 3: Undeformed (f) Sol. 3: Final deformed

(g) Sol. 4: Undeformed (h) Sol. 4: Final deformed

(i) Sol. 5: Undeformed (j) Sol. 5: Final deformed

Fig. 11. Non-dominated light in weight topologies of compliant mechanisms tracing curvilinear path.

solution 5 as shown in Figure 11(i) exhibits the maximum diverse structure with respect to the *reference* design among the set of light weight solutions (1 to 5), but it is evolved as a heavy in weight solution. Topologically, all the structures are same because these designs comprise of open loops of material between three regions of interest (support, loading and output regions). But, the different distribution of material in the design domain of each solution results in 'trade-off' between the posed objectives.

As discussed in Section 3.1, the flexibility is provided to the customized NSGA-II algorithm to identify the support and loading positions, and magnitude of input displacement. As the previous studies of the authors of this paper [5,6] suggest, these conditions can assist the customized NSGA-II algorithm to evolve non-dominated solution's based topologies. The optimum set of applied and boundary conditions of this example are tabulated in Table 1. It is observed from the table that the solutions 1 to 5 have identical applied and boundary conditions.

Table 1
Evolved support and loading positions and, input displacement magnitude of curvilinear path generating compliant mechanism.

Study → Conditions ↓	Single-objective (Ref. design)	Bi-objective (Solutions 1 to 5)
Support position (*mm*) (*from the origin*)	2	18
Loading position (*mm*) (*from the origin*)	24	32
Input displacement (*mm*)	5	7

The progress of the applied and boundary conditions for each non-dominated feasible solutions is shown in Figure 12 during the NSGA-II run. In the present case, the variety of these conditions can be seen during the initial iterations of NSGA-II. But, as the algorithm progresses, the elastic structures with a few applied and boundary conditions are feasible and non-dominated. After the termination of above algorithm, the elastic structures have identical conditions as mentioned in the last paragraph. The Figure 12 also shows that only a few set of applied and boundary conditions during the NSGA-II run evolve feasible solutions. It reveals that any arbitrary set of these conditions cannot evolve the feasible designs which trace-out the given prescribed path. Also, all those sets of applied and boundary conditions which generate such feasible solutions are not emerged as non-dominated. Hence, the provided flexibility of identifying the applied and boundary conditions not only evolves the non-dominated solutions based topologies but also, explores the possibility of non-optimum boundary conditions which might be considered in the previous practice of the designers or decision makers.

In the above topology optimization problem, the first feasible solutions are appeared after a few generation of NSGA-II algorithm because the optimization procedure starts with random initial population. To make the population members feasible, first the NSGA-II algorithm tries to satisfy the given set of constraints of the problem as described in Equation 1. As soon as, the first few feasible solutions are evolved, they start dominating other solutions based structures with same or different sets of applied and boundary conditions based on their objective function values. Meanwhile, if any solution based structure is alive and becomes non-dominated during NSGA-II run, then only it can propagate to further generations. In the present case of tracing curvilinear path by compliant mechanisms, only one set of applied and boundary conditions dominates the rest and evolves as the optimum conditions.

4.1.3. *Prescribed Curvilinear Path and Paths Traced*

A pictorial view of paths generated by the solutions obtained after the local search of single and two-objective studies is shown in Figure 13 along

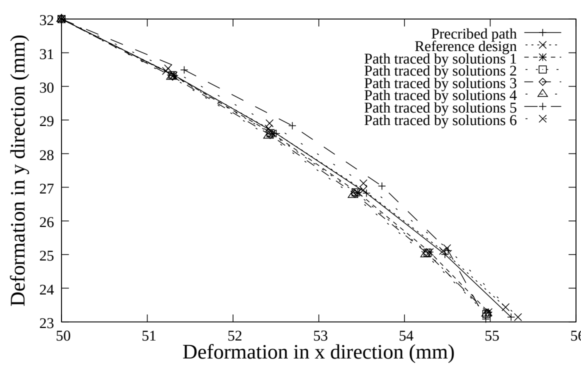
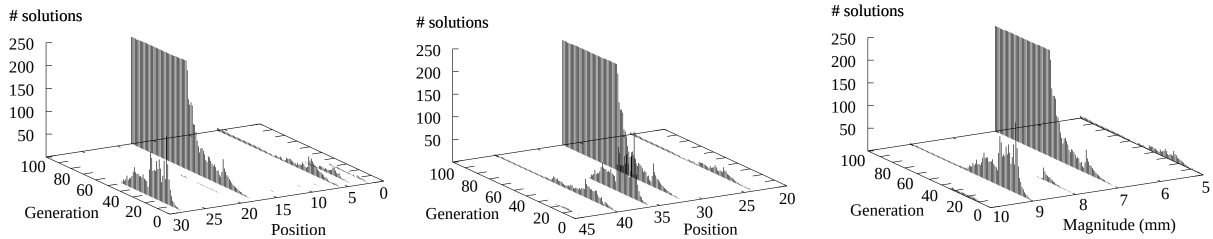


