Deciphering Innovative Principles for Designing Electric Brushless D.C. Permanent Magnet Motors

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***Abstract*— This paper shows how a routine design optimization task can be enhanced to decipher important and innovative design principles which provide far-reaching knowledge about the problem at hand. Although the so-called 'innovization' task is proposed by the first author elsewhere for this task, the application to a brushless D.C. permanent magnet motor design is the first real application to a discrete optimization problem. The model for cost and peak-torque objectives and associated constraints are borrowed from an existing study. The extent of knowledge gained in designing a high-performing and optimal motors achieved in this study is phenomenal and should motivate other practitioners to pursue similar studies in other design and optimization related activities.**

I. Introduction

Contemporary economies are knowledge-driven. New challenges in business, global markets with new competitors, shorter product life cycles and demanding customers accent the need of innovation for companies to be competitive. Innovation, increases bottom-line profitability, reduce costs and raise productivity. The customer, benefits from innovation by having a consistent product and service value delivery. In the context of engineering design, a new tool for design and innovation is inevitable. Usually, companies depend on intelligence derived from past experiences. Unfortunately, there exist very few scientific and systematic procedure for achieving innovations. Goldberg [1] narrates that a competent genetic algorithm - a search and optimization procedure on natural evolution and natural genetics - can be an effective mean to arrive at an innovative design for a single objective scenario.

Deb and Srinivasan [2] extended Goldberg's argument and describe a systematic procedure *'innovization'* involving a multi-objective task and a subsequent analysis of optimal solutions to arrive at a deeper understanding of the problem. Literature has a plethora of classical and evolutionary approaches, which arrive to a set of Pareto-optimal solutions iteratively and reliably [3], [4], [5], [6]. This set of Pareto-optimal solutions represent bits and pieces of facts, a hidden treasury of principles for optimal design. Pooling of these bits and facts using innovization shall unveil innovative design principles which are common to optimal trade-off solutions. Such commonality principles, provides a reliable *'recipe'* for optimal problem solving. The above procedure is illustrated in this paper considering a case study of brushless D.C permanent magnet (BDCPM) motors. This procedure offers valuable insights into the design of BDCPM motors.

In the remainder of this paper, we briefly discuss the BDCPM motor design problem and present a bi-objective optimization problem formulation in Section II. Section III describes the proposed local search based evolutionary optimization procedure. Section IV presents the results obtained by the proposed procedure and lists the *innovized* design principles obtained for the BDCPM motor design problem. Sections V and VI presents results for two modifications of the original problem, providing a set of overall design principles obtained for the same motor design task. Finally, conclusions are made in Section VII.

II. Design Optimization of BDCPM motors

With the advances in permanent-magnet and power-electronic technology, BDCPM motors are fast gaining in popularity, particularly as energy efficient motors. The motor essentially comprises of an outer stator assembly (windings on a frame), as shown in Figure 1 and an inner rotor assembly (permanent magnets mounted on a rotor), as shown in Figure 2. It is not surprising that a real-world product, such

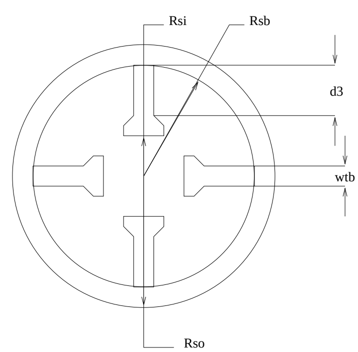

Fig. 1. Stator assembly.

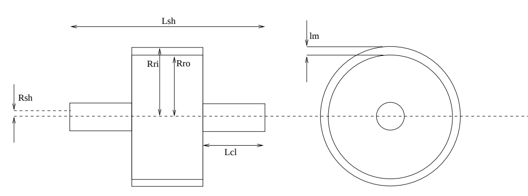

Fig. 2. Rotor assembly.

as the BDCPM motor, will have many design parameters

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(including various dimensions, materials, and manufacturing processes) which ideally must all be considered in the optimization task. However, due to manufacturing conveniences, not every parameter is considered as a ‘variable’. Instead, most parameters are kept fixed to reasonable values and only a few of them are allowed to vary during the design optimization task. Here, we borrow the formulation of the BDCPM motor design optimization problem detailed elsewhere [7].

