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Advancement of Path Generating Compliant Mechanisms (PGCM) Topologies by Initial Population Strategy of Customized Evolutionary Algorithm

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Abstract The present work demonstrates the advancement of evolved topologies of path generating compliant mechanisms by introducing initial population strategy with customized NSGA-II algorithm. The strategy is based on the categorization of design domain of compliant mechanisms that generates initial population of elastic continuum structures. It is anticipated in this paper that this strategy further customizes the evolutionary algorithm to efficiently deal with the compliant mechanism problems. In this work, constraint single- and bi-objective optimization formulations are used which were proposed in the previous studies of authors of this paper. Two examples of compliant mechanisms tracing downward curvilinear path and straight line path are solved and their respective topologies are presented. This study provides a platform to the designers and decision makers to understand the topological changes which result in trade-off between the posed-objectives. The present work also offers them to choose any PGCM topology as per their requirement from the non-dominated set of solutions.

Keywords Compliant Mechanisms · Multi-Objective Topology Optimization · Evolutionary Algorithm

1 Introduction

Compliant mechanisms (CM) are flexible elastic structures which can deform to transmit the force and/or generating some desired path on the application of applied load. The two approaches for designing the CM can be found in literature. In the Ist approach, the designs are inspired by traditional kinematic synthesis of rigid-body mechanism called *pseudo-rigid-body mechanisms* (Howell and Midha (1996); Hetrick and Kota (1999)). The IInd approach is a continuum mechanics based approach which generates monolithic structures called *compliant mechanisms*. These mechanisms have shown many advantages over pseudo-rigid-body mechanisms as jointless and monolithic structures, involved less friction, wear and noise (Howell and Midha (1994)), ease of manufacturing without assembly, light weight devices (Ananthasuresh and Kota (1995)) etc. The applications of compliant mechanism are in product design, off-shore structures, smart structures, MEMS (Ananthasuresh (2003)) etc.

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1.1 Approaches to Synthesis CM Designs

Several studies have been done based on continuum mechanics approach by considering homogenization method (Bendsoe and Kikuchi (1988); Nishiwaki et al (1998)) or material density approach (Yang and Chuang (1994)) in which the discrete nature of a designing problem is converted into the continuous variable problem. It results an easy handling of the problem solving but simultaneously, a threshold value is required for each assigned variable. Sometimes, any arbitrary assignment of the threshold value may lead to non-optimum designs. The classical methods of optimization are used to deal with the continuous variables problems but these methods can stuck at some local optimum design while solving the non-linear problems. Therefore to overcome the present issue, another approach is discussed in the next paragraph.

An approach of using a binary (0-1) representation of material for the continuum mechanics based approach can help to preserve the discrete nature of the structural and CM related problems (Chapman et al (1994); Chapman and Jakiela (1996); Duda and Jakiela (1997); Jakiela et al (2000); Kane and Schoenauer (1996)). The binary, material-void design domain results in a discrete, typically non-convex space

(Anagnostou et al (1992)) and allow for a precise, although discretized, topology boundary but this representation usually results in 'checker-board' pattern problem and 'floating elements' of material which are disconnected from the main part of structure in a design domain. This approach can easily integrate with any evolutionary algorithm to evolve the optimum designs and structures because these algorithms can handle the non-linearity involved in the topology optimization of structures and compliant mechanism and can also deal with the multi-objective optimization. Generally, the evolutionary algorithms consume much higher time to evolve the optimum designs.

1.2 Measures of Designing Compliant Mechanisms

The different measures of providing the flexibility and required stiffness have been incorporated in the past studies. A few common measures which are formulated using either a single- or multi-objective sets are listed as, (i) minimization of weighted linear combination of deformation at the prescribed output port and strain energy (Ananthasuresh et al (1994)), (ii) minimization of least-square errors between prescribed and actual values of geometric (Sigmund (1997), Min and Kim (2004)) and mechanical advantages (Larsen et al (1997)), (iii) maximizing the ratio of mutual energy to strain energy (Frecker et al (1997), Luo et al (2005)), (iv) maximization of mechanical efficiency, geometric and mechanical advantage, and minimization of the maximum compressive load (Parsons and Canfield (2002)), (v) maximization of mutual potential energy (Lu and Kota (2006)), (vi) simultaneous minimization of weight and input energy to elastic structures (Sharma et al (2006, 2008c,a)), (vi) simultaneous minimization of weight and maximization of diversity of elastic structures (Sharma et al (2008d,b)) etc. With the help of these measures, various mechanical aspects of compliant mechanisms are incorporated.

1.3 Binary Representation

In this paper, a few important studies using binary representation of material for the synthesis of compliant mechanisms are discussed which are modeled using either truss/frame ground structures or two-dimensional continuum structures and are optimized using an evolutionary optimization. In truss/frame ground structures, presence of a truss/frame element depends on the value of a binary bit. With an additional approaches of flexible building blocks (Bernardoni et al (2004)), spanning tree theory (Zhou and Ting (2005)) and load path synthesis (Lu and Kota (2006)), topologies of compliant mechanisms are generated which are well-connected and free from gray scale and hence, results in the improved designs.

For representing a two-dimensional continuum structure using a Boolean variables, a design domain is discretized into finite elements and each element of a structure is either represented by material or void depending on the boolean variable value. Using a modified evolutionary structural optimization (ESO) procedure (Ansola et al (2007)), genetic programming (Parsons and Canfield (2002)) and genetic algorithms (Tai and Chee (2000); Tai and Akhtar (2005); Tai and Prasad (2007); Tai et al (2002); Hull and Canfield (2006); Sharma et al (2006, 2008d,b,c,a)), compliant mechanisms are designed with different objectives and tasks. Using a morphological technique of representing a structure, various problems of compliant mechanisms and structural optimization are solved in which Bezier curves are used to represent the shape of structures (Tai and Chee (2000); Tai and Akhtar (2005); Tai and Prasad (2007); Tai et al (2002)).