Fig. 13. Prescribed path and path traced by the *reference* design and by the minimum weight local search solutions.

with the prescribed path. This figure signifies the adherence between the prescribed path and paths traced by all local search solutions. For quantitative study of these paths, a Table 2 is drawn in which the values of maximum allowed d_1 at each precision point and distance d_2 between the precision



(a) Support position (b) Loading position (c) Magnitude of input displacement

Fig. 12. Progress of non-dominated feasible solutions during NSGA-II run.

Table 2
Deviation at precision points.

Precision points (PP)	1	2	3	4	5
Maximum allowed $\mathbf{d}_1$	0.3196	0.3142	0.3093	0.3056	0.3027
Reference design					
$\mathbf{d}_2$	0.1662	0.1714	0.1017	0.1088	0.3026
Two-objective study					
Solution 1: $\mathbf{d}_2$	0.0097	0.0439	0.1027	0.1903	0.3025
Solution 2: $\mathbf{d}_2$	0.0233	0.0664	0.1278	0.2086	0.3026
Solution 3: $\mathbf{d}_2$	0.0417	0.0841	0.1374	0.2110	0.3027
Solution 4: $\mathbf{d}_2$	0.0586	0.1145	0.1672	0.2269	0.3026
Solution 5: $\mathbf{d}_2$	0.1994	0.2987	0.2788	0.1214	0.3012
Solution 6: $\mathbf{d}_2$	0.2208	0.3142	0.2941	0.1851	0.0809

point and corresponding point on the actual path are given. The d_2 value of the *reference* design shows that it first increases till precision point 2 and then, its value is decreased at precision point 3. Finally, it makes the precision point 5 critical because d_2 value is close to maximum d_1 value. A similar behavior can be seen in the d_2 value of solution 5, but the traced path intersects the prescribed path after precision point 4 (refer Figure 13). Rest of the solutions show the increasing trend of d_2 value which make the precision point 5 critical. This shows that the precision points defining the extreme part of prescribed path become critical. But an importance of the constraints on initial precision points can not be ignored that assist the optimization algorithm to come up with the feasible PGCM topologies [5]. Overall, the figure and table of this section show the fulfillment of the task of tracing the prescribed using the precision points based constraints formulation. The same set of constraints were used with the different objective set in the studies [1,4–6] by the authors of this paper. This indicates that the precision point based constraints can be coupled with any single and bi-objective sets to satisfy the functional aspect of path generating compliant mechanisms. The prescribed path here is designed in such a way that an output point of each structure has to deform to 10.48% in x-direction and 17.72% in y-direction with respect to the size of design domain.

4.1.4. *Time Complexity*

The parallel computing platform is used in the present study to deal with the computational extensive problem of compliant mechanisms. The maximum time of optimization algorithm is consumed in the non-linear finite element analysis whereas, the other function evaluations and the communication among the processors take a smaller time. The times taken by the given optimization procedure for solving the single and bi-objective optimization problems are shown in Table 3. During the NSGA-II run, 24 processors are used which helps in reducing the computational time almost in proportion to the number of processors available. The local search is performed individually in different processors which take a considerable amount of time of the given optimization procedure to refine the representative NSGA-II solutions.

Table 3
Time taken by given optimization procedure.

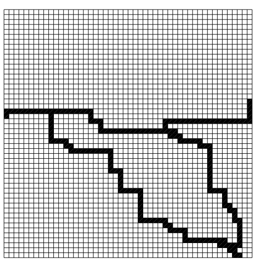
Problem	NSGA-II Time (hrs)	Local search Time (hrs)
Single-objective	5.81	8.65
Two-objective	7.23	Solution 1: 14.71
		Solution 2: 16.18
		Solution 3: 9.43
		Solution 4: 6.34
		Solution 5: 24.21
		Solution 6: 32.45

4.2. *Compliant Mechanisms Tracing Straight Line Path*

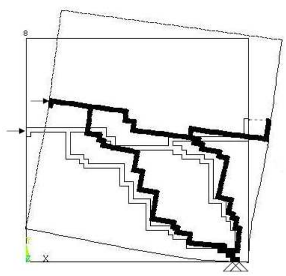
The design domain of this example is same as the previous one but instead of tracing the curvilinear path, now the compliant mechanism has to generate the straight line path at the same output point. It makes the problem even more difficult to solve because the categorization of design domain in the study promotes those elastic structures which follow some curvilinear paths [2,6].