Two conflicting objectives are considered for the design: (i) Minimization of manufacturing cost (in \$) of the motor and (ii) maximization of peak torque (N-m). Five design variables are considered: $\mathbf{x} = (n_l, N, L_{\rm type}, M_{ph}, A_{\rm gauge})$, where $n_l \in [44, 45, \ldots, 132]$ is the number of laminations used in the motor, $N \in [20, 21, \ldots, 80]$ is the number of turns in each coil, $L_{\rm type} \in [\texttt{X}, \texttt{Y}, \texttt{Z}]$ is one of three types of laminations allowed to build the motor, $M_{ph} \in [\texttt{Y}, \Delta]$ is one of the two types of electric connections and $A_{\rm gauge} \in [16, 16.5, ..., 23.5]$ is one of the 16 wire gauges used in the windings. Importantly, all five variables are discrete in nature, of which M_{ph} is a Boolean variable and $L_{\rm type}$ is a ternary variable. The cost term takes care of various manufacturing and materials costs and each term is obtained from regression analysis of data obtained from practice [8]. Here, we do not describe how the cost and peak torque terms are developed (interested readers may refer to the original study [7]), instead we present the problem formulation:

$$
\begin{aligned}
&\text{Minimize} \quad C_{total}\ (n_l, N, L_{\rm type}, M_{ph}, A_{\rm wire}) = \\
&\begin{cases} 2.6(\frac{n_l}{44})^{0.25}, & \text{for } L_{\rm type} = \text{X} \\ 0.38 + 2.42(\frac{n_l}{44})^{0.37}, & \text{for } L_{\rm type} = \text{Y} \\ 1.07 + 1.83(\frac{n_l}{44})^{0.58}, & \text{for } L_{\rm type} = \text{Z} \end{cases} \\
&+ [\{0.026, 0.028, 0.03\}n_l \cong L_{\rm type} = \{\texttt{X}, \texttt{Y}, \texttt{Z}\}] + \frac{n_l}{44} \\
&+ 9.8 \times 10^5 A_{\rm wire} N [5.8 \times 10^{-4} n_l \\
&\quad + \frac{\pi}{2}\{w_{tb} + \frac{5\pi}{12}(R_{si} + \frac{d_3}{2})\}] \\
&+ 0.3035 + 0.876N[5.8 \times 10^{-4} n_l + \\
&\quad \frac{\pi}{2}\{w_{tb} + \frac{5\pi}{12}(R_{si} + \frac{d_3}{2})\}] \\
&+ \pi[\{31.2(R_{si} + d_3 + w_{bi}) + 0.0312\}\{L_{sh} - 5.08 \times 10^{-3}\}] \\
&+ \begin{cases} 0.7(\frac{5.8\times10^{-4} n_l + 0.0792}{0.1047})^{1.21}, & \text{for } L_{\rm type} = \text{X} \\ 0.54 + 0.26(\frac{5.8\times10^{-4} n_l + 0.0802}{0.1057})^{3.74}, & \text{for } L_{\rm type} = \text{Y} \\ 0.58 + 0.32(\frac{5.8\times10^{-4} n_l + 0.0905}{0.1160})^{4.23}, & \text{for } L_{\rm type} = \text{Z} \end{cases} \\
&+ 9 + \begin{cases} 1.3(\frac{5.8\times10^{-4} n_l + 0.0792}{0.1047})^{0.38}(\frac{n_l}{44})^{0.18}, & \text{for } L_{\rm type} = \text{X} \\ 1.6(\frac{5.8\times10^{-4} n_l + 0.0802}{0.1057})^{0.38}(\frac{n_l}{44})^{0.40}, & \text{for } L_{\rm type} = \text{Y} \\ 1.8(\frac{5.8\times10^{-4} n_l + 0.0905}{0.1160})^{0.41}(\frac{n_l}{44})^{0.52}, & \text{for } L_{\rm type} = \text{Z} \end{cases} \\
&+ (1.536R_{si} - 6.26 \times 10^{-3})n_l + (2.014R_{si} - 8.21 \times 10^{-3})n_l \\
&+ \pi\{29.4(R_{si} - 7.25 \times 10^{-3})^2 n_l + 1.014 \times 10^5 R_{sh}^2 L_{cl}\} \\
&+ 0.1862 + 5.31 \times 10^4 \pi [R_{st}^2 L_{st} - 2R_{sh}^2 L_{cl} \\
&\quad - 5.8 \times 10^{-4} n_l (R_{si} - 7.25 \times 10^{-3})^2] + 0.01 \\
&+ 0.1473\pi(R_{si} - 7.25 \times 10^{-3})n_l + \begin{cases} 1.2, & \text{for } T_p \leq 3.5 \\ 1.6, & \text{for } T_p > 3.5 \end{cases} \\
&+ [\{0.5, 0.55, 0.60\} \cong L_{\rm type} = \{\texttt{X}, \texttt{Y}, \texttt{Z}\}] \\
&+ \begin{cases} 3.2(\frac{5.8\times10^{-4} n_l + 0.0792}{0.1047})^{-1.92}(\frac{n_l}{44})^{0.92}, & \text{for } L_{\rm type} = \text{X} \\ 3.4(\frac{5.8\times10^{-4} n_l + 0.0802}{0.1057})^{-1.92}(\frac{n_l}{44})^{1.06}, & \text{for } L_{\rm type} = \text{Y} \\ 3.7(\frac{5.8\times10^{-4} n_l + 0.0905}{0.1160})^{-3.25}(\frac{n_l}{44})^{1.60}, & \text{for } L_{\rm type} = \text{Z} \end{cases}
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
&\text{Maximize} \quad T_p = 87300 C_{tor} N R_{si} A_{\rm wire} n_l \\
&\text{Subject to} \\
&g_1 : T_p \geq 0.83 \\
&g_2 : T_p \leq 5.27 \\
&g_3 : A_{\rm wire} N \leq \{150, 240, 280\} \times 10^{-7} \cong L_{\rm type} = \{\texttt{X}, \texttt{Y}, \texttt{Z}\}
\end{aligned}
$$

