Developing Multiple Topologies of Path Generating Compliant Mechanism (PGCM) using Evolutionary Optimization

Deepak Sharma, Kalyanmoy Deb, N. N. Kishore
KanGAL Report No. 2009002
Kanpur Genetic Algorithms Laboratory (KanGAL)
Department of Mechanical Engineering
Indian Institute of Technology Kanpur, U.P., India-208016
Email: {dsharma,deb,nnk}@iitk.ac.in

The present work aims to evolve the multiple topologies of path generating compliant mechanisms. The trade-off solution's based topologies are developed by simultaneously minimization of weight and supplied input energy to elastic structures. The functional aspect of these compliant mechanisms is accomplished by imposing the constraints on precision points to generate the user-defined path. These hard constraints are designed with some possibility of violation and are based on one user-defined parameter called allowable deviation (η). The present constraint bi-objective optimization formulation ensures the tracing of prescribed path by all feasible compliant mechanism topologies. The sensitivity of the evolved topologies is also investigated in this paper which is based on different η values. The evolutionary algorithm (NSGA-II) is customized in the present work to efficiently deal with the constraint bi-objective, non-linear and discrete problem of compliant mechanisms. The obtained non-dominated solutions after the termination of NSGA-II algorithm are further refined by a binary-variable based local search method. Overall, this study is expected to provide a platform to the designers and decision makers to understand the topological changes and the flexibility to choose a particular design of compliant mechanism as per their requirement from the non-dominated set of solutions.

1 Introduction

Compliant mechanisms (CM) are flexible structures which undergo an elastic deformation on the application of load to accomplish the assigned task. The two approaches for designing the CM can be found in literature. In the Ist approach, the designs are inspired by traditional kinematic synthesis of rigid-body mechanism called *pseudo-rigid-body mechanisms* [1, 2]. The IInd approach is a continuum mechanics based approach which generates monolithic structures called *compliant mechanisms*. These mechanisms have shown many advantages over the pseudo-rigid-body mechanisms as jointless and monolithic structures, less friction, wear and noise [3], ease of manufacturing without assembly, light weight devices [4] etc. The applications of compliant mechanism are in the areas of product design, off-shore structures, smart structures, MEMS [5] etc.

Several studies have been done based on the continuum mechanics approach by considering the homogenization method [6,7], the material density approach [8] and by using the contours of 'shape density' function [9]. In these methods, the discrete nature of designing problem is converted into the continuous variable problem. It results an easy handling of the problem solving but simultaneously, a threshold value is required for each assigned variable. Sometimes, any arbitrary assignment of the threshold value may lead to non-optimum designs.

For the topology optimization of compliant mechanisms, different measures of flexibility and required stiffness have been incorporated in the past studies which were optimized using various techniques of optimization. In this paper, a few important studies which dealt with the single or multi-objective optimization formulation of compliant mechanism synthesis are presented. In an early methods of topology optimization, Ananthasuresh et al. [10] used a multi-criteria formulation, wherein the weighted linear combination of deformation at the prescribed output port and strain energy were optimized using sequential quadratic programming. Similarly, the flexibility was measured as the minimization of least-square errors between prescribed and actual values of geometric [11] and mechanical advantages [12]. In another multi-objective study, the weighted sum of minimum deviation between specified and current geometrical advantage, and minimum of mean compliance at an input and output ports were dealt [13], simultaneously.

In the study [14], a multi-criteria optimization problem was formulated by maximizing the ratio of mutual energy to strain energy subjected to equilibrium equation of loading conditions and a constraint on the total material resource. The same two objectives were used in the study [15] to construct the weighted sum formulation for the continuum structure based complaint mechanisms. The above mentioned studies used the classical methods of optimization to deal with the continuous variables problems but these methods can stuck at some local optimum design while solving the non-linear problems. Therefore to overcome the present issue, another approach is discussed in the next paragraph.

An approach of using a binary (0-1) representation of material for the continuum mechanics based approach can help to preserve the discrete nature of structural and CM related problems [16–19]. The binary, material-void design domain results in a discrete, typically non-convex space [20] and allow for a precise, although discretized, topology boundary but this representation usually results in 'checkerboard' pattern problem and 'floating elements' of material which are disconnected from the main part of structure in a design domain.

The above approach can easily integrate with any evolutionary algorithm to evolve the optimum designs and structures because these algorithms can handle the non-linearity involved in the topology optimization of structures and compliant mechanisms and can also directly deal with the multi-objective optimization. Using the binary (0-1) representation, the study [21] dealt with a multi-objective optimization of CM designs by considering the maximization of mechanical efficiency, geometric and mechanical advantage, and minimization of the maximum compressive load. Luo et al. [22] used maximization of mutual potential energy subjected to path length, stiffness, connectivity requirement, stress, variable mesh geometry, size, and mixed variable constraints for a unique approach of load path synthesis for topology and dimensional synthesis of compliant mechanisms.

All the aforementioned studies discussed various formulations used to generate the compliant mechanisms which were designed to perform the assigned task. These studies dealt with the classical and evolutionary approaches with linear finite element models. But when large displacement complaint mechanisms [23–26] and path generating / tracing complaint mechanisms (PGCM) are designed, then it involves non-linear FE models which makes the designing problem more difficult to solve. As the present work concentrates on the PGCM designing, a few important studies are discussed here which use non-linear FE models and optimized using evolutionary algorithms.

Initially, a single-objective optimization based PGCM design was formulated by minimizing the deviation between the distance of desired and corresponding actual trajectories after dividing them into N precision points [27]. This study used a novel morphological representation scheme [28] and generated the PGCM design using genetic algorithm. Later, a multi-objective formulation [29] was used in which the least square objective of actual and desired output responses at each precision point was minimized using NSGA-II algorithm [30]. In this study, the number of objectives were equal to the number of precision points used to represent the prescribed path. Finally from the evolve set of non-dominated solutions, a solution that minimizes the sum of individual least square objectives was chosen.

The precision points which are used to represent the prescribed path are characterized either by a same level of input load or input displacement which can result in an artificial constraint in the design problem. Also, when an objective of minimizing the Euclidean distance is considered, it might misrepresent the nature of design problem by requiring the shape, size, orientation and position of the prescribed path to be optimized all at once. This problem was overcome by using Fourier shape descriptor-based objective [31, 32]. The formulation involved many user-defined parameters which might effect the optimum solution based designs.

Authors of this paper thought that it is compulsory task of the compliant mechanisms to trace the prescribed path. When an Euclidean distance error based objective function is used, the optimum design may not follow the prescribed path adequately. Thus, a formulation is required which can ensure the generation of prescribed path by all compliant mechanisms. In this work, a constraint bi-objective optimization formulation is used which can evolve the multiple topologies of compliant mechanisms. The functional aspect of generating the prescribed path by these mechanisms is fulfilled by imposing the constraints on precision points. The details of PGCM formulation is discussed in Section 2.

To solve the constraint bi-objective, non-linear and discrete problem of compliant mechanisms, the evolutionary algorithm (NSGA-II [30]) is used as global optimizer. In this paper, the optimization technique is not used as a black box. Rather, the NSGA-II algorithm is customized to efficiently deal with the topology optimization problem. The local search method is coupled with the optimization procedure to further refine the topologies obtained from NSGA-II algorithm. The details of the customized evolutionary algorithm is given in Section 3. In the later part of this paper, the compliant mechanism's topologies and associated results are presented in Section 4.

2 PGCM Problem Definition

The problem formulation of path generating compliant mechanisms requires primary attention because the task of generating/tracing the user-defined path is an essential functional aspect of these mechanisms. Hence in the PGCM problem definition, the functional and other aspects have to be incorporated which are discussed later in this section. First of all, the design domain of compliant mechanism (50 mm by 50 mm) is explained as shown in Figure 1 which is categorized into three regions of interest. The Ist region is called support region where the nodes of an element of the elastic structure are restrained with zero displacement. In the IInd region (loading region i.e. a node of an element), some input displacement boundary condition is applied. The output region is the IIIrd region, that is, a fixed point on the elastic structure which traces out the desired path defined by user.

In this work, the origin of the design domain is fixed on its left hand side and the output region is positioned at the coordinate $(50, 32)$ of the structure. As Figure 1 shows, a spring of constant stiffness (κ = 0.4 KN/m) is attached at the

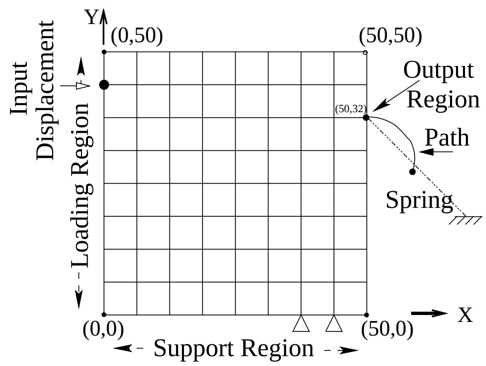


Fig. 1. A design-domain with loading, output and support regions.

output point for providing some resistance during the deformation of elastic structure.