4.2.1. *Reference Design Tracing Straight Line Path*

The evolved *reference* design tracing straight line path is shown in Figure 14. The design contains one



(a) Undeformed topology



(b) Deformed topology

Fig. 14. The deformed and undeformed *reference* design.

closed loop of material and has a weight of 0.565 gm. The design is supported at an element which is located at 46 mm away from the origin. The input load of 5 mm is applied at a node which is positioned at 28 mm away from the origin. These applied and boundary conditions which are evolved by the customized NSGA-II algorithm, are also given in the Table 4.

4.2.2. *Light-in-Weight Topologies Tracing Straight Line Path*

The multiple light-in-weight topologies of compliant mechanisms are evolved in the present section which generate straight line path. The position of representative NSGA-II solutions (a to f) and their position after local search (1 to 6) are shown in the two-objective space of Figure 15. Based on primary and helper objectives, all the six local search solutions become the part of non-dominated front. The position of *reference* design is also shown in Figure 15 and it indicates that the *reference* design is evolved as minimum weight topology. Hence, the non-dominated front is represented by the *reference*

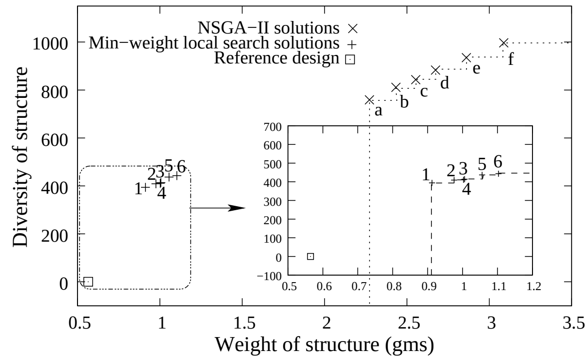


Fig. 15. The NSGA-II and minimum weight local search solutions.

design and local search solutions (1 to 6) of compliant mechanisms tracing straight line path.