Here, $C_{tor} = \{\frac{2}{3}, \frac{1}{3}\} \cong M_{ph} = \{\texttt{Y}, \Delta\}$. It is clear from these expressions that objective and constraints are highly discontinuous and largely varies with the type of lamination used to design the motor. It is needless to say that it is this kind of discontinuities which will prohibit classical gradient based optimization techniques to fail to solve such problems. Although non-smooth optimization techniques [9] can be applied to handle such discontinuities by using subdifferentials, an additional difficulty of variables being discrete in this problem makes evolutionary approaches ideal in handling this problem.

Constraints g_1 and g_2 bounds the peak-torque within a lower and upper bound, respectively. The constraint g_3 incorporates the ’in practice’ winding constraint used by manufacturers that prevents magnetic saturation and demagnetization. Each lamination has a thickness $t = 5.8 \times 10^{-4}$ m, so that the overall frame length becomes $L = 5.8 \times 10^{-4} n_l$ m (where n_l is the number of laminations, one of the variables of our study). The laminations (and the motor frame) have $N_s = 24$ slots. The defining dimensions in of these laminations are summarized in Table I. The length of the shaft L_{sh} (which is obtained from the side clearance as $L_{sh} = L + 2L_{cl}$) and the shaft radius R_{sh} are determined by the lamination types used for the BDCPM motor family. The shafts of the BDCPM motor family are machined from the bar-stock of length L_{st} and radius R_{st}. The length of the bar stock corresponds to the length of the shafts L_{sh}. The above parameters are used as fixed and not used as variables in the optimization study.

III. Proposed Evolutionary Procedure

In this section, we describe the bi-objective evolutionary optimization procedure used to solve the BDCPM motor design problem.

A. Representation and EA Operators

An efficient representation of the variables is one of the prerequisites for an ideal performance of an evolutionary algorithm on any problem. Let us recall that this problem involves five discrete variables $\mathbf{x} = (n_l, N, L_{\rm type}, M_{ph}, A_{\rm gauge})$. In this study, we treat these variables as they are. For example, variables n_l and N are integer-valued and we directly code them as integers. Initial population and subsequent EA operators (recombination and mutation) are designed to create and maintain their integer restrictions. For these two variables, the discrete version of Simulated Binary Crossover (SBX) and Polynomial Mutation (PM) [5] are used, so that integers within specified bounds are always created. The variable $L_{\rm type}$ is a ternary variable representing $\{\texttt{X}, \texttt{Y}, \texttt{Z}\}$ type of connection within

TABLE I

DIMENSIONS OF THE BDCPM MOTOR FAMILY.