1.4 Studies Based on PGCM Designing

The large displacement complaint mechanisms (Buhl et al (2000); Pedersen et al (2001); Saxena and Ananthasuresh (2001); Saxena (2005b)) and path generating / tracing complaint mechanisms (PGCM) synthesis require non-linear FE models which make the designing problem even-more difficult to solve. As the present work concentrate on the PGCM designing, a few important studies are discusses here which used the non-linear FE models and optimized using evolutionary algorithms. Initially, a single-objective optimization based PGCM design is formulated by minimizing the deviation between the distance of desired and corresponding actual trajectories after dividing them into N precision points (Tai et al (2002)). This study uses a novel morphological representation scheme (Tai and Chee (2000)) and generates the PGCM design using genetic algorithm. Later, a multi-objective formulation (Saxena (2005a)) is used in which the least square objective of actual and desired output responses at each precision point is minimized using NSGA-II (Deb et al (2002) algorithm). In this study, the number of objectives are equal to the number of precision points which are used to represent the prescribed path. Finally from the evolve set of non-dominated solutions, a solution that minimizes the sum of individual least square objectives is chosen.

The precision points which are used to represent the prescribed path are characterized either by a same level of input load or input displacement which can result in an artificial constraint in the design problem. Also, when a objective of minimizing the Euclidean distance are considered, it might misrepresent the nature of design problem by requiring the shape, size, orientation and position of the prescribed path to be optimized all at once. This problem is overcome by using Fourier shape descriptor-based objective (Ullah and Kota (1997); Rai et al (2007)). The formulation involve many user-defined parameters which can effect the optimum solution based designs.

The authors of this paper also suggested a bi-objective optimization formulation for path generating compliant mechanisms (Sharma et al (2006, 2008d,b,c,a)). The formulation uses the two-objective set to evolve trade-off multiple topologies of compliant mechanisms. The functional aspect of generating the prescribed path by these mechanisms is achieved by imposing the constraints at precision points of the prescribed path. These hard constraints are designed with some possibility of violation and are dependent on user-defined allowable deviation (η). During the successive attempts of evolving the multiple topologies of path generating compliant mechanisms, the evolutionary algorithm (NSGA-II, (Deb et al (2002))) is customized and the value of η is suggested between 10% to 20% (Sharma et al (2008a)). In the present work, the advancement of the topologies of path generating compliant mechanisms is introduced using initial population strategy. This strategy is coupled with the customized NSGA-II algorithm to deal with the constraint single- and bi-objective studies of path generating compliant mechanisms. Two examples of compliant mechanisms tracing (i) downward curvilinear path, and (ii) straight line path are solved and their respective evolved topologies are presented. In the remaining part of this paper, the PGCM formulation is discussed in Section 2, the details of customized NSGA-II algorithm is given in Section 3. The evolved topologies and the discussion are presented in Section 4. Finally, the study is concluded in Section 5 with some future works.

2 Description of Problem Formulation

In this section of paper, we target the optimization formulation of path generating compliant mechanism which was proposed by the authors of this paper in their previous studies (Sharma et al (2006, 2008d,b,c,a)). In this formulation, the functional aspect of generating the prescribed path by compliant mechanisms is accomplished by imposing the constraints at precision points. Before going into the details of the formulation, first the design domain of compliant mechanism (50 mm by 50 mm) is explained as shown in Figure 1 which is categorized into three regions of interest. The Ist region is called support region where the nodes of an element of the elastic structure are restrained with zero displacement. In the IInd region (loading region i.e. a node of an element), some input displacement boundary condition is applied. The output region is the IIIrd region, that is, a fixed point on the elastic structure which traces out the desired path defined by user.

In this work, the origin of the design domain is fixed on its left hand side and the output region is positioned at the coordinate $(50,32)$ of the structure. As Figure 1 shows, a spring of constant stiffness (κ = 0.4 KN/m) is attached at

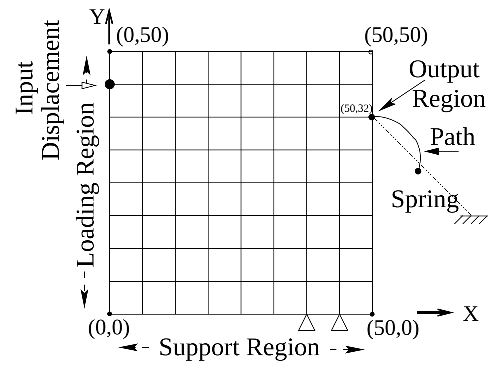


Fig. 1 A design-domain with loading, output and support regions.

the output point for providing some resistance during the deformation of elastic structure.

As discussed earlier, the essential functional aspect of path generating compliant mechanisms is to trace the prescribed path. Thus, the same compulsory task of compliant mechanism is accomplished by imposing the constraints at precision points. These hard constraints bound the maximum distance between the prescribed and actual paths for all feasible designs. A hypothetical case is shown is Figure 2 in which a prescribed path and an actual path traced by

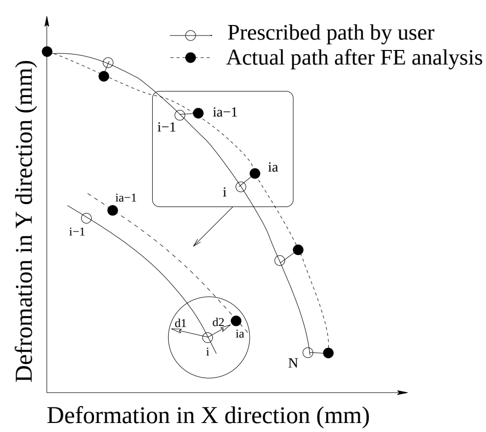


Fig. 2 The prescribed path and an actual path traced by the elastic structure after FE analysis.

the elastic structure after FE analysis are drawn. Here, the prescribed path is represented by N precision points. The corresponding points on an actual path traced by an elastic structure is evaluated from geometrical non-linear FE analysis based on equal load steps.

To physically represent the constraints, first an euclidean distance (say d_1) is evaluated by estimating the distance between the current (i) and previous ($i-1$) precision points and get multiplied by a factor η called as *percent of allowable deviation*. Then, another euclidean distance (say d_2) between the current precision point (i) and the corresponding point (i_a) of actual path is calculated. Based on these calculations, a constraint is imposed at each precision point which ensures that $d_2 \leq d_1$. A pictorial significance is shown in Figure 2 in which, if a circle of radius d_1 at the current precision point (i) is drawn, then the corresponding point (i_a) of actual path must lie within or on the circle to satisfy the constraint on each precision point. The mathematical representation of constraints at each N precision points is given in Equation 1. Any elastic structure which satisfies these constraints can guarantee to accomplish the task of tracing the path based on user-defined allowable deviation (η).

In the present study, the single- and bi-objective studies are done which are subjected to the constraints as defined in the last paragraph. An additional constraint of limiting the stress is also included into the formulation. The constraint single and bi-objective formulations of path generating compliant mechanisms are given in Equation 1.

Single-objective optimization:
Minimize: Weight of structure

Bi-objective optimization:
Minimize: Weight of structure (primary obj.),
Minimize: Supplied Input energy to structure (secondary obj.),

Both problems are subjected to:

$$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta \times \sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, \quad i = 1,2,...,N$$
$$\sigma_{flexural} - \sigma \geq 0, \tag{1}$$

where $\eta = 15\%$ is the permissible deviation (kept fixed in this paper, (refer (Sharma et al (2008a)) for more details), and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the structure, respectively.

3 Customized NSGA-II Algorithm

In this section, we draw our attention to customize the evolutionary algorithm for effectively solving the optimization problems defined in Equation 1. In this paper, a popularly used elitist non-dominated sorting genetic algorithm (known as NSGA-II (Deb et al (2002)) which is developed by second author of this paper and his students) is used as a global search and optimizer which has shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front for two objective problems. A detailed description of NSGA-II algorithm can be found in the study (Deb et al (2002)). In short, NSGA-II is population based evolutionary optimization procedure which uses mathematical partial-ordering principle to emphasize non-dominated population members and a crowding distance scheme to emphasize isolated population members in every iteration. An elite-preserving procedure also ensures inclusion of previously found better solutions to further iterations. The overall procedure with N population members has a computational complexity of O(NlogN) for two and three objectives problems and has been popularly used in many studies. NSGA-II is also adopted by a few commercial softwares (such as iSIGHT and modeFRONTIER). A code implementing NSGA-II is available at *http://www.iitk.ac.in/kangal/codes.shtml* website.

A local search method is coupled with the customized NSGA-II algorithm to further refine the non-dominated solution's based topologies of compliant mechanisms. The basic details of local search based customized NSGA-II algorithm is shown in Figure 3. It shows various schemes

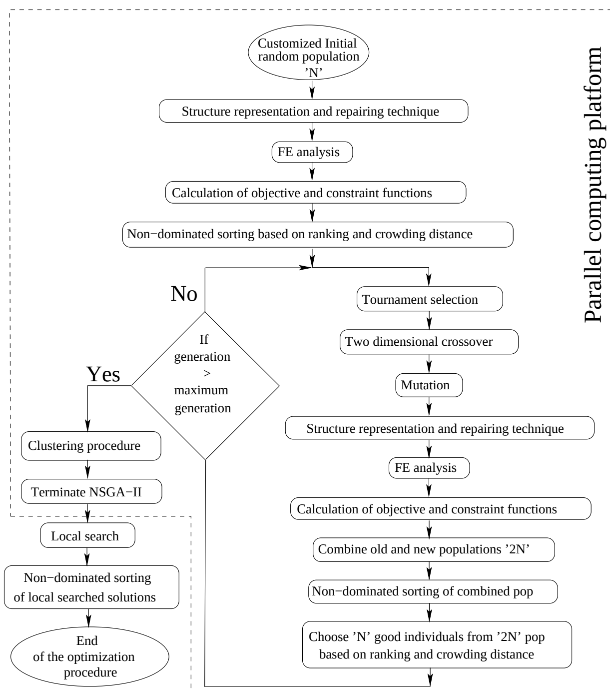


Fig. 3 A flow chart of customized NSGA-II algorithm

like initial population, structure representation and repairing techniques, two-dimensional crossover operator, mutation operator etc. which are expected to efficiently deal with the topology optimization problems of structures and compliant mechanisms.

3.1 GA Parameters

A population of 240, crossover probability of 0.95 and mutation probability of (1/string length) are assigned and the NSGA-II algorithm is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 637 bits is used as shown in Figure 4. This string is made of two sets in which the Ist set of 625 bits represents

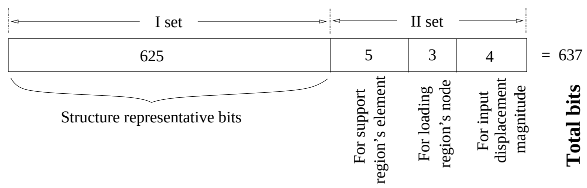


Fig. 4 A binary string comprises of two sets.

the shape of structure (described in Section 3.2) whereas, the decoded value of IInd set identifies the support and loading positions in their respective regions (refer Figure 1), and magnitude of input displacement. For the same, the remaining 12 bits of IInd set are further divided into three sets of five, three and four bits as shown in Figure 4. The decoded value of first five bits indicates the location of an element from the origin where the elastic structure is to be supported. The decoded value of subsequent three bits helps in determining the loading position, that is, a node where the input load is applied. The decoded value of last four bits are used to evaluate the magnitude of input displacement which can vary from 1 mm to 16 mm at step of 1 mm. The above mentioned flexibility is implemented to come-up with the optimum combinations of support and loading positions, and input displacement magnitude to promote the non-dominated solutions through the run of NSGA-II algorithm. A detailed significance of additional bits will be discussed later along with the results presented in the study.

3.2 Structure Representation Scheme

Before starting the details of initial population strategy, the structure representation scheme is described here. A binary string of 625 bits (refer Figure 4) is used to represent the shape of structure. First, a binary string is copied to two dimensional representation as shown in Figure 5. Thereafter,

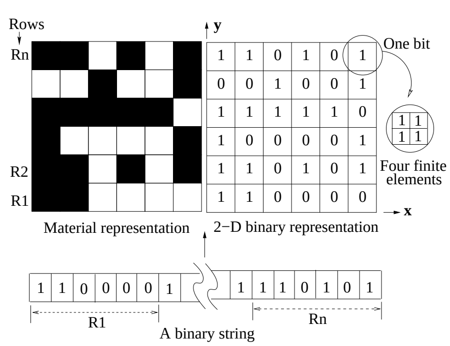


Fig. 5 A representation of structure using binary string and material.

the material-void representation of each grid is chosen based on the binary bit value, for example, bit value 1 signifies that material is present whereas, 0 represents the void. This scheme divides a design domain of structure into 25×25 $(= 625)$ grids in x and y directions, respectively.

3.3 Initial Population Strategy

In the later studies of structures and compliant mechanisms (Deb and Goel (2001); Deb and Chaudhuri (2005); Sharma et al (2006, 2008d,b,a)), an initial random population was generated by flipping a coin and decided whether the bit value at each gene is 0 or 1. Therefore, the gene value (0-1) was assigned at random to the whole binary string which resulted in impracticalities such as 'checker-board' pattern and disconnected 'floating' material. The connectivity among the three regions of interest (refer Figure 1) was also checked before FE analysis of structure.

In the present work, a strategy of generating an initial random population using a (0-1) binary representation of elastic continuum structures is developed in which the three regions of interest are connected through the intermediate points (Sharma et al (2008c)). A pictorial view is drawn in Figure 6 to show a connectivity between the support and

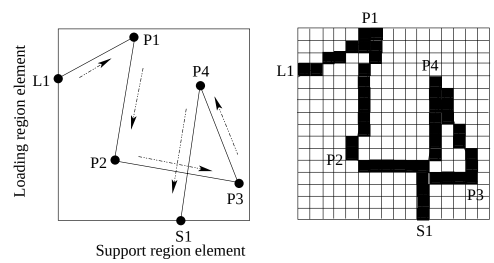


Fig. 6 Connectivity between support and loading regions.

loading region's elements. Here, the positions of support and loading region's elements are calculated after decoding the II nd set of binary string (refer Figure 4) whereas, the location of output region's element is fixed through-out the study.

For connecting the support and loading regions, a random integer number between 1 to 5 is generated to decide the number of intermediate points through which these two regions get connected. Depending upon the number of intermediate points, the coordinates of each intermediate point is randomly generated within the design domain. The points P1, P2, P3 and P4 in Figure 6 show the location of intermediate points and, the support (S1) and loading (L1) region's elements are connected through these points by straight lines. Thereafter, a material is assigned to those elements where these straight lines pass. A material connectivity of the above mentioned regions is also shown in Figure 6. Similarly, a set of piece-wise linear line segments between the support and output regions and another set between the loading and output regions are explained. Therefore depending on the randomly generated intermediate points, an initial population for the NSGA-II algorithm is developed.

A repairing technique is also employed in which if two elements generate a point connection, then the given procedure puts one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity.

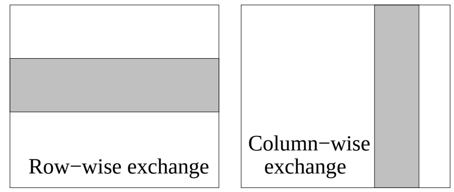


Fig. 7 A two dimensional crossover which swaps a patch of row/column between two parent solutions.

3.4 Finite Element Analysis

After the custom initialization, structure representation, and repairing techniques, the elastic structure is analyzed for stress and deformation by FE analysis. In this study, one grid of a structure (as described in Section 3.2) is further discretized into four finite elements with same binary variable value as shown in Figure 5. Therefore in the present process, the structure is discretized with 4×625 $(= 2500)$ 4-node rectangular finite elements and analyzed through a non-linear large deformation FE analysis using *ANSYS* package. But, the GA operations are performed on the same structure represented by 625 bits.

3.5 GA Operators

Crossover is an important genetic algorithms (GA) operator which is responsible for the search aspect of the algorithm. It creates new solutions which differ from the parent solutions. In this study, a two-dimensional crossover operator is used which has shown the successfully applications in shape optimization (Deb and Goel (2001); Deb and Chaudhuri (2005)) and in the designing of compliant mechanisms problems by authors (Sharma et al (2006, 2008d,c,a)). In the present recombination operator, two parent solutions are selected and a coin is flipped to decide for row or column-wise crossover. If a row crossover is done, a row is chosen with an equal probability of (P_{xover}/no. of rows) for swapping. The same is done if a column-wise crossover has to be done. During crossover, a random number is generated to identify the number of rows (columns) to be swapped and then, another generated random number helps in getting the first row (column) number of patches. A range of row (column) index is calculated and swapped with other parent. A pictorial view of the crossover on the structures represented by I^{st} set of binary string is shown in Figure 7. For the crossover of 12 bits of II^{nd} set, a standard single point crossover is used in the present study.

Mutation operator is another GA operator which generates new solutions in the population but usually it is done with a low probability. Here, it is done with a probability of (1/string length) on each bit of a string of I^{st} set to change from a void to a filled or from a filled to a void grid. A detailed discussion of these crossover and mutation operators are given elsewhere (Goel (2001); Chaudhuri (2002)). For mutating the remaining 12 bits of II^{nd} set, first the decoded values of support and loading regions, and magnitude of input displacement are evaluated and then, these values are perturbed within the range of $\{-2, 2\}$ at their original values. Here, it is ensure that the perturbed values of above three boundary conditions do not fall outside their respective bounds. This mutation operator helps to get the nearest integer value at the original one. After perturbation, these mutated values are again coded into the binary string of 12 bits.

3.6 Parallel Computing

A distributed computing platform is used in the present study to reduce the computational time of designing and synthesis of compliant mechanisms. In this parallelization process, the root processor first initializes a random population. Then, it divides the entire population into different sub-populations in proportion to the number of processors available. After this, each sub-population is sent to different slave processors. These slave processors further evaluate the objective functions and constraints values, and send them to the root processor. Thereafter, root processor performs the GA operators, like selection, crossover and mutation operators, non-dominated front ranking etc. on the population and replaces it with good individuals. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through *ANSYS* FE package which consumes the maximum time of the optimization procedure (Sharma et al (2006, 2008d,b,c,a)). A MPI based Linux cluster with 24 processors is used in the present study. A detailed specification and configuration of the Linux cluster are given at *http://www.iitk.ac.in/kangal/facilities.shtml* website.

3.7 Clustering Procedure

For an adequate convergence near to the global 'Pareto-optimal' front, the evolutionary algorithms (EA) need a fairly large number of population members and generations depending upon the problem complexity. Thus, the number of feasible solutions after the EA run are usually close to the population size. It is not advisable to represent so many solutions to the end user for a subsequent decision-making task. Therefore, the clustering procedure is employed in the study in which the neighboring solutions are grouped together and solutions from each group representing that region of the non-dominated front are chosen as representative solutions (Zitzler (1999)). Figure 8 shows the procedure pictorially. After clustering, the parallel NSGA-II algorithm is terminated. Thereafter, the local search method (described in next section) is employed on each representative NSGA-II solutions based designs to improve them locally.

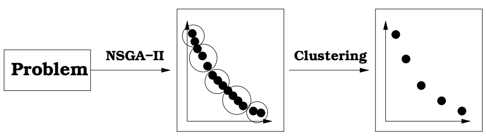


Fig. 8 A clustering Procedure.

3.8 Local Search Method

The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is given in Equation 2.

$$Minimize\ F(x) = Minimize \sum_{j=1}^{n} \frac{\overline{w}_j^x (f_{j_{max}}^x - f_j^x)}{f_{j_{max}}^x - f_{j_{min}}^x}, \qquad (2)$$

where, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of j^{th} objective function in the population, respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the j^{th} objective function which is computed as:

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \qquad (3)$$

where M is the number of representative solutions after clustering procedure. In Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space (Deb (2001)). As Equation 3 suggests, these weights are calculated based on their positions in two-objective space after the termination of NSGA-II algorithm.

In the local search method, first the weighted sum of scaled fitness of a selected representative solution is evaluated as given in Equation 2. Thereafter, one bit of representative solution is mutated at a time and the design is extracted from the new string. This new string's based structure is analyzed by FE package and then, the objective and constraint functions are evaluated. If the new design does not satisfy any constraint, then the change in new string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string values. In case of mutating '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that of old string, or else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old string's weighted sum value, then it is accepted or else the change is discarded. In case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the grids having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the change is accepted. This process is repeated till all bits are mutated once. If there is no change in the value of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As discussed in Section 3.4, one binary bit represents four elements for FE analysis. Therefore, a binary string of 625 bits represents a structure which is discretized with 2500 finite elements. In case of local search, the previous binary strings (625 bit) of representative solutions are reconstructed into the new binary strings of 2500 grids. These grids represent the same structure of 2500 elements and the local search search is performed on these 2500 grids.

4 Evolved Topologies and Discussion

In this section, the evolved topologies of single- and bi-objective optimization studies are presented. First, we deal with the single-objective optimization of path generating compliant mechanism and thereafter, the two-objective study is done. Two examples of compliant mechanisms generating (i) curvilinear path and, (ii) straight line path are solved. The evolved topologies of both examples are shown and their associated significance are discussed in the subsequent subsections.

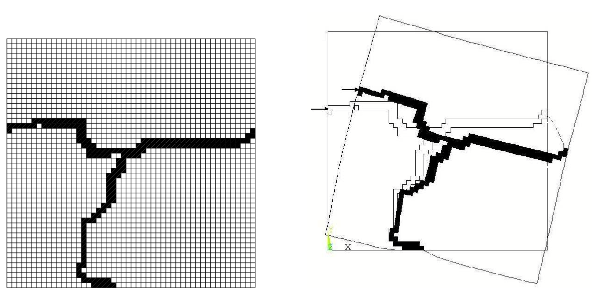
A few parameters are kept constant during the whole study such as, a material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for synthesis of compliant mechanism. The direction of input displacement is fixed along x direction. Here, the prescribed path is represented by five precision points and the trajectory traced by output region's node of the elastic structure is evaluated through a geometric nonlinear FE analysis using *ANSYS* package. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. After the termination of NSGA-II algorithm, maximum six representative solutions are chosen from the non-dominated set of NSGA-II solutions with the help of clustering procedure.

4.1 Curvilinear Path Generating Compliant Mechanisms

Using an improved initial population strategy along with the provided flexibility of identifying the applied and boundary conditions to the customized evolutionary algorithm, the topologies of compliant mechanism tracing downward curvilinear path are evolved. First, a minimum weight design is evolved using single-objective study of minimizing the weight of continuum structure. Later, the multiple non-dominated topologies of compliant mechanism are generated using a two-objective study.

4.1.1 Minimum Weight Design

A minimum weight design of 0.525 gm is evolved after solving the constraint single-objective optimization problem of Equation 1. The undeformed and final deformed topology of minimum weight design is shown in Figure 9. The optimum applied and boundary conditions identified by the customized NSGA-II algorithm are tabulated in Table 1. These conditions are evaluated after decoding the IInd set of binary string. The table indicates that the minimum weight design of compliant mechanism is supported at an element which is located at 20 mm away from the origin. The input load of 7 mm is applied at the loading position, that is, at 32 mm away from the origin.



(a) Undeformed topology (b) Final deformed topology

Fig. 9 A minimum weight design.

4.1.2 Non-dominated Solutions based Compliant Mechanism Topologies

In this section, the non-dominated topologies of compliant mechanism generating curvilinear path are evolved after solving the constraint two-objective optimization problem (refer Equation 1) using the customized NSGA-II algorithm. A two-objective space is drawn in Figure 10 to display the locations of representative NSGA-II solutions (a to f) and local search solutions (1 to 6). It shows that out of six local search solutions, five of them (1 to 5) become a part of non-dominated front.

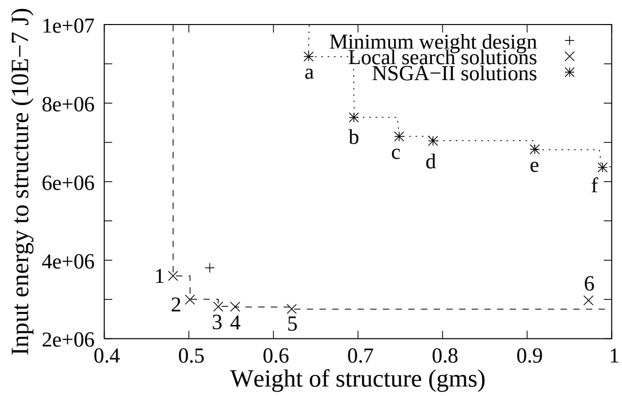


Fig. 10 NSGA-II solutions before and after the local search and compared with the single-objective 'minimum-weight' design.

The Figure 10 also shows the position of minimum weight design corresponding to its supplied input energy value in two-objective space. It indicates that solutions 1 and 2 are evolved as lighter-in-weight solutions and also require less supplied energy in comparison with the minimum weight design solution. Hence, these solutions dominate the minimum weight solution of single-objective study. It reveals a fact that the secondary objective of minimizing the supplied input energy not only helps in evolving multiple topologies in one run of optimization but also generates non-dominated solutions with the primary objective of minimizing the weight of structures. On the other hand, a single objective study only deals with the optimization of one objective and may result in a premature sub-optimal solution.

The undeformed and final deformed topologies of these non-dominated solutions (1 to 5) are shown in Figure 11. Here, the solution 1 evolves as minimum weight topology but at the same time it requires larger input energy to deform the elastic structure and follow the prescribed path. If we look on the other extreme of non-dominated front of Figure 10, the topology of solution 5 requires minimum supplied input energy but it comes up as heavier elastic structure. Similarly, other solutions show 'trade-off' between the posed objectives. Topologically, all these non-dominated solutions are same because they consist only open loops of material between the three regions of interest (support, loading and output). But at the same time, we can see the changes in the designs which result in 'trade-off' solutions based topologies. Here, the solution 5 has a 'winding shape' near to the junction of each loop of material from the three regions of interest (refer Figure 11(i)). This 'winding shape' helps to reduce the requirement of input energy. Similarly, the solutions 3 and 4 show another kind of distribution of material near the support position which makes trade-off designs. On the other hand, solutions 1 and 2 seem quite rigid but evolve as lighter weight topologies.

As discussed in Section 3.1, the flexibility is provided to the customized NSGA-II algorithm to identify the support and loading positions, and magnitude of input displacement conditions. As the previous studies of the authors of this paper (Sharma et al (2008c,a)) suggest, these conditions can assist the customized NSGA-II algorithm to evolve non-dominated solution's based topologies. Therefore, the optimum set of applied and boundary conditions are tabulated in Table 1. It is observed that the loading position of all

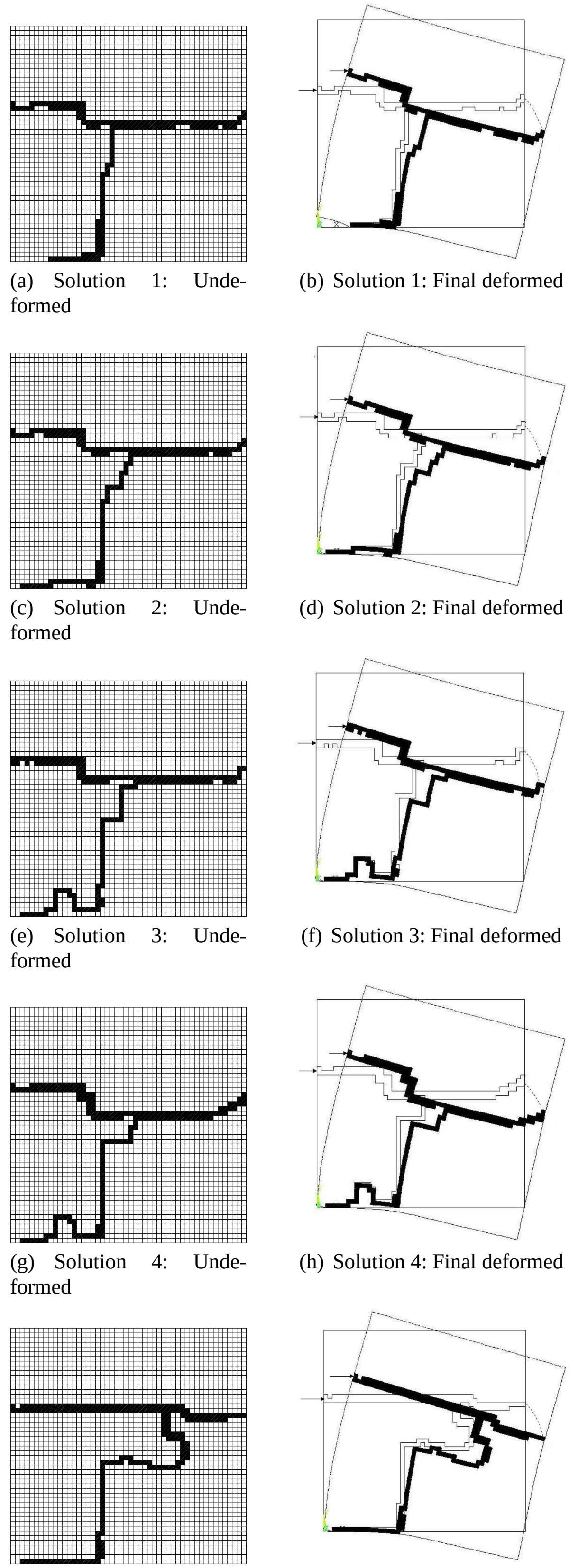
Table 1 Evolved support and loading positions and, input displacement magnitude of curvilinear path generating compliant mechanism.

Conditions → Study ↓	Support position (*mm*) (*from the origin*)	Loading position (*mm*) (*from the origin*)	Input displacement (*mm*)
Single-objective	20	32	7
Bi-objective	Solution 1 at: 8 Solutions 2 to 5 at: 2	32	7

non-dominated topologies is identified at 32 mm away from the origin and these solutions require 7 mm of input displacement to deform them and also, to follow the downward curvilinear prescribed path. However, these non-dominated feasible elastic structures are supported at different elements, for example, the solution 1 gets supported at 8 mm and the rest of non-dominated solutions are supported at 2 mm. Two more advantages of providing the flexibility to the evolutionary algorithm can be seen here; first, it can work in the scenario of unknown conditions of support and loading positions, and magnitude of input displacement. Second, it can help the designers and decision makers to appreciate the optimum combination of these conditions for evolving the non-dominated PGCM topologies that explores the possibility of non-optimum conditions which might be considered in their previous practices.

4.1.3 Traced Paths & Prescribed Path

A pictorial view of paths generated by the solutions obtained after the local search of single and two-objective studies is shown in Figure 12 along with a prescribed path. This fig-



(a) Solution 1: Undeformed
(b) Solution 1: Final deformed
(c) Solution 2: Undeformed
(d) Solution 2: Final deformed
(e) Solution 3: Undeformed
(f) Solution 3: Final deformed
(g) Solution 4: Undeformed
(h) Solution 4: Final deformed
(i) Solution 5: Undeformed
(j) Solution 5: Final deformed

Fig. 11 Non-dominated topologies of compliant mechanisms generating curvilinear path.

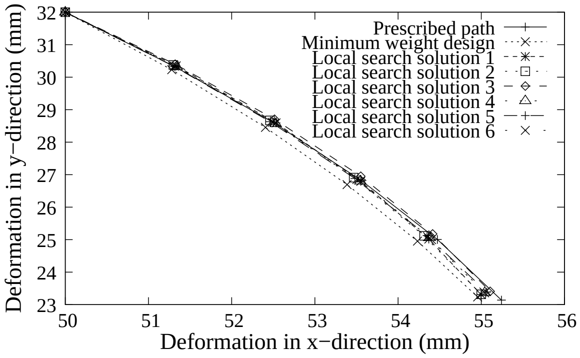


Fig. 12 Prescribed path and path traced by the minimum weight design and local search solutions.

ure signifies the adherence between the prescribed path and paths traced by all local search solutions. For quantitative study of these paths, a Table 2 is drawn in which the values

Table 2 Deviation at precision points.

Precision points (PP)	1	2	3	4	5
Maximum allowed $\mathbf{d}_1$	0.3196	0.3142	0.3074	0.3084	0.3092
Single-objective study 'Minimum weight design'					
$\mathbf{d}_2$	0.1032	0.1739	0.2127	0.2470	0.3091
Two-objective study					
Solution 1: $\mathbf{d}_2$	0.0290	0.0301	0.0324	0.1047	0.3058
Solution 2: $\mathbf{d}_2$	0.0531	0.0829	0.1220	0.1979	0.3091
Solution 3: $\mathbf{d}_2$	0.0761	0.1084	0.1262	0.1812	0.3091
Solution 4: $\mathbf{d}_2$	0.0226	0.0407	0.0256	0.1064	0.3091
Solution 5: $\mathbf{d}_2$	0.0278	0.0516	0.0939	0.1766	0.3091
Solution 6: $\mathbf{d}_2$	0.0161	0.0217	0.0588	0.1502	0.3083

of maximum allowed d_1 at each precision point and distance d_2 between the precision point and corresponding point on the actual path are given. It can be seen here that the value of d_2 increases for all solutions as these topologies follow the precision points from 1 to 5. This shows that the precision points defining the extreme part of prescribed path become critical. But the importance of constraints on initial precision points can not be ignored that assist the evolutionary algorithm to come up with the feasible compliant mechanism topologies (Sharma et al (2008a)). Overall, the figure and table presented here, show the fulfillment of the functional aspect of tracing the prescribed using the precision points based constraints formulation with both single and bi-objective sets. The prescribed path here is designed in such a way that an output point of each elastic structure has to deform to 10.48% in x-direction and 17.72% in y-direction with respect to the size of design domain.

4.1.4 Comparison

In the present study, a comparison is also made between the solutions of two-objective studies which are obtained by incorporating a custom improved initial population strategy and by an usual way of creating an initial population of continuum structures in which the material in each element of a structure is assigned at random (authors refer this initialization as a 'random initialization', Sharma et al (2006, 2008d,a)). The Figure 13 shows the representative NSGA-II

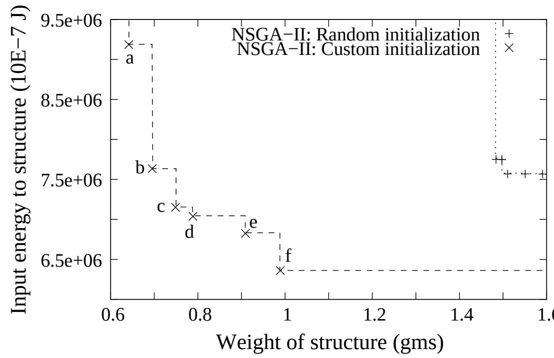


Fig. 13 NSGA-II solutions of both initial population initializations.

solutions of both two-objective studies and it clearly reveals that the NSGA-II solutions of an improved initial population strategy dominate and explore the larger area of two-objective space as well, in comparison with the NSGA-II solutions of random initialization. We know that the performance of a local search procedure is dependent on the position of representative NSGA-II solutions in the objective space. Hence, the non-dominated local search solutions of improved strategy outperform that of random initialization study as shown in Figure 14. Therefore, solutions 1 to 5 become the part 'Pareto-optimal' front.

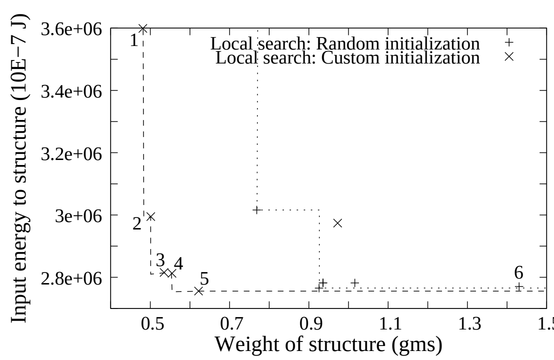


Fig. 14 Local search solutions of both initial population initializations.

4.1.5 Computation Time

The parallel computing platform is used in the present study to deal with the computationally extensive problem of compliant mechanisms. The maximum time of optimization procedure consumes in the non-linear finite element analysis whereas, the other function evaluations and the communication among the processors take a smaller time. The times taken by the given optimization procedure for solving the single and bi-objective optimization problems are shown in Table 3. During the NSGA-II run, 24 processors are used

Table 3 Time taken by given optimization procedure.

Problem	NSGA-II Time (hrs)	Local search Time (hrs)
Single-objective	5.59	12.19
Two-objective	5.47	Solution 1: 8.65 Solution 2: 9.15 Solution 3: 9.61 Solution 4: 10.93 Solution 5: 11.52 Solution 6: 25.07

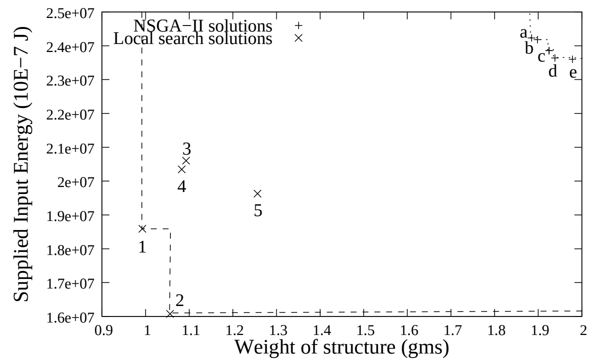


Fig. 15 NSGA-II and local search solutions for compliant mechanisms tracing straight line path.

which helps in reducing the computational time almost in proportion to the number of processors available. The local search is performed individually in different processors that takes a considerable amount of time of a given procedure to refine the representative NSGA-II solutions.

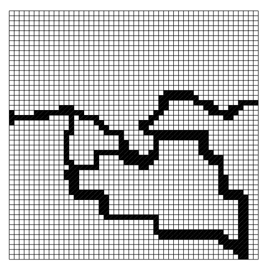
4.2 Straight Line Path Generating Compliant Mechanisms

In the last section, the simple example of curvilinear path generating compliant mechanism was dealt with using the proposed formulation and customized optimization procedure. Here, one can argue that many continuum structures can easily trace the given prescribed path because of its simplicity. On this basis, the competency of both, the PGCM formulation and the customized optimization procedure, can not be justified with the desired level of confidence. Therefore in this section, the non-intuitive topologies of complaint mechanisms tracing straight line path are evolved. Here, only a bi-objective formulation using an improved initial random population is exercised because we have already seen the significance of multi-objective optimization over single-objective optimization.

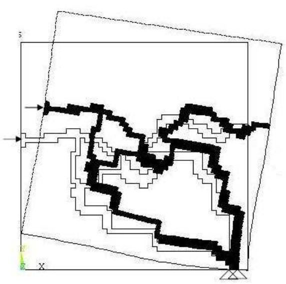
4.2.1 Non-dominated Compliant Mechanism Topologies

The present example of complaint mechanism differ from the previous example that instead of tracing the a curvilinear path (refer Section 4.1.3), now the compliant mechanism traces-out the straight line path at same output point. This makes the problem even more difficult to solve because the continuum structures usually trace-out some curvilinear path as per the categorization of design domain defined in the present work (refer Figure 1).

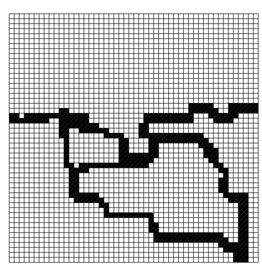
While dealing with the present compliant mechanism problem, the customized evolutionary algorithm evolves the six representative NSGA-II solutions (*a* to *e*) which are shown in 2-D objective space of Figure 15. The local search solutions (1 to 5) are also shown in the same figure. Here, only two solutions (1 and 2) out of five local search solutions become the part of non-dominated front. The undeformed and final deformed topologies of these non-dominated solutions are shown in Figure 16. Topologically, both PGCM designs



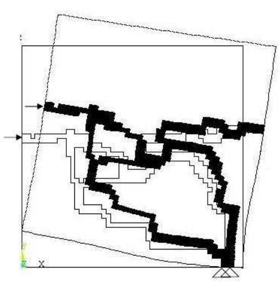
(a) Solution 1: Undeformed



(b) Solution 1: Final deformed



(c) Solution 2: Undeformed



(d) Solution 2: Final deformed

Fig. 16 Non-dominated topologies of compliant mechanisms generating straight line path.

are same with two closed loops of materials, but the kind of arrangements of material in the design domain result in trade-off between the two-objective. The topologies of both the non-dominated solutions can reveal a fact of the necessity of those compliant mechanisms that generate straight line path.

In this example, the compliant mechanisms are supported at an element which is positioned at 46 mm away from the origin. The input load of 5 mm is applied at a node which is positioned at 28 mm away from the origin, to deform them and to follow the straight line path. These conditions are also evolved from the IInd set of binary string. An interesting thing can be seen here that these topologies are supported on their right-hand side while tracing the straight line path. However in the previous example of curvilinear path tracing, the compliant mechanisms are supported on their left-hand side. In both examples, only the change was implemented in terms of nature of prescribed path and rest of the conditions were same. But, the flexibility provided by the additional bits of IInd set to the customized NSGA-II algorithm identifies these support positions because the structures supported on their right-hand side show minimum tendency to generate higher curvilinear paths. On the other hand, left hand side supported structures tends to trace the higher curvilinear trajectory.

4.2.2 Traced Paths and Prescribed Straight Line Path

The straight line path traced by all local search solutions along with the prescribed path is shown in Figure 17. It

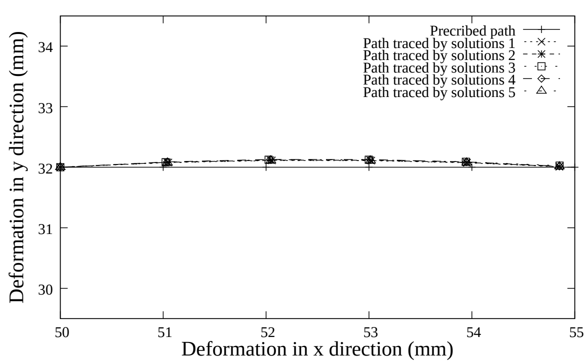


Fig. 17 Prescribed path and path traced by all local search solutions.

shows that the elastic continuum structures do not trace the exact straight line path because it is too optimistic condition to generate the same path as it is prescribed for the given categorization of design domain. However, the imposed constraints at precision point which allow a fixed percentage of deviation in terms of η value, assist the optimization procedure to evolve the feasible topologies. The Table 4 is also drawn for quantitative analysis in which the d_2 value for all local search solutions increases till the precision point 2 and then, decreases. Finally, the maximum d_2 value can be seen at precision point 5 which makes it critical. Here, we can clearly see the significance of imposed constraints on precision points which guides the optimization procedure to evolve such non-intuitive PGCM topologies. The prescribed straight line path here is designed in such a way that an output point of each structures has to deform to 10.00% in x-direction and 0.0% in y-direction with respect to the size of design domain.

Table 4 Deviation at precision points.

Precision points (PP)	1	2	3	4	5
Maximum allowed $\mathbf{d}_1$	0.15	0.15	0.15	0.15	0.15
Two-objective study					
Solution 1: $\mathbf{d}_2$	0.0930	0.1260	0.1119	0.0876	0.1498
Solution 2: $\mathbf{d}_2$	0.0901	0.1257	0.1169	0.0957	0.1497
Solution 3: $\mathbf{d}_2$	0.0874	0.1262	0.1243	0.1071	0.1499
Solution 4: $\mathbf{d}_2$	0.0962	0.1350	0.1277	0.1045	0.1498
Solution 5: $\mathbf{d}_2$	0.0869	0.1209	0.1113	0.0911	0.1499

5 Conclusions

In this paper, the improved initial population strategy was successfully coupled with the customized NSGA-II algorithm. The non-dominated solutions obtained from this strategy outperformed the solutions of random initialization. This advancement in the evolution of multiple topologies further customized the NSGA-II algorithm to efficiently deal with the topology optimization of compliant mechanisms. The present paper also showed the significance of evolving the multiple non-dominated topologies of compliant mechanism over single-objective optimization. The same set of constraints were used with single and bi-objective studies which justified its competency to couple with any objective set. Also, different path generating compliant mechanisms can be evolved using the same constraint formulation. In the future work, the emphasis can be given to other GA operators to introduce more diversity into the topologies of compliant mechanisms. Path generating compliant mechanisms can also be designed with another multi-objective set which further assists the optimization procedure for multiple diverse topologies.

6 Acknowledgment

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