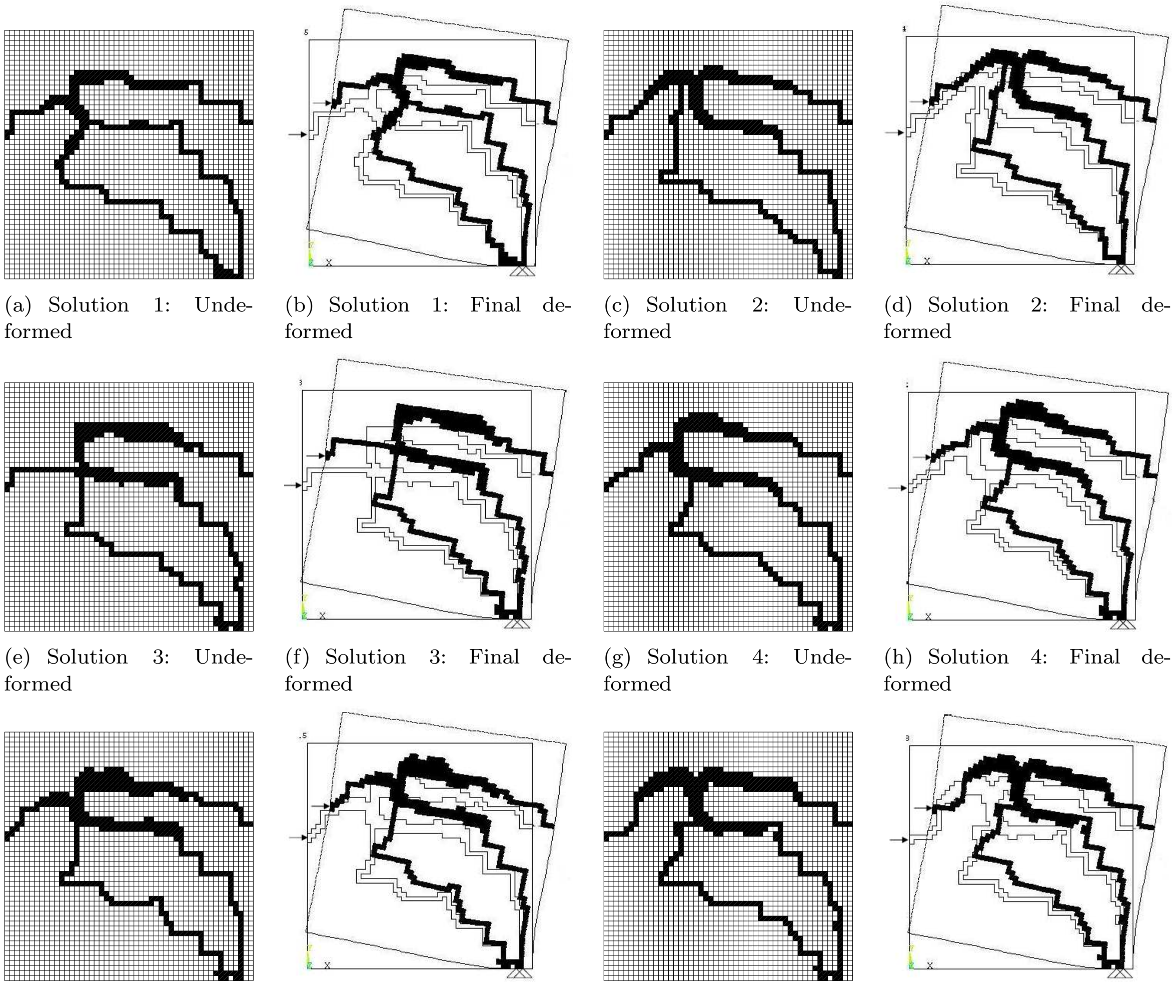
The undeformed and final deformed topologies of local search solutions (1 to 6) are shown in Figure 16. The solution 1 is evolved as minimum weight topology among the six local search solutions but exhibits minimum diversity with respect to the *reference* design. Another extreme solution 6 shows maximum diversity with respect to the *reference* design but evolves as heavy-in-weight solution. Topologically, all the local search solutions consist of one closed loop topology but, the kind of distribution of material joining the loop and, the loading and output regions results in 'trade-off' between the posed objectives. An interesting thing can be ponder here that the *reference* design and all six local search solutions have same one closed loop topology which signifies the necessity of these compliant mechanisms to trace the given straight line path [6].

The optimum applied and boundary conditions of above local search solutions are Tabulated in Table 4. These conditions are identical to the conditions

Table 4
Evolved support and loading positions and, input displacement magnitude of straight line path generating compliant mechanism.

Study → Conditions ↓	Single-objective (Ref. Design)	Bi-objective (Solutions 1 to 6)
Support position (mm) (*from the origin*)	46	46
Loading position (mm) (*from the origin*)	28	28
Input displacement (mm)	5	5

of *reference* design. The same applied and boundary conditions were also observed in the previous study



(a) Solution 1: Undeformed (b) Solution 1: Final deformed (c) Solution 2: Undeformed (d) Solution 2: Final deformed

(e) Solution 3: Undeformed (f) Solution 3: Final deformed (g) Solution 4: Undeformed (h) Solution 4: Final deformed

(i) Solution 5: Undeformed (j) Solution 5: Final deformed (k) Solution 5: Undeformed (ℓ) Solution 5: Final deformed

Fig. 16. Non-dominated light-in-weight topologies of compliant mechanisms tracing straight path.

of authors [6] when the different bi-objective set was used to evolve the compliant mechanisms tracing the same straight line path. It signifies that irrespective of the nature of bi-objective sets, the customized NSGA-II algorithm identifies the above optimum applied and boundary conditions which helps in generating the non-dominated PGCM topologies. This additional information might be useful to the designers and decision makers in their future practice. An interesting thing can also be seen here that the topologies are supported on their right-hand side while tracing the straight line path whereas in the previous example of curvilinear path tracing, the compliant mechanisms are supported on their left-hand side. The flexibility provided by the additional bits of IInd set to the customized NSGA-II algorithm identifies these support positions because the elastic structures supported on their right-hand side show minimum tendency to generate higher curvilinear paths. On the other hand, left hand side supported elastic structures tend to trace the curvilinear trajectories.