L_{type}	R_{si}(mm)	d_3(mm)	w_{tb}(mm)	w_{bi}(mm)	L_{cl}(cm)	R_{sh}(mm)	R_{st}(m)
X	21.90	12.0	2.39	5.23	4.215	4.11	0.016
Y	22.22	15.1	2.39	5.23	4.265	4.11	0.016
Z	25.40	15.1	2.80	5.50	4.775	4.45	0.019

our EA. This variable is subjected to mutation only. When mutated, a value is changed to other two options with an equal probability. The variable A_{wire} has 16 options, thus, we use a four-bit binary substring for this variable and employ standard binary crossover and mutation operators. The decoded values are first mapped between 16 (for the substring (`0000`)) and 23.5 (for the substring `(1111)`) to compute the gauge number, $\kappa = A_{\text{gauge}}$. Thereafter, the true cross-sectional area of the wire is computed as follows: $A_{\text{wire}} = (13.097(10^{-7})1.123^{-2(\kappa-16)}$ m^2. Thus, higher the gauge number (κ) of the wire, the smaller is the cross-sectional area. The next variable M_{ph} is binary-valued and a single bit is used to represent connections Y and Δ. A mutation operator is used to handle this variable. Thus, a typical EA string representing all five variables is illustrated, as follows:

$$(101,\ 54,\ \texttt{Z},\ \texttt{Y},\ (\texttt{1000}))$$

This string signifies a BDCPM motor having 101 laminations (with a length of 58.58 mm), 54 turns in each coil, Z-type lamination, Y-type electric connection, and a gauge $\kappa = 19.5$ wire having $5.178(10^{-7})$ m^2 cross-sectional area of the wire. All other parameters are fixed as they are described in the problem formulation. For this particular motor, the cost and peak-torque computed using our code is \$ 59.88 and 4.17 N-m, respectively. In the original study [8], the same solution is evaluated and the above two values are reported as \$ 60.88 and 4.07 N-m, respectively. It is worth mentioning here that after modifying a number of obvious typing errors in the expressions for the objectives and constraint functions presented in the original study, we obtained an identical cost of \$ 59.88 and peak-torque of 4.17 N-m. A match of these values gives us confidence in our implementation.

B. *Local search based NSGA-II Procedure*

An evolutionary optimization procedure is repeatedly shown to find a near-optimal solution quickly, but from there a convergence to the true optimal solution may be time-consuming. This is also true for evolutionary multi-objective optimization (EMO) procedures. To improve the convergence properties of EMO procedures, often they are hybridized with a local search procedure [5], [10]. In this study, we suggest a local search based NSGA-II for obtaining true Pareto-optimal solutions. The method is generic and can be used for solving other discrete optimization problems as well.

Step 1: Obtain a discrete set of near Pareto-optimal solutions using NSGA-II [11]. The obtained non-dominated solutions are copied to an archive.

Step 2: For each obtained solution, perform a local search as follows:

- 2a: Create all discrete neighboring solutions with $(2H + 1)$ perturbations in each variable (of which H of them are positive perturbations, another H are negative perturbations and one being the solution itself). With n discrete variables in a problem, we shall have at most $(2H + 1)^n$ discrete solutions.
- 2b: The non-dominated solutions of this set are identified and then copied to the archive. Any duplicates are removed and new archive members are evaluated.
- 2c: Any dominated solutions of the archive is deleted.

Step 3: The archive is declared as the set of obtained Pareto-optimal solutions.

The accuracy and computational time of the above local search procedure depends on the chosen value of H. In this study, we use $H = 3$ (barring on variables L_{type} and M_{ph}). This requires at most 7^3 or 343 neighboring solutions to be evaluated for each NSGA-II solution. But since such a study would be done on a problem only once, a larger computational time can be allocated to perform such an important study.

The innovization task is an extension of a multi-objective optimization study [2]. After a set of trade-off optimal or near-optimal solutions are found by a local search based NSGA-II procedure, the nature of change in variable values are analyzed as a function of one of the objectives. Ideally, trade-off optimal solutions are likely to follow some similar properties due to their optimality conditions. The innovization procedure attempts to unveil such important properties which are likely to provide important knowledge, particularly in design related problems.

IV. RESULTS FOR THE ORIGINAL PROBLEM

NSGA-II parameters used in this study are as follows: population size = 100, maximum generations = 100, η_c =5, η_m =10, $p_c = 0.9$ for both real-valued and binary variables, $p_m = 0.33$ and 0.02, for real-valued and binary variables, respectively. The Pareto-optimal front obtained using the hybrid local search based NSGA-II is shown in Figure 3. Since in this problem, peak-torque is maximized and cost is minimized, the obtained Pareto-optimal front is the bottom-right boundary of the feasible objective space, as shown in

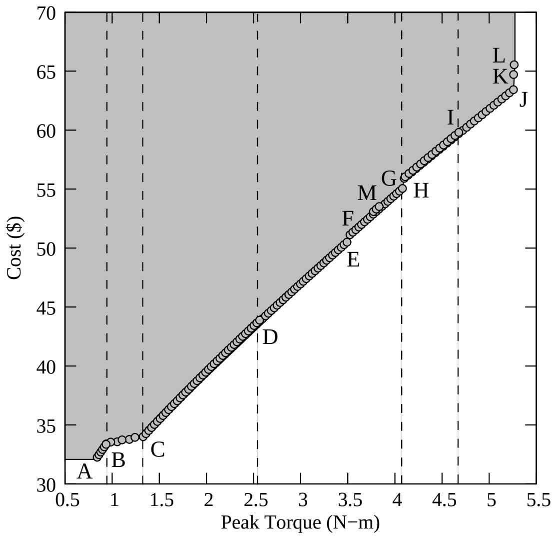

Fig. 3. Pareto-optimal Front for the original BDCPM motor design problem.

the figure. The figure shows that motors ranging from 0.842 to 5.258 N-m peak-torque and cost value ranging from $ 32.261 to $ 63.438 are obtained. The minimum-cost solution is as follows: $n_l = 45$, $N = 20$, $L_{\rm type} = \mathtt{X}$, $M_{ph} = \mathtt{X}$, and $A_{\rm gauge} = 18$ gauge and the maximum peak-torque solution is $n_l = 127$, $N = 24$, $L_{\rm type} = \mathtt{Z}$, $M_{ph} = \mathtt{Y}$, and $A_{\rm gauge} = 16$ gauge. Also, an interesting aspect of the shape of the Pareto-optimal front is that it is non-convex. Thus, some popular weighting multi-objective optimization algorithms will not be able to find intermediate solutions in this problem. As shown in the figure, non-convexity does not cause a difficulty to the NSGA-II procedure.

Interestingly, the front seems to have jumps and discontinuities at several locations. Interesting ones are from E to F and another from G to H. Although not entirely clear from this plot, but there exists two adjacent non-dominated fronts around regions CD and HI. There are also three non-dominated solutions at region M. These solutions make a small trade-off with the main-stream trade-off solutions (such as CE, HJ, and FG). We make an attempt to understand the reasons for these jumps and presence of adjacent non-dominated fronts.

A jump in the cost value at peak-torque (T_p) equal to 3.5 N-m (from E to F) occurs due to a term in the cost expression (15-th term in the cost expression in equation (1)). A jump of $ 0.4 is added when the peak-torque crosses a value of 3.5 N-m. The second jump (from G to H) occurs due to the optimality properties around these solutions, a matter which we shall discuss a little later.

Let us now discuss the reason for having an adjacent non-dominated front at three different locations (CD, M, and HI). For this purpose, we enlarge a part of the region in CD in Figure 4. The figure shows that there are two sets of non-dominated solutions placed side by side. Each pair has certain unique characteristics (which we shall reveal a little later) and seems to appear in three distinct regions in the entire Pareto-optimal front, causing an adjacent non-dominated front.

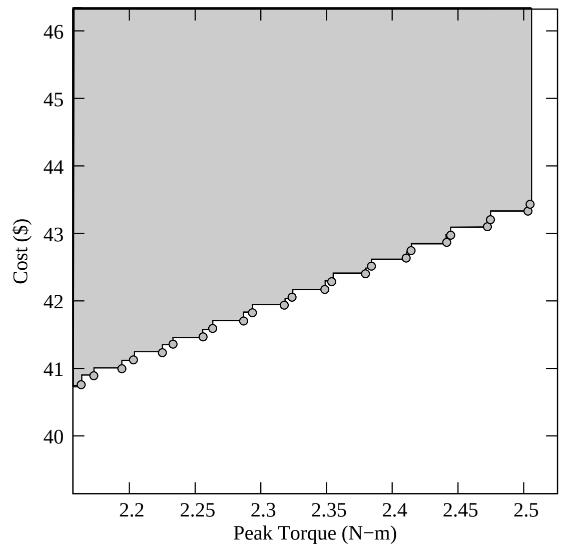

Fig. 4. Adjacent non-dominated fronts in the region CD.

Now, we investigate how the obtained variable values change as the peak-torque values change from a small to a large value. This task revealing properties among Pareto-optimal solutions is termed as an 'innovization' task elsewhere [2].

A. Innovized Principles

The following important observations are made:

1) It is quite interesting to note that all obtained Pareto-optimal solutions has one unique property: They all have Y type electric connection. As discussed elsewhere [12], with a Y connection, comparatively smaller number of turns are needed to be wound to get the same power output. We discover this important fact of design of BDCPM motors through our optimization study and without explicitly providing such information.
2) Figure 5 shows that the variation of number of laminations increases linearly with the peak-torque. The reasons for the jump from G to H and adjacent non-dominated fronts discussed above will be clear from this figure. First, it is interesting to observe that a jump from G to H in the Pareto-optimal front occurs due to a change in the number of laminations at around 4.08 N-m. Recall that the bounds used for the number of laminations were 44 and 132, respectively. Figure 5 reveals that a recipe to design a motor having a large peak-torque is increase the number of of laminations from 44 (solution C) to 132 (solution G) in a linear manner. Since no further increase in this variable is allowed to achieve more peak-torque beyond 4.08 N-m, the optimization procedure invents a different way of designing the motor. This causes a jump to take

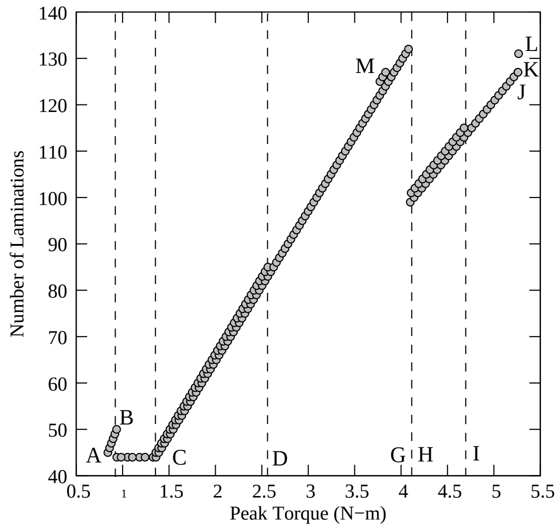

Fig. 5. Variation of number of laminations.

place in the Pareto-optimal front. Figure 6 shows that for all solutions in the CG region, 23 turns are needed. However, when the number of laminations hit the upper

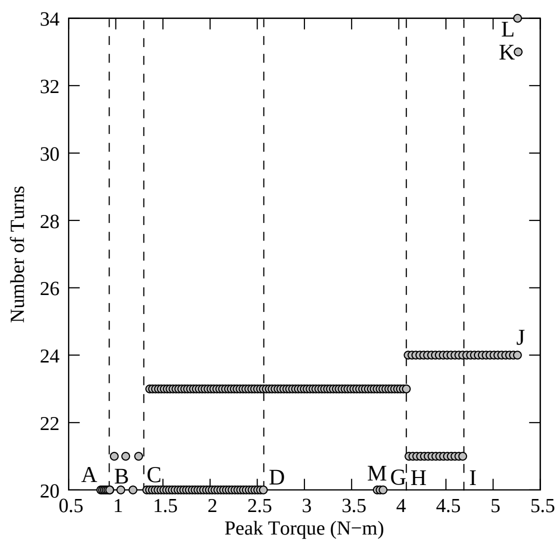

Fig. 6. Variation of number of turns per coil.

bound (132), 24 turns are called for, but then the number of laminations can now be reduced to 99. Higher torque motors can again be designed by keeping 24 turns per coil and by steadily increasing the number of laminations in a linear fashion.

3) Based on these properties of the obtained solutions, we can divide the entire Pareto-optimal front into six regions, as shown in the above figures. Each region has certain common properties which keep solutions in the respective trade-off region and certain variations which provide the trade-off in them. We discuss more about these properties in the next subsection.

4) Following generic properties of the obtained solutions can be established from above figures and Figures 7 and 8:
   - More the requirement of the peak-torque, more number of laminations must have to be used (Figure 5).
   - More the requirement of the peak-torque, more turns per coil must have to be used (Figure 6).
   - More the requirement of the peak-torque, more cross-sectional area must be used for the wire, as shown in Figure 7.

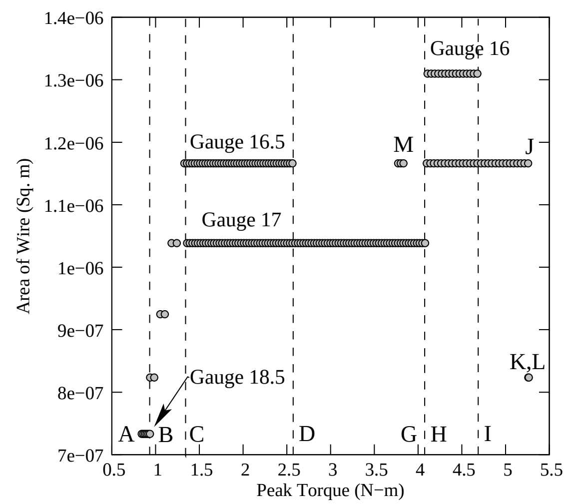

Fig. 7. Variation of cross-sectional area of wire.

   - More the requirement of the peak-torque, laminations must be used from type X to Z, as shown in Figure 8.

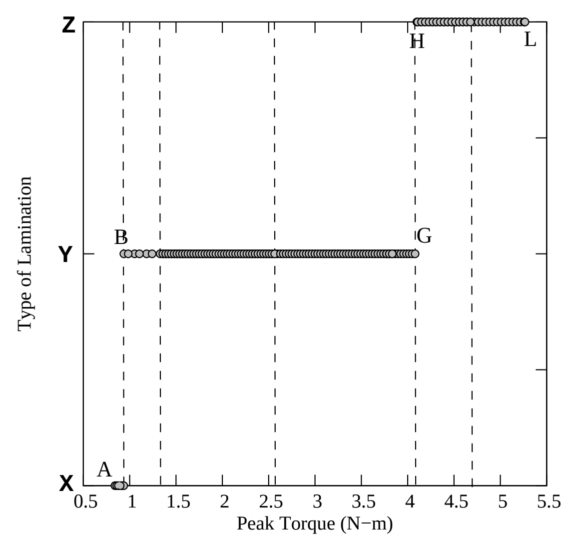

Fig. 8. Variation of number of turns per coil.

5) Another interesting observation is that the entire range of n_l (number of laminations) in [44, 132] appears

in the solutions of the Pareto-optimal front, whereas only a few (20 to 33) of the chosen range (20 to 80) for number of turns (N) is needed to constitute the Pareto-optimal front. Similarly, although 16 different wire gauges are considered in the optimization study, only six most thick wires are found to be optimal. As mentioned earlier, of the two electric connections, Y is found to appear in all Pareto-optimal solutions and all three lamination types find a place on the Pareto-optimal front. These information can be very useful, not only in understanding the problem better, but also in managing a better inventory in a manufacturing plant.

The adjacent non-dominated fronts observed in Figure 3 is clear from these variable plots. Around the regions CD, M and HI, a slightly higher values of N, lower values of n_l, and lower gauges of wire can be used to constitute motors which get marginally non-dominated to solutions of CG and HJ. It can be seen from the figures that both adjacent fronts (CD and CG) maintain all other variables the same, except variable n_l. Figure 6 also shows that the difference in n_l values between the adjacent fronts widen as peak-torque is increased in these regions. But at D, the difference becomes so large that along with other variables values the resulting solution gets dominated by a solution from CG, thereby prohibiting the adjacent front CD to proceed further. A similar phenomenon occurs at right-most point in M and also at I. Realizing that these adjacent front solutions are marginally non-dominated with the main-stream Pareto-optimal solutions, we now remove the adjacent non-dominated solutions from the obtained front, so as to conceive a clear picture of the trade-off between cost and peak-torque for the BDCPM motor design problem.

B. *Innovizations from a Partial Pareto-optimal Set*

Figure 9 shows the Pareto-optimal front without the adjacent optimal solutions for clarity. Now the entire front can be divided into four divisions, as marked in the figure. Figures 10 and 11 reveal the basis for these divisions. An exploration of these filtered Pareto-optimal solutions yields following interesting insights:

1) Region I has fixed $L_{\rm type}$ = X, $A_{\rm wire}$ = $7.33(10^{-7})$ m^2, and N = 20. The trade-off is solely obtained by varying n_l linearly to produce different motors at lower costs.
2) Region II has fixed $L_{\rm type}$ = Y, n_l = 44, and N = 20. The parameter $A_{\rm wire}$ varies monotonically to generate different motors.
3) Region III has fixed $L_{\rm type}$ = Y, $A_{\rm wire}$ = $1.03(10^{-6})$ m^2, and N = 23. The parameter n_l varies linearly to produce distinct motors.
4) Region IV has fixed $L_{\rm type}$ = Z, $A_{\rm wire}$ = $41.16(10^{-6})$ m^2, and N = 24. The parameter n_l also varies linearly to produce different motors.

All Pareto-optimal solutions need a Y-type electric connection. Above observations clearly indicate that all four regions vary with only one of the two decision variables

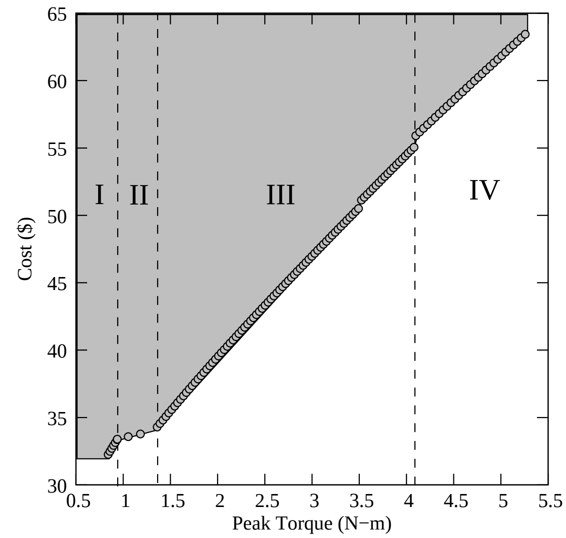

Fig. 9. Pareto-optimal front with main-stream solutions.

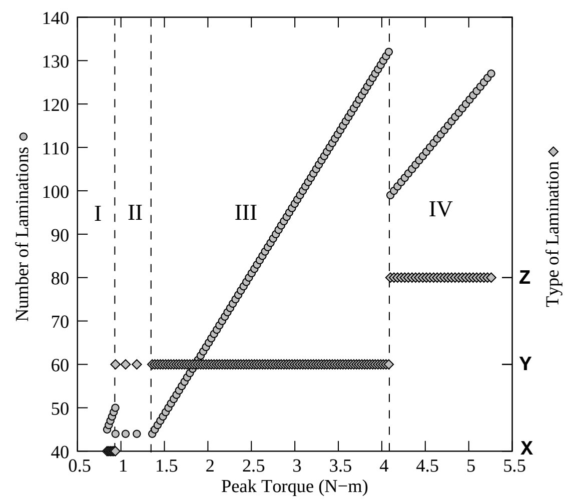

Fig. 10. Number and type of laminations for the partial Pareto-optimal front.

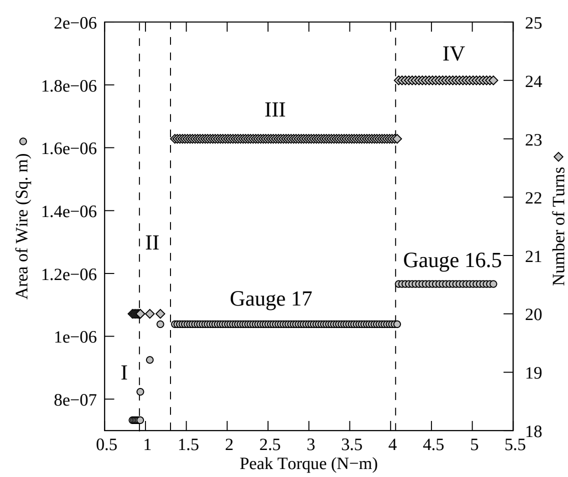

Fig. 11. Area of wire and number of turns for the partial Pareto-optimal front.

(n_l and $A_{\rm wire}$). For most part of the Pareto-optimal front, the variable n_l (number of laminations) seem to be the only parameter responsible for the trade-off among the solutions. Thus, overall we learn from this study that the cost-optimal way to design BDCPM motors having different peak-torques is to mainly change the number of laminations and keep fixed the other variables to certain values. Thus, a recipe to design a high torque motor is to make the length of the motor large. Importantly, other combinations of variables can also produce a motor having a desired peak-torque, but such designs will not be cost-optimal. For example, the Pareto-optimal solution (98, 23, X, Y, (0010)) has a cost value of \$ 47.185 and peak-torque of 3.029 N-m, whereas another feasible solution (137, 33, X, Y, (1000)) corresponds to a cost value of \$ 56.039 and peak-torque of 3.029 N-m. Thus, although both motors have the same peak-torque, the non-optimal solution is \$ 8.854 more costly than our obtained optimal solution. Since we first found a set of Pareto-optimal solutions corresponding to both cost minimization and peak-torque maximization and then analyzed the obtained solutions to decipher such properties, the revealed properties have shown to provide critical information about how to design a motor in an optimal manner.

Furthermore, in most Pareto-optimal solutions, only two wire gauges are required, thereby suggesting to maintain a standardized and minimal inventory. Each of these two wire gauges seem to have an associated lamination type and the number of turns. For region III, the combination