As discussed earlier, the essential functional aspect of path generating compliant mechanisms is to trace the prescribed path. Thus, the same compulsory task of compliant mechanism is accomplished in the present paper by imposing the constraints at precision points. These hard constraints bound the maximum distance between the prescribed and actual paths for all feasible designs. A hypothetical case is shown is Figure 2 in which a prescribed path and an actual path traced by the elastic structure after FE analysis are drawn. Here, the prescribed path is represented by N precision points. The corresponding points on an actual path traced by an elastic structure is evaluated from geometrical non-linear FE analysis based on equal load steps.

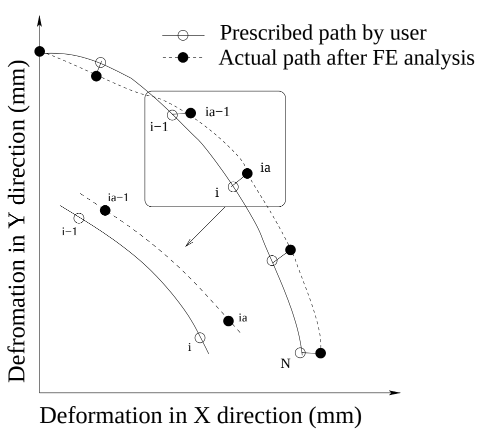


Fig. 2. The prescribed path and an actual path traced by the elastic structure after FE analysis.

To physically represent the constraints, first an euclidean distance (say d_1) is evaluated by estimating the distance between the current (i) and previous ($i-1$) precision points and get multiplied by a factor η called as *percent of allowable deviation*. Then, another euclidean distance (say d_2) between the current precision point (i) and the corresponding point (i_a) of actual path is calculated. Based on these calculations, a constraint is imposed at each precision point which ensures that $d_2 \leq d_1$. A pictorial significance is shown in Figure 2 in which, if a circle of radius d_1 at the current precision point (i) is drawn, then the corresponding point (i_a) of actual path must lie within or on the circle to satisfy the constraint on each precision point. The mathematical representation of constraints at each N precision points is given in Equation 1. Any elastic structure which satisfies these constraints can guarantee to accomplish the task of tracing the path based on user-defined allowable deviation (η).

In the present study, an additional constraint limiting the maximum stress developed in the elastic structure is also taken into the consideration for the feasible PGCM designs. The bi-objective constraint optimization formulation of PGCM is given in Equation 1 in which the primary objective of minimizing the weight and secondary objective of minimizing the supplied input energy to the continuum elastic structure [33, 34] are used to evolve 'trade-off' solution's based compliant mechanism designs.

Bi-objective optimization:
Minimize: Weight of structure (primary obj.),
Minimize: Supplied Input energy to structure (secondary obj.),

Problem is subjected to:

$$1 - \frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta\times\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}} \geq 0, \quad i = 1,2,...,N$$
$$\sigma_{flexural} - \sigma \geq 0, \tag{1}$$

where η is the percentage of allowable deviation, N is number of precision points representing prescribed path and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the elastic structure, respectively.

In the given formulation, η is only a user-defined parameter which signifies the adherence between the actual and prescribed paths. The different η values symbolize the relaxation in constraint formulation and have an effect on the evolved topologies of PGCM. Therefore in this paper, the multiple topologies of PGCM are evolved after solving Equation 1 and simultaneously, their sensitivity towards different η values are investigated. The obtained results are then compared and further justified with the results of region-wise analysis. The multi-objective evolutionary algorithm is customized in this paper to efficiently deal with such structural topology optimization problem which is described in the subsequent section.

3 Customized Evolutionary Algorithm

Applications of evolutionary algorithms (EA) in topology optimization of structure and compliant mechanism designs [16–19, 21, 22, 37] are mostly preferred as these problems are discrete in nature and involve non-linearity. Many times, these algorithms are modified according to the nature of problems so that they can efficiently deal such problems and generate improved solutions [28, 33–36, 38–42]. In this paper, a popularly used elitist non-dominated sorting genetic algorithm (known as NSGA-II [30] which is developed by second author of this paper and his students) is used as a global search and optimizer which has shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front for two objective problems. A detailed description of NSGA-II algorithm can be found in the study [30]. In short, NSGA-II is population based evolutionary optimization procedure which uses mathematical partial-ordering principle to emphasize non-dominated population members and a crowding distance scheme to emphasize isolated population members in every iteration. An elite-preserving procedure also ensures inclusion of previously found better solutions to further iterations. The overall procedure with *N* population members has a computational complexity of O(NlogN) for two and three objectives problems and has been popularly used in many studies. NSGA-II is also adopted by a few commercial softwares (such as iSIGHT and modeFRONTIER). A code implementing NSGA-II is available at *http://www.iitk.ac.in/kangal/codes.shtml* website.

A local search method is coupled with the evolutionary optimization procedure to further refine the non-dominated solution's based designs of compliant mechanism. The basic details of local search based customized NSGA-II algorithm is shown in Figure 3. It shows various schemes like structure representation, connectivity analysis, two-dimensional crossover operator, mutation operator etc. which is expected to efficiently deal with the topology optimization problems of structures and compliant mechanisms.

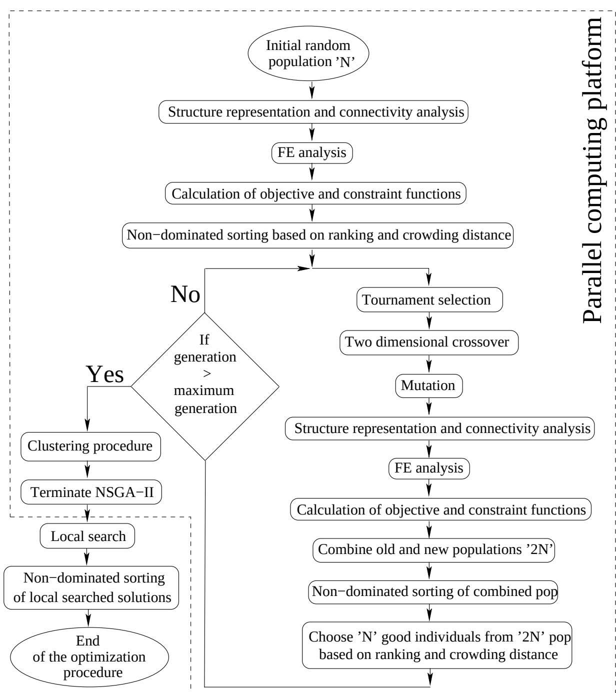


Fig. 3. A flow chart of customized NSGA-II algorithm

3.1 GA Parameters

A population of 240, crossover probability of 0.95 and mutation probability of $(1/\text{string length})$ are assigned and the NSGA-II algorithm is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 637 bits is used as shown in Figure 4. This string is made of two sets in which the Ist set of 625 bits represents the shape of structure (described in Section 3.2) whereas, the decoded value of IInd set identifies the support and loading positions in their respective regions (refer Figure 1), and magnitude of input displacement. For the same, the remaining 12 bits of IInd set are further divided into three sets of five, three and four bits as shown in Figure 4. The decoded value of first five bits indicates the location of an element from the origin where the elastic structure is to be supported. The decoded value of subsequent three bits helps in determining the loading position, that is, a node where the input load is applied. The decoded value of last four bits are used to evaluate the magnitude of input displacement which can vary from 1 mm to 16 mm at step of 1 mm. The above mentioned flexibility is implemented to come-up with the optimum combinations of support and loading positions, and input displacement magnitude to promote the non-dominated solutions through the run of NSGA-II algorithm. Additionally, it can also help in eliminating the issue of characterization of precision points using same level of input load as mention elsewhere [31]. A detailed significance of additional bits will be discussed later along with the results presented in the study.

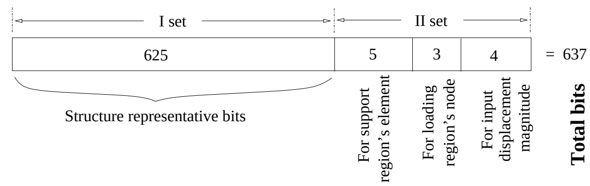


Fig. 4. A binary string comprises of two sets.

3.2 Structure Representation Scheme

A binary string of 625 bits is used to represent the shape of structure. First, a binary string is copied to two dimensional representation as shown in Figure 5. Thereafter, the material-void representation of each grid is chosen based on the binary bit value, for example, bit value 1 signifies that material is present whereas, 0 represents the void. This scheme divides a design domain of structure into 25×25 $(= 625)$ grids in x and y directions, respectively.

3.3 Connectivity Analysis and Repairing Techniques

In the structure representation scheme, values (0-1) at each grid are initially assigned in a random manner. This ensures that the material present in a design domain does not follow any particular pattern. But here, it is not guaranteed that the three regions of interest (support, loading and output

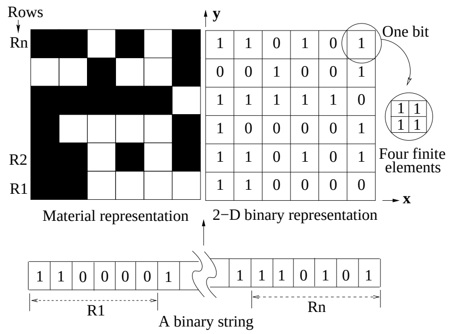


Fig. 5. A representation of structure using binary string and material.

regions of a structure) are connected to each other by material. Therefore, a connectivity analysis is done to make the designs meaningful. For the same, the first task is to find the clusters of material at all three regions of a structure and check whether they are connected or disconnected to each other. In the disconnected scenario, the individual distances are calculated for each grid of material of first cluster with each grid of material of another cluster. Then, a straight line is drawn between those two grids which show a minimum distance between the above two clusters. Thereafter, material is assigned to those grids where the above straight line passes. In the same way, connectivity among support, loading and output regions of a structure is checked and ensured.

When all the three regions are connected either directly or indirectly to each other, the given optimization procedure fills those grids with the material which are void and surrounded by the neighboring eight grids of material. If two grids generate a point connection, then the given procedure puts one extra material at the nearby grid to eliminate the problem of high stress at the point connectivity. If any cluster of material which is not a part of any clusters of three regions of interest as mentioned above, then it is deleted from the structure (assigned '0' to each grid of this cluster). Hence, the type of representation and connectivity eliminates the problems of 'checker-board' pattern and 'floating elements' of material. The detailed description of connectivity analysis and repairing techniques can be found elsewhere [43].

3.4 Finite Element Analysis

After the random initialization, structure representation, connectivity analysis and repairing techniques, the elastic structure is analyzed for stress and deformation by FE analysis. In this study, one grid of a structure (as described in Section 3.2) is further discretized into four finite elements with same binary variable value as shown in Figure 5. Therefore in the present process, the structure is discretized with $4 \times 625\,(=2500)$ 4-node rectangular finite elements and analyzed through a non-linear large deformation FE analysis using *ANSYS* package. But, the GA operations are performed on the same structure represented by 625 bits.

3.5 GA Operators

Crossover is an important genetic algorithms (GA) operator which is responsible for the search aspect of the algorithm. It creates new solutions which differ from the parent solutions. In this study, a two-dimensional crossover operator is used which has shown a successfully applications in shape optimization [44,45] and in the designing of compliant mechanisms problems by authors [33–35]. In the present recombination operator, two parent solutions are selected and a coin is flipped to decide for row or column-wise crossover. If a row crossover is done, a row is chosen with an equal probability of (P_{xover}/no. of rows) for swapping. The same is done if a column-wise crossover has to be done. During crossover, a random number is generated to identify the number of rows (columns) to be swapped and then, another generated random number helps in getting the first row (column) number of patches. A range of row (column) index is calculated and swapped with other parent. A pictorial view of the crossover on the structures represented by Ist set of binary string is shown in Figure 6. For the crossover of 12 bits

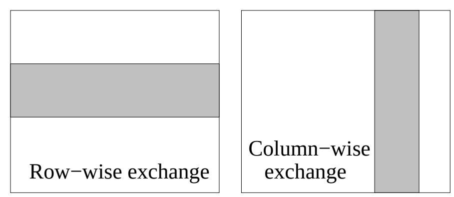


Fig. 6. A two dimensional crossover which swaps a patch of row/column between two parent solutions.

of IInd set, a standard single point crossover is used in the present study.

Mutation operator is another GA operator which generates new solutions in the population but usually it is done with a low probability. Here, it is done with a probability of (1/string length) on each bit of a string of Ist set to change from a void to a filled or from a filled to a void grid. A detailed discussion of these crossover and mutation operators are given elsewhere [43,46]. For mutating the remaining 12 bits of IInd set, first the decoded values of support and loading regions, and magnitude of input displacement are evaluated and then, these values are perturbed within the range of $\{-2,2\}$ at their original values. Here, it is ensure that the perturbed values of above three boundary conditions do not fall outside their respective bounds. This mutation operator helps to get the nearest integer value at the original one. After perturbation, these mutated values are again coded into the binary string of 12 bits.

3.6 Parallel Computing

A distributed computing platform is used in the present study to reduce the computational time of designing and synthesis of compliant mechanisms. In this parallelization process, the root processor first initializes a random population. Then, it divides the entire population into different sub-populations in proportion to the number of processors available. After this, each sub-population is sent to different slave processors. These slave processors further evaluate the objective functions and constraints values, and send them to the root processor. Thereafter, root processor performs the GA operators, like selection, crossover and mutation operators, non-dominated front ranking etc. on the population and replaces it with good individuals. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through *ANSYS* FE package which consumes the maximum time of the optimization procedure [33–36]. A MPI based Linux cluster with 24 processors is used in the present study. A detailed specification and configuration of the Linux cluster are given at *http://www.iitk.ac.in/kangal/facilities.shtml* website.

3.7 Clustering Procedure

For an adequate convergence near to the global 'Pareto-optimal' front, the evolutionary algorithms (EA) need a fairly large number of population members and generations depending upon the problem complexity. Thus, the number of feasible solutions after the EA run are usually close to the population size. It is not advisable to represent so many solutions to the end user for a subsequent decision-making task. Therefore, the clustering procedure is employed in the study in which the neighboring solutions are grouped together and solutions from each group representing that region of the non-dominated front are chosen as representative solutions [47]. Figure 7 shows the procedure pictorially. After clustering, the parallel NSGA-II algorithm is terminated. Thereafter, the local search method (described in next section) is employed on each representative NSGA-II solutions based designs to improve them locally.

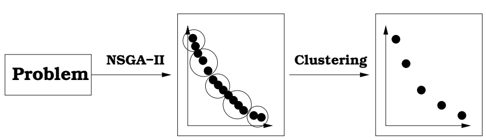


Fig. 7. A clustering Procedure.

3.8 Local Search Method

The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is given in Equation 2.

$$Minimize\ F(x) = Minimize \sum_{j=1}^{n} \frac{\overline{w}_j^x (f_{j_{max}}^x - f_j^x)}{f_{j_{max}}^x - f_{j_{min}}^x}, \qquad (2)$$

where, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of j^{th} objective function in the population, respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the j^{th} objective function which is computed as:

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \qquad (3)$$

where M is the number of representative solutions after clustering procedure. In Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space [48]. As Equation 3 suggests, these weights are calculated based on their positions in two-objective space after the termination of NSGA-II algorithm.

In the local search method, first the weighted sum of scaled fitness of a selected representative solution is evaluated as given in Equation 2. Thereafter, one bit of representative solution is mutated at a time and the design is extracted from the new string. This new string's based structure is analyzed by FE package and then, the objective and constraint functions are evaluated. If the new design does not satisfy any constraint, then the change in new string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string values. In case of mutating '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that of old string, or else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old string's weighted sum value, then it is accepted or else the change is discarded. In case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the grids having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the change is accepted. This process is repeated till all bits are mutated once. If there is no change in the value of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As discussed in Section 3.4, one binary bit represents four elements for FE analysis. Therefore, a binary string of 625 bits represents a structure which is discretized with 2500 finite elements. In case of local search, the previous binary strings (625 bit) of representative solutions are reconstructed into the new binary strings of 2500 grids. These grids represent the same structure of 2500 elements and the local search search is performed on these 2500 grids.

4 Topologies of Curvilinear Path Generating Compliant Mechanism

In the present section, the multiple topologies of curvilinear path generating compliant mechanism are evolved by solving the constraint bi-objective formulation using the customized evolutionary optimization. The sensitivity of the compliant mechanism topologies are also investigated for different values of η.

A few parameters are kept constant during the whole study such as, a material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for synthesis of compliant mechanism. The direction of input displacement (i.e. along x direction) and the prescribed path are also fixed. Here, the prescribed path is represented by five precision points and the trajectory traced by output region's node of the elastic structure is evaluated through a geometric nonlinear FE analysis using *ANSYS* package. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. After the termination of NSGA-II algorithm, maximum six representative solutions are chosen from the non-dominated set of NSGA-II solutions with the help of clustering procedure.

4.1 Sensitivity of PGCM topologies towards η values

The factor η is an important parameter for PGCM designing because it signifies an adherence between the prescribed path and the actual path traced by compliant mechanism designs. A pictorial view showed in Figure 2 signifies the closeness of two paths which is checked by imposing the constraint at each precision point. These constraints are illustrated with the help of drawing a circle of radius d_1 at the current precision point which is described in Section 2. For smaller values of η, the condition of adherence is strict and finding any feasible solution would be a difficult task for any optimization procedure. On the other hand, larger value of η increases the feasible region of the problem and thus, relaxes the constraints at precision points. But at the same time, the prescribed and actual paths can be apart from each other. Thus, it is important to find a suitable range of η so that the designers and decision makers can choose the PGCM topologies with the desired inspiration level. Therefore, this study not only helps in investigating the sensitivity of PGCM topologies towards user-defined allowable deviation (η) but also, it is expected to provide a platform to the designers and decision makers to understand the topological changes and flexibility to choose a particular design of PGCM among the obtained non-dominated topologies as per their requirement.

4.1.1 Study for different η values

In this section, PGCM designs are evolved for different user-defined allowable deviation such as $\eta = 5\%, 10\%, 15\%, 20\%$ and 25% by solving Equation 1 using the customized NSGA-II algorithm as described in Section 3.

This study starts with $\eta = 5\%$ in which not a single feasible solution is found by the customized evolutionary algorithm. This situation arises because of the condition of close adherence between the actual and prescribed paths. The feasible region around each precision point is very small and therefore, the optimization procedure fails to find any feasible solution. Hence, it is too optimistic condition to evolve any compliant mechanism tracing almost the same path as it is prescribed.

In the second attempt, PGCM designs are evolved for $\eta = 10\%$. As the feasible region around each precision point has increased, the customized NSGA-II algorithm evolves six feasible non-dominated NSGA-II solutions (a to f) whose positions in the two-objective space are shown in Figure 8. It also shows the position of these solutions after local search, after which only five solutions (1 to 5) are evolved because solutions e and f converged to a single solution 5. Here, only three solutions (1, 2 and 3) out of five local search solutions become non-dominated (clearly shown in a small window of Figure 8) and thus, become a part of non-dominated front for $\eta = 10\%$. The deformed and final undeformed topologies of these three solutions are shown in Figure 9. In this figure, solution 1 (refer Figure 9(a)) is a minimum weight solution as compared to all non-dominated solutions but at the same time it requires higher input energy to deform the structure and to trace the prescribed path (refer Figure 8). On the other hand, solution 3 (refer Figure 9(e)) requires minimum supplied input energy but it evolves as heavier structure. In a similar way, solution 2 shows trade-off between the two-objectives of minimizing the weight of structure and minimizing the supplied input energy to the elastic structure. Topologically, solution 1 consists three closed loops of material whereas, solutions 2 and 3 comprise of one and two closed loops topologies, respectively.

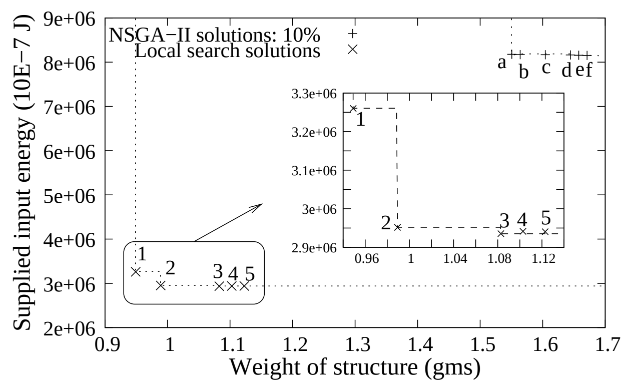
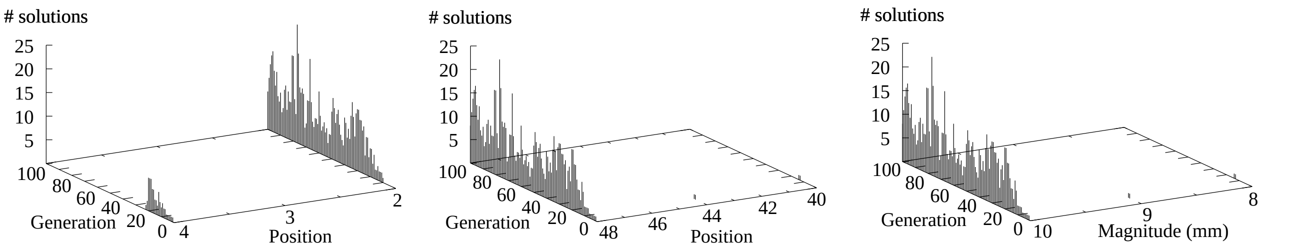


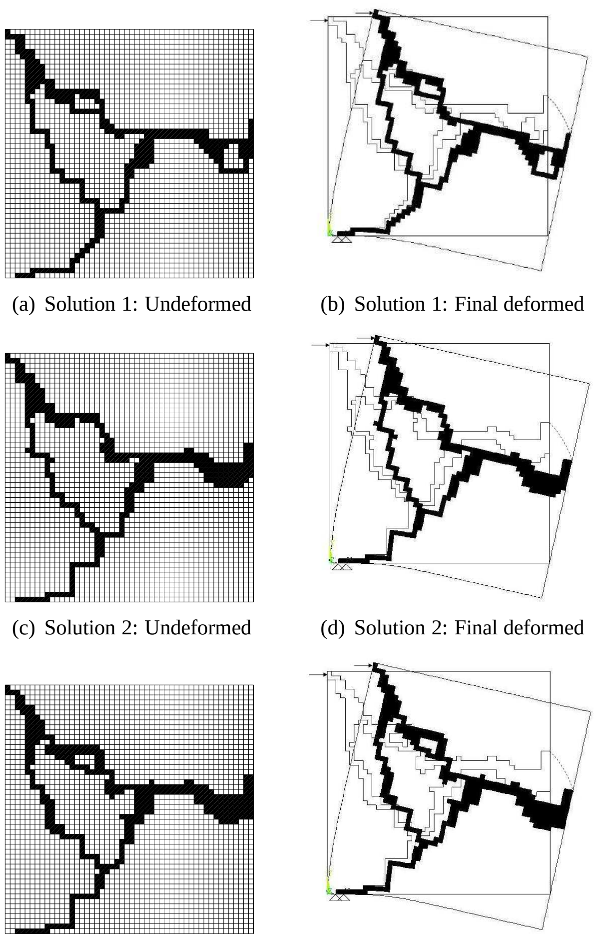
Fig. 8. The NSGA-II and local search solutions for $\eta = 10\%$.

As discussed in Section 3.1, the flexibility is provided to the customized NSGA-II algorithm to identify the optimum combinations of support and loading positions, and magnitude of input displacement. The progress of these conditions for feasible non-dominated solutions during the NSGA-II run is shown in Figure 10. The support positions of solutions are shown in Figure 10(a) which are evaluated after decoding the first five bits of IInd set of binary string of each so-



(a) Support position (b) Loading position (c) Magnitude of input displacement

Fig. 10. The progress of feasible non-dominated solutions for $\eta = 10\%$.



(a) Solution 1: Undeformed (b) Solution 1: Final deformed

(c) Solution 2: Undeformed (d) Solution 2: Final deformed

(e) Solution 3: Undeformed (f) Solution 3: Final deformed

Fig. 9. The non-dominated topologies of PGCM for $\eta = 10\%$.

lution. As this figure shows, two different support positions are evolved in the beginning of NSGA-II run, but at time of termination of the algorithm, all the feasible non-dominated solutions are supported at same position, that is, 2 mm away from the origin. Similarly, the loading positions (refer Figure 10(b)) and magnitude of input displacement (refer Figure 10(c)) are calculated in each iteration of NSGA-II after decoding the next three bits and last four bits of IInd set of binary string, respectively. These conditions are also tabulated in Table 1 which indicates the loading position at 48 mm and required input displacement magnitude of 10 mm. The above mentioned set of conditions emerges as optimum for PGCM designing for $\eta = 10\%$.

Table 1. Evolved support and loading positions and, input displacement magnitude for different η values.

Conditions → η values ↓	Support position (*mm*) (*from the origin*)	Loading position (*mm*) (*from the origin*)	Input displacement (*mm*)
10%	2	48	10
15%	2	48 46	10 9
20%	2	48	10
25%	2	40	8

In the subsequent attempt, an Equation 1 is solved for $\eta = 15\%$. In this case, the compliant mechanisms are designed with relatively larger allowable deviation at each precision point. Here, the customized NSGA-II algorithm evolves five feasible non-dominated solutions (*a* to *e*, refer Figure 11) and thereafter, these solutions are further refined by local search method. From a set of five local search solutions (1 to 5), only two solutions (1 and 2) become a part of non-dominated front for $\eta = 15\%$. The solution 1 is evolved

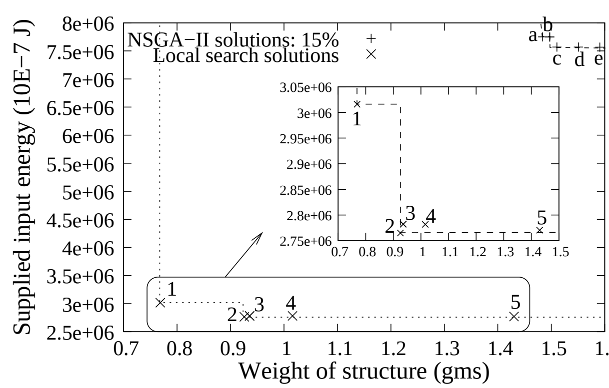
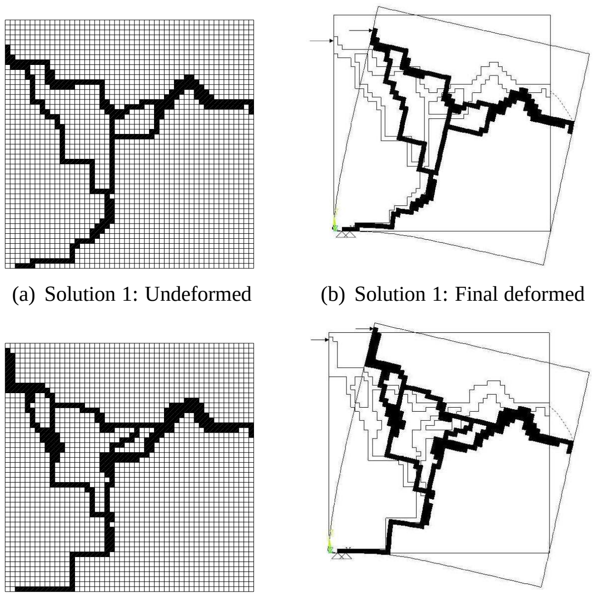


Fig. 11. The NSGA-II and local search solutions for $\eta = 15\%$.

as a lighter in weight but requires relatively larger input energy as compared to that of solution 2 which is heavy in weight. The different looking topologies of these solutions are shown in Figure 12 with their final deformed and un-



(a) Solution 1: Undeformed (b) Solution 1: Final deformed

(c) Solution 2: Undeformed (d) Solution 2: Final deformed

Fig. 12. The non-dominated topologies of PGCM for $\eta = 15\%$.

deformed stages. Here, solution 1 (refer Figure 12(a)) consists of two closed loops topology whereas, solution 2 has three closed loops of material (refer Figure 12(d)). These arrangements of material in the design domain of both solutions make trade-off between the two-objectives.

For $\eta = 15\%$, the progress of feasible non-dominated solutions with respect to support and loading regions, and input displacement is similar as observed during the study of $\eta = 10\%$. The optimum set of these conditions are also given in Table 1 which indicates that the solutions 1 and 2 are supported at 2 mm but the load is applied at different positions. For the solution 1, the load is applied at 48 mm and 46 mm position is evolved for solution 2. As these solutions have different loading positions, the solutions 1 and 2 require 10 mm and 9 mm of input displacement to deform the topologies to trace the prescribed path, respectively.

In the next attempt, the PGCM design problem is dealt with $\eta = 20\%$. The feasible region around each precision point is further increased in comparison with the previous attempts. In this case, the NSGA-II algorithm evolves several feasible non-dominated solutions and among them, six representative solution are chosen with the help of clustering procedure as described in Section 3.7. The Figure 13 shows the positions of representative NSGA-II solutions (a to f) and after local search solutions (1 to 6) in which only three solutions (1, 2 and 3) out of six become a part of non-dominated front for $\eta = 20\%$. The support and loading positions, and input displacement condition of these solutions are given in Table 1. The undeformed and final deformed topologies of

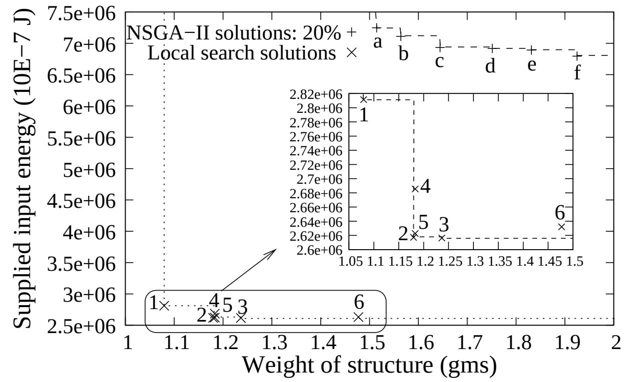
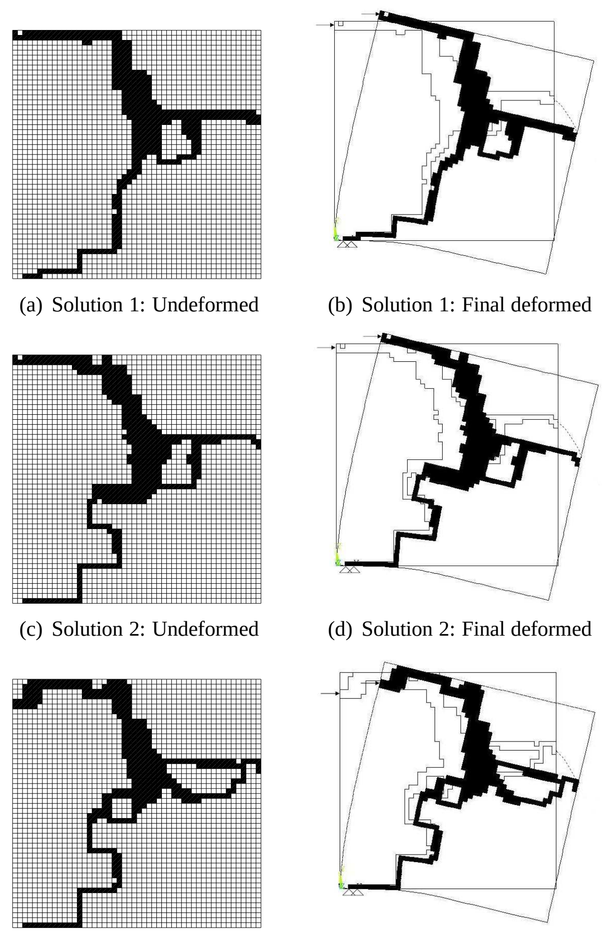


Fig. 13. The NSGA-II and local search solutions for $\eta = 20\%$.

these solutions are shown in Figure 14 in which solutions



(a) Solution 1: Undeformed (b) Solution 1: Final deformed

(c) Solution 2: Undeformed (d) Solution 2: Final deformed

(e) Solution 3: Undeformed (f) Solution 3: Final deformed

Fig. 14. The non-dominated topologies of PGCM for $\eta = 20\%$.

1 and 2 consist one closed loop topology whereas, solution 3 has two closed loops of material. The solutions 2 and 3 also show a different kind of material's distribution near to the support region which causes the less requirement of input energy to deform the elastic structures in-comparison to

the solution 1. Hence, the number of closed loops and the arrangements of material joining support, loading and output regions in the design domain result in trade-off between the two objectives.

In the last attempt, η is set to 25%, which signifies the larger feasible region around each precision point but at the same time, the prescribed and actual paths can be apart from each other. In this case, only two solutions are evolved by the customized NSGA-II algorithm as shown in Figure 15 and rest of the population members are the copies of above

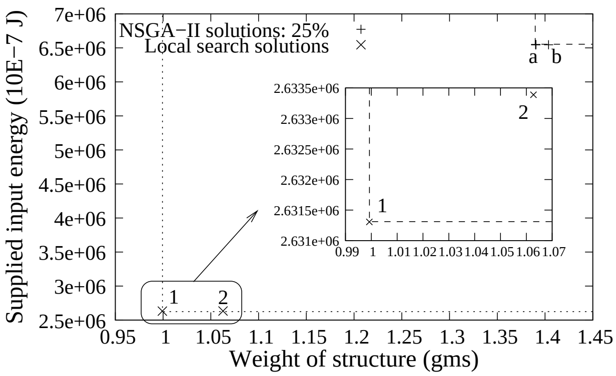
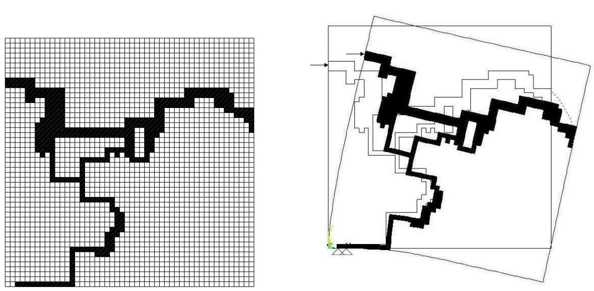


Fig. 15. The NSGA-II and local search solutions for $\eta = 25\%$.

two solutions. After the local search of these solutions, solution 1 dominates solution 2 because it has minimum weight and requires minimum supplied input energy to deform the structure and to generate the prescribed path. The final deformed and undeformed PGCM design of solution 1 which consists of two closed loops of material is shown in Figure 16. In this case, the solution 1 is supported at 2 mm, the load



(a) Solution 1: Undeformed

(b) Solution 1: Final deformed

Fig. 16. The non-dominated designs of PGCM for $\eta = 25\%$.

is applied at 40 mm and requires 8 mm of input displacement to deform the compliant mechanism.

4.1.2 Discussion

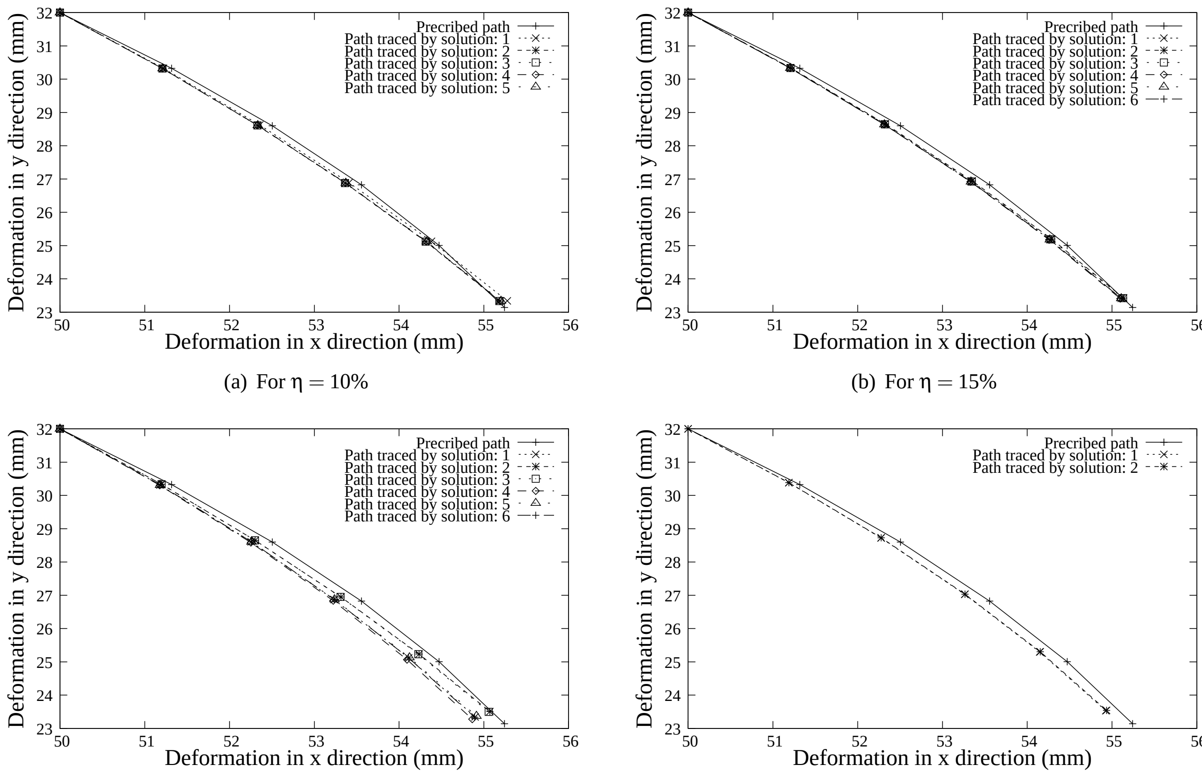
For $\eta = 10\%$, 15%, 20% and 25%, the customized NSGA-II algorithm starts evolving the elastic structures with varying boundary and applied conditions in its initial generation but at the end of algorithm, it identifies one or two optimum sets of these conditions. In all problems, the first feasible solutions are appeared after a few generation of NSGA-II algorithm because the optimization procedure starts with random initial population. To make the population members feasible, first the NSGA-II algorithm tries to satisfy the given set of constraints of the problem as described by Equation 1. As soon as, the first few feasible solutions are evolved, they start dominating other solutions based structures with same or different sets of boundary and applied conditions. The domination is based on their objective function values. Meanwhile, if any solution based structure is alive and becomes non-dominated during NSGA-II run, then only it can propagate to further generations. Therefore in all problems, elastic structures with a few sets of boundary and applied conditions dominate the rest and assists the evolution of non-dominated designs. As several non-dominated feasible solutions dominate the whole population members, the selection operator of NSGA-II makes a copy of these non-dominated solutions in every generation. After a while, all population members become the copies of these non-dominated designs, as it can be observed in the study of $\eta = 25\%$. If individual studies of different η values are analyzed, the compliant mechanism designs are topologically different because of the number of closed loops of material present in the design domain but they look similar except for $\eta = 15\%$.

From the above study, it can be seen that for very small value of η, the customized NSGA-II algorithm fails to evolve any feasible solution. As the η value increases, the feasible non-dominated solutions get evolved. In case of $\eta = 10\%$, the customized NSGA-II procedure generates only six non-dominated solution's based topologies but with similar looking designs. For $\eta = 15\%$, again the optimization algorithm evolves six solutions of diverse topologies. When the η value is increased to 20%, a local search based NSGA-II procedure generates several non-dominated solutions and from the evolved set of these solutions, six representative solutions are chosen for local search. Here, when several non-dominated solutions are generated but at the same these solutions are evolved as bulky designs. When $\eta = 25\%$ is assigned, it allows a large amount of relaxation in the constraint violation which results in the domination of couple of PGCM designs and finally, it concludes at one non-dominated topology.

Overall, the effect of η has been observed on the generated topologies which now helps in deciding a suitable range of η. From the above discussions, the $\eta = 10\%$ to 20% is a suitable range for designing the PGCM topologies with the desired aspiration level. In the present study, $\eta = 15\%$ is chosen for the rest of the study because it assists the optimization procedure to evolve a diverse set of light weight topologies of PGCM.

4.1.3 Generated paths

The paths traced by all local search solutions of $\eta = 10\%, 15\%, 20\%$ and 25% studies are shown in Figure 17 along with the prescribed path. It shows the closeness be-



(a) For $\eta = 10\%$

(b) For $\eta = 15\%$

(c) For $\eta = 20\%$

(d) For $\eta = 25\%$

Fig. 17. The prescribed path and paths traced by local search solutions.

tween the paths for different η values, pictorially. The Table 2 gives quantitative specification of these paths in which the maximum allowable distance d_1 (refer Figure 2) at each precision point and distance d_2 between the precision point of a prescribed path and corresponding point of actual path are given. For $\eta = 10\%$, precision points 3, 4 and 5 are critical because d_2 value is close to the maximum value of d_1. A closer look into the Figure 17(a) shows that the paths traced by all local search solutions intersect the prescribed path in-between the precision points 4 and 5. In case of $\eta = 15\%$, the maximum value of d_1 increases at each precision point and also, each solution based design shows an increasing trend in d_2 value. This makes the precision point 5 to be critical. An increasing trend of d_2 value can also been seen for $\eta = 20\%$ and 25%. Paths traced by solutions 2 and 3 for $\eta = 20\%$ are comparatively closer to the prescribed path (smaller d_2 value) which can also be seen in Figure 17(c). Overall, the figure and table show an accomplishment of tracing the prescribed which is successfully achieved by using the precision points based constraints formulation. Also, a different behavior of paths traced by local search solutions can also be appreciated which either intersect or below the prescribed path.

A close observation on the values of d_2 for the precision point 1 for all cases of η shows that its minimum and maximum values lie between 0.0964 to 0.1411. These values are even less than the smallest value of d_1 ($= 0.2130$ for $\eta = 10\%$) at precision point 1. This indicates that constraint on precision point 1 is not really a critical one. As, all these elastic structures get deformed to follow the remaining part of prescribed path, constraints on the respective precision points become critical. Therefore, the intermediate and last precision points are crucial for the designs to trace the prescribed path. But, if any elastic structure makes the constraint critical on precision point 1, then it is unlikely that the structure can generate the prescribed path by satisfying the constraints on all precision points. Therefore, the constraint on initial precision points can not be ignored because these constraints assist the optimization procedure to evolve the feasible designs. Here, the prescribed path is same for all η values based studies and it is designed such that the output point of every structure has to deform 10.48% in x-direction and 17.72% in y-direction with respect to the size of design domain.

4.2 Region wise analysis

In the last section, the non-dominated topologies of compliant mechanism tracing downward curvilinear path were evolved and their sensitivity towards different η values were discussed. The flexibility of identifying the conditions helped in evolving the optimum combinations of support and loading positions, and the magnitude of input displacement for generating the given prescribed path.

At this stage, one thing can be ponder that the non-dominated solutions of all η based studies are supported at one position after the termination of NSGA-II, that is, at the

Table 2. Deviation between precision points and corresponding points of actual path.

Precision points	1	2	3	4	5
For $\eta = 10\%$					
Max. allowed $\mathbf{d}_1$	0.2130	0.2094	0.2062	0.2037	0.2018
Solution 1: $\mathbf{d}_2$	0.0964	0.1495	0.1600	0.1527	0.2018
Solution 2: $\mathbf{d}_2$	0.1077	0.1741	0.1978	0.1924	0.2017
Solution 3: $\mathbf{d}_2$	0.1086	0.1759	0.2001	0.1945	0.2018
Solution 4: $\mathbf{d}_2$	0.1073	0.1733	0.1966	0.1911	0.2018
Solution 5: $\mathbf{d}_2$	0.1063	0.1710	0.1928	0.1867	0.2017
For $\eta = 15\%$					
Max. allowed $\mathbf{d}_1$	0.3196	0.3142	0.3093	0.3056	0.3027
Solution 1: $\mathbf{d}_2$	0.1118	0.1913	0.2389	0.2677	0.3027
Solution 2: $\mathbf{d}_2$	0.1107	0.1891	0.2363	0.2660	0.3025
Solution 3: $\mathbf{d}_2$	0.1098	0.1870	0.2327	0.2617	0.3027
Solution 4: $\mathbf{d}_2$	0.1138	0.1953	0.2443	0.2726	0.3026
Solution 5: $\mathbf{d}_2$	0.1128	0.1924	0.2395	0.2676	0.3027
For $\eta = 20\%$					
Max. allowed $\mathbf{d}_1$	0.4261	0.4189	0.4124	0.4075	0.4036
Solution 1: $\mathbf{d}_2$	0.1387	0.2485	0.3264	0.3768	0.4036
Solution 2: $\mathbf{d}_2$	0.1181	0.2075	0.2718	0.3285	0.4036
Solution 3: $\mathbf{d}_2$	0.1189	0.2103	0.2767	0.3335	0.4036
Solution 4: $\mathbf{d}_2$	0.1409	0.2529	0.3326	0.3828	0.4036
Solution 5: $\mathbf{d}_2$	0.1375	0.2455	0.3217	0.3720	0.4036
Solution 6: $\mathbf{d}_2$	0.1360	0.2448	0.3234	0.3755	0.4036
For $\eta = 25\%$					
Max. allowed $\mathbf{d}_1$	0.5326	0.5236	0.5156	0.5093	0.5045
Solution 1: $\mathbf{d}_2$	0.1409	0.2599	0.3568	0.4373	0.5045
Solution 2: $\mathbf{d}_2$	0.1411	0.2600	0.3567	0.4370	0.5045

element which is positioned at 2 mm from the origin. Here, two conclusions can be drawn that either the present support position is unique for the compliant mechanisms to trace the given path or the optimization procedure fails to maintain a diversity in identifying the support positions during the NSGA-II run.

Therefore, to answer the present issues and for the justification of previous results, the support region is divided into four sub-regions as shown in Figure 18. In Section 4.1, the support position was identified by decoding the first five bits of IInd set of a binary string. In that case, the decoded value can vary from 0 mm to 50 mm. But here, the structure can be supported at any element which belongs to only one sub-support region throughout the optimization run. Rest of the boundary and applied conditions are same as defined in the previous section. Here, the support positions of sub regions I, II, III and IV spread between 0 to 11 mm, 12 to 23 mm, 24 to 35mm and 36 to 50 mm, respectively.

The progress of feasible non-dominated solutions during the NSGA-II run for all sub-region studies is similar as observed in the study of η = 10%. Therefore, the evolved support positions of all sub-regions are tabulated in Table 3. An interesting thing can be seen that the elastic structures are supported at the first starting element of sub regions II, III and IV whereas in case of sub region I, the compliant mechanisms are again supported at an element which is located at 2 mm away from origin. The Table 3 also shows the magnitudes of input displacement for all four sub-regions. It is observed here that the magnitude of input displacement increases as the elastic structures get supported away from the origin.

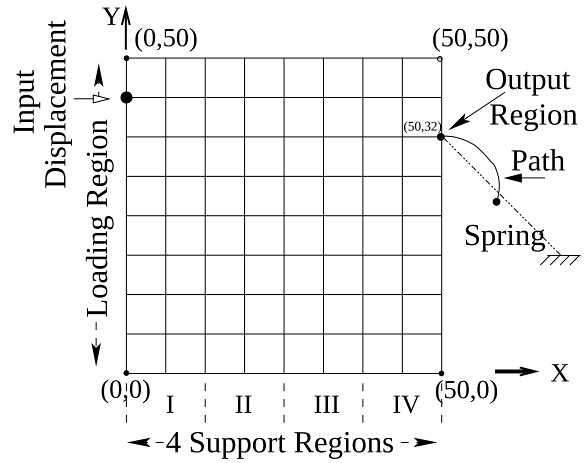


Fig. 18. A design domain with sub divided support region.

Table 3. Evolved support positions and input displacement magnitudes of region-wise analysis.

Conditions → Sub-regions ↓	Support position (*mm, from the origin*)	Input displacement (*mm*)
I	2	7
II	12	10
III	24	12
IV	36	14

At this point, the comparison of NSGA-II solutions of each sub-region study is necessary for better understanding. The Figure 19 shows the dominance of NSGA-II solutions

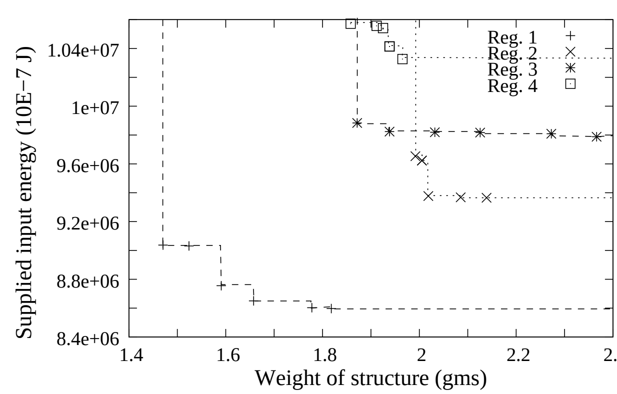


Fig. 19. A comparison between the NSGA-II solutions evolved from different sub-region analyses.

of sub region I (support position at 2 mm) over the others because the design supported in sub-region I are light in weight and require minimum input energy to trace the prescribed path. The figure also reveals that the optimization procedure can generate the elastic structures which are supported at different sub-regions. But, as these structures are supported away form the origin, they require large input displacement. Hence, larger amount of input energy has to be supplied to deform the elastic structures to generate the prescribed path.

As the principle of domination suggests [48], the NSGA-II algorithm assists those sub-regions and conditions which can generate the non-dominated topologies throughout its run. Here, an argument can be given that when any path generating complaint mechanism is designed, the conditions of support and loading position, and magnitude of input displacement are always defined a priori. But in this study, two important advantages of providing the flexibility to the evolutionary algorithm can be seen: first, it can work in the scenario of unknown conditions of support and loading positions, and magnitude of input displacement. Second, it can help the designers and decision makers to appreciate the optimum combination of these conditions for evolving the non-dominated PGCM topologies that explores the possibility of non-optimum conditions which might be considered in their previous practices.

5 Conclusions

The constraint bi-objective formulation was successfully implemented in the present study to evolve multiple 'trade-off' solution's based topologies of path generating complaint mechanisms. The idea of imposing the constraints on precision points of prescribed path seems acceptable to design the compliant mechanisms generating prescribed path. Based on the evolved topologies, the range of η is suggested between 10% to 20%. In this paper, the customized evolutionary algorithm not only solved the present constraint bi-objective problem efficiently, but also evolved the diverse topologies of compliant mechanism. These multiple topologies provide a platform to the designers and decision makers to choose any compliant mechanism from the non-dominated set of solutions. Also, they can understand the topological changes which result in trade-off between the objectives. The flexibility of identifying the support and loading positions and, magnitude of input displacement explored the possibility of non-optimum conditions which might be considered in the previous practices of designers and decision makers. Some more advantages of this flexibility are: (i) it assisted the evolutionary algorithm in the scenario of unknown applied and boundary conditions, and (ii) helped to come-up with the optimum set of these conditions.

Although in the present work, multiple topologies of path generating compliant mechanism were evolved but further attention can be given to the bi- or multi-objective set that can ensure the evolution of diverse and different looking topologies of compliant mechanism. Some new or modified GA operators can also be suggested in the future work to further customize the evolutionary algorithm.

Achnowledgements

Authors acknowledge the support from the Academy of Finland (grant 118319) and Foundation of Helsinki School of Economics. Authors would like to thank Dr. Shamik Chaudhuri for his regular guidance during the development of parallel code of NSGA-II.

References

[1] Howell, L. L., and Midha, A., 1996. "A loop-closure theory for the analysis and synthesis of compliant mechanisms". *ASME, Journal of Mechanical Design,* **118**, March, pp. 121–125.

[2] Hetrick, J. A., and Kota, S., 1999. "An energy formulation for parametric size and shape optimization of complaint mechanisms". *ASME, Journal of Mechanical Design,* **121**, June, pp. 229–234.

[3] Howell, L. L., and Midha, A., 1994. "A method for the design of compliant mechanisms with small-length flexural pivots". *ASME, Journal of Mechanical Design,* **116**, March, pp. 280–290.

[4] Ananthasuresh, G. K., and Kota, S., 1995. "Designing compliant mechanism". *Mechanical Engineering,* **117**(11), November, pp. 93–96.

[5] Ananthasuresh, G. K., 2003. *Optimal Synthesis Methods for MEMS*. Kluwer Academic Publishers.

[6] Bendsoe, M. P., and Kikuchi, N., 1988. "Generating optimal topologies in structural design using a homogenization method". *Computer Methods in Applied Mechanics and Engineering,* **71**, pp. 197–224.

[7] Nishiwaki, S., Frecker, M. I., Min, S., and Kikuchi, N., 1998. "Topology optimization of compliant mechanisms using the homogenization method". *International Journal for Numerical Methods in Engineering,* **42**, pp. 535–559.

[8] Yang, R. J., and Chuang, C. H., 1994. "Optimal topology design using linear programming". *Computers and Structures,* **52**(2), pp. 265–275.

[9] Kumar, A. V., and Gossard, D. C., 1996. "Synthesis of optimal shape and topology of structures". *ASME, Journal of Mechanical Design,* **118**, March, pp. 68–74.

[10] Ananthasuresh, G. K., Kota, S., and Kikuchi, N., 1994. "Strategies for systematic synthesis of compliant mems". In Proceedings of the 1994 ASME Winter Annual Meeting, pp. 677–686.

[11] Sigmund, O., 1997. "On the design of compliant mechanisms using topology optimization". *Mechanics Based Design of Structures and Machines,* **25**(4), pp. 493–524.

[12] Larsen, U. D., Sigmund, O., and Bouwstra, S., 1997. "Design and fabrication of compliant micromechanisms and structures with negative poisson's ratio". *Journal of Miceroelectromechanical Systems,* **6**(2), June, pp. 99–106.

[13] Min, S., and Kim, Y., 2004. "Topology optimization of compliant mechanism with geometrical advantage". *International Journal of JSME, Series C,* **47**(2), pp. 610–615.

[14] Frecker, M. I., Ananthasuresh, G. K., Nishiwaki, S., Kikuchi, N., and Kota, S., 1997. "Topological synthesis of compliant mechanisms using multi-criteria optimization". *ASME, Journal of Mechanical Design,* **119**, June, pp. 238–245.

[15] Luo, Z., Chen, L., Yang, J., and Abdel-Malek, K., 2005. "Compliant mechanism design using multi-objective topology optimization scheme of continuum structures". *Structural and Multidisciplinary Optimization,* **30**, pp. 142–154.

[16] Chapman, C. D., Saitou, K., and Jakiela, M. J., 1994. "Genetic algorithms as an approach to configuration and topology design". *ASME, Journal of Mechanical Design,* **116**, December, pp. 1005–1012.

[17] Chapman, C. D., and Jakiela, M. J., 1996. "Genetic algorithm-based structural topology design with compliance and topology simplification considerations". *ASME, Journal of Mechanical Design,* **118**, March, pp. 89–98.

[18] Duda, J. W., and Jakiela, M. J., 1997. "Generation and classification of structural topologies with genetic algorithm speciation". *ASME, Journal of Mechanical Design,* **119**, March, pp. 127–131.

[19] Jakiela, M. J., Chapman, C. D., Duda, J., Adewuya, A., and Saitou, K., 2000. "Continuum structural topology design with genetic algorithms". *Computer Methods in Applied Mechanics and Engineering,* **186**, March, pp. 339–356.

[20] Anagnostou, G., Ronquist, E., and Patera, A., 1992. "A computational procedure for a part design". *Computer Methods in Applied Mechanics and Engineering,* **97**, pp. 33–48.

[21] Parsons, R., and Canfield, S. L., 2002. "Developing genetic programming techniques for the design of compliant mechnanisms". *Structural and Multidisciplinary Optimization,* **24**, pp. 78–86.

[22] Lu, K.-J., and Kota, S., 2006. "Topology and dimensional synthesis of compliant mehcanisms using discrete optimization". *ASME, Journal of Mechanical Design,* **128**, September, pp. 1080–1091.

[23] Buhl, T., Pedersen, C. B. W., and Sigmund, O., 2000. "Stiffness design of geometrically nonlinear structures using topology optimization". *Structural and Multidisciplinary Optimization,* **19**, pp. 93–104.

[24] Pedersen, C. B. W., Buhl, T., and Sigmund, O., 2001. "Topology synthesis of large-displacement compliant mechanisms". *International Journal for Numerical Methods in Engineering,* **50**, pp. 2683–2705.

[25] Saxena, A., and Ananthasuresh, G. K., 2001. "Topology synthesis of compliant mechanisms for nonlinear force-deflection and curved path specifications". *ASME, Journal of Mechanical Design,* **123**, March, pp. 33–42.

[26] Saxena, A., 2005. "Topology design of large displacement compliant mechanisms with multiple material and multiple output ports". *Structural and Multidisciplinary Optimization,* **30**, pp. 477–490.

[27] Tai, K., Cui, G. Y., and Ray, T., 2002. "Design synthesis of path generating compliant mechanisms by evolutionary optimization of topology and shape". *ASME, Journal of Mechanical Design,* **124**, September, pp. 492–500.

[28] Tai, K., and Chee, T. H., 2000. "Design of structures and compliant mechanisms by evolutionary optimization of morphological representations of topology". *ASME, Journal of Mechanical Design,* **122**, December, pp. 560–566.

[29] Saxena, A., 2005. "Synthesis of compliant mechanisms for path generation using genetic algorithm". *ASME, Journal of Mechanical Design,* **127**, July, pp. 745–752.

[30] Deb, K., Agrawal, S., Pratap, A., and Meyarivan, T., 2002. "A fast and elitist multi-objective genetic algorithm: NSGA-II". *IEEE Transactions on Evolutionary Computation,* **6**(2), pp. 182–197.

[31] Ullah, I., and Kota, S., 1997. "Optimal synthesis of mechanisms for path generation using fourier descriptors and global search methods". *ASME, Journal of Mechanical Design,* **119**, December, pp. 504–510.

[32] Rai, A. K., Saxena, A., and Mankame, N. D., 2007. "Synthesis of path generation compliant mechanisms using initially curved frame elements". *ASME, Journal of Mechanical Design,* **129**, October, pp. 1056–1063.

[33] Sharma, D., Deb, K., and Kishore, N. N., 2006. "Evolving path generation compliant mechanisms (PGCM) using local-search based multi-objective genetic algorithm". In International Conference on Trends in Product Life Cycle, Modeling, Simulation and Synthesis (PLMSS), pp. 227–238.

[34] Sharma, D., Deb, K., and Kishore, N. N., 2008. "An improved initial population strategy for compliant mechanism designs using evolutionary optimization". In ASME International Design Engineering Technical Conferences (IDETC) and Computers and Information in Engineering Conference (CIE), New York, USA, pp. Paper No. DETC2008–49187.

[35] Sharma, D., Deb, K., and Kishore, N. N., 2008. "Towards generating diverse topologies of path tracing compliant mechanisms using a local search based multi-objective genetic algorithm procedure". In IEEE Congress on Evolutionary Computation (CEC-08), pp. 2004–2011.

[36] Sharma, D., Deb, K., and Kishore, N. N., 2008. "A domain-specific crossover and a helper objective for generating minimum weight compliant mechanisms". In Genetic and Evolutionary Computation Conference (GECCO-08), pp. 1723–1724.

[37] Hull, P. V., and Canfield, S., 2006. "Optmial synthesis of compliant mechanisms using subdivision and commercial fea". *ASME, Journal of Mechanical Design,* **128**, March, pp. 337–348.

[38] Bernardoni, P., Bidaud, P., Bidard, C., and Gosselin, F., 2004. "A new compliant mechanism design methodology based on flexible building blocks". In Proceeding of SPIE Smart Structures and Materials, Modelling, Signal Processing and Control (SanDiego, CA, USA), pp. 244–254.

[39] Zhou, H., and Ting, K., 2005. "Topology synthesis of compliant mechanisms using spanning tree theory". *ASME, Journal of Mechanical Design,* **127**, July, pp. 753–759.

[40] Sharma, D., Deb, K., and Kishore, N. N., 2008. Developing multiple topologies of path generating compliant mechanism (PGCM) using evolutionary optimization. Tech. Rep. KanGAL Report No.2009002., Indian Institute of Technology Kanpur, India.

[41] Sharma, D., Deb, K., and Kishore, N. N., 2008. On the way to evolve topologies of compliant mechanisms using evolutionary optimization. Tech. Rep. KanGAL Report No.2009003., Indian Institute of Technology Kanpur, India.

[42] Sharma, D., Deb, K., and Kishore, N. N., 2008. A customized evolutionary algorithm to generate multiple light-weight compliant mechanisms topologies. Tech. Rep. KanGAL Report No.2009004., Indian Institute of Technology Kanpur, India.

[43] Chaudhuri, S., 2002. "Optimal shape design of mechanical components using local search based multi-objective genetic algorithm". Master's thesis, Indian Institute of Technology Kanpur.

[44] Deb, K., and Goel, T., 2001. "A hybrid multi-objective evolutionary approach to engineering shape design". In Proceedings of the First International Conference on Evolutionary Multi-Criterion Optimization (EMO-01), pp. 385–399.

[45] Deb, K., and Chaudhuri, S., 2005. "Automated discovery of innovative designs of mechanical components using evolutionary multi-objective algorithms". In *Evolutionary Machine Design: Methodology and Applications*, N. Nedjah and L. de Macedo M, eds. Nova Science Publishers, Inc, New York, ch. 6, pp. 143–168.

[46] Goel, T., 2001. "Optimal shape design of mechanical components for single and multi-objectives using genetic algorithms". Master's thesis, Indian Institute of Technology Kanpur.

[47] Zitzler, E., 1999. "Evolutionary agorithms for multiobjective optimization: Methods and applications". PhD thesis, Zürich, Switzerland: Swiss Federal Institute of Technology ETH.

[48] Deb, K., 2001. *Multi-objective optimization using evolutionary algorithms*. Chichester, UK: Wiley.