4.2.3. *Prescribed Straight Line Path and Paths Traced*

The straight line path traced by all local search solutions along with the prescribed path is shown in Figure 17. It shows that the continuum elastic structures do not trace the exact straight line path because it is too optimistic condition to generate the same path as it is prescribed for the given categorization of design domain. At the same time, the im-

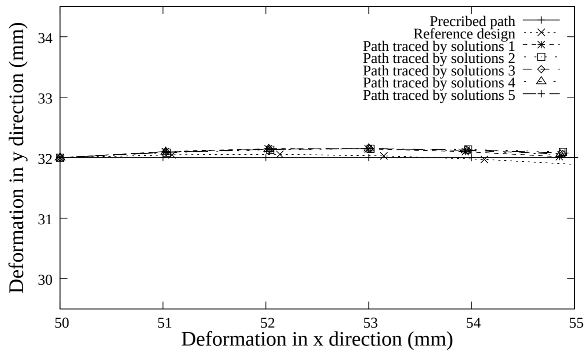


Fig. 17. Prescribed path and path traced by all local search solutions.

posed constraint at each precision point which limits the maximum deviation in terms of η value, assists the customized NSGA-II algorithm to evolve the feasible solutions. Table 5 is also drawn for quantitative analysis in which the d_2 value of all local search solutions first increases till the precision point 3 and then, its value is decreased at precision point 4. The d_2 value is again risen at precision point 5. Here, the maximum d_2 value is observed at precision point 3 which makes it critical along with precision point 5 for the solutions $1, 2, 5$ and 6. The *reference* design traces out the different nature of path (refer Figure 17) which intersect the prescribed path near to the precision point 4 with the same nature of d_2 value as described above. The straight line prescribed path here is designed in such a way that an output point of each structures has to deform to 10.00% in x-direction and 0.0% in y-direction with respect to the size of design domain.

Table 5
Deviation at precision points.

Precision points (PP)	1	2	3	4	5
Maximum allowed $\mathbf{d}_1$	0.15	0.15	0.15	0.15	0.15
Reference design					
$\mathbf{d}_2$	0.1006	0.1473	0.1496	0.1267	0.1350
Two-objective study					
Solution 1: $\mathbf{d}_2$	0.0918	0.1385	0.1489	0.1398	0.1485
Solution 2: $\mathbf{d}_2$	0.1039	0.1497	0.1460	0.1179	0.1487
Solution 3: $\mathbf{d}_2$	0.0939	0.1421	0.1499	0.1303	0.1207
Solution 4: $\mathbf{d}_2$	0.0982	0.1457	0.1500	0.1274	0.1287
Solution 5: $\mathbf{d}_2$	0.0974	0.1443	0.1499	0.1350	0.1492
Solution 6: $\mathbf{d}_2$	0.1002	0.1472	0.1498	0.1302	0.1473

The computation time involved to generate the compliant mechanisms tracing straight line is similar as presented in Section 4.1.4. The maximum time of the NSGA-II algorithm is consumed in the nonlinear FE analysis which is done in parallel computing platform using 24 processors. The local search again takes a considerable amount of time of the whole optimization algorithm to refine the topologies of representative NSGA-II solutions.

5. Conclusions

In this paper, the aim of evolving the multiple light-in-weight topologies of compliant mechanism was achieved by customized evolutionary optimization using (i) problem specific bi-objective formulation, (ii) modified NSGA-II algorithm and (iii) minimum weight local search method. The helper objective assisted the customized NSGA-II procedure to evolve 'trade-off' solutions with the primary objective. Although, the evolved solutions were topologically same, but the kind of distribution of material in the design domain of each non-dominated solution resulted in the conflicting scenario between the posed objectives. The existing NSGA-II algorithm was successfully customized with various schemes that solved the compliant mechanism problems, efficiently. The flexibility of identifying the applied and boundary conditions also assisted the customized algorithm to promote the non-dominated solutions. This is useful in the scenario of unknown applied and boundary conditions and also, it explores the possibilities of non-optimum conditions which might be considered in the previous practice of designers and decision makers. Overall, this study presents the guidelines of evolving the multiple compliant mechanism topologies on the optimization frame-work. It also opens the door of customizing the engineering design problem and optimization technique to solve them.

In the future work, the same algorithm can further be customized by incorporating with some new schemes. Another multi-objective set which can assist the optimization technique to evolve diverse cum light-in-weight topologies, can be coupled with the precision point based constraints to evolve of path generating compliant mechanism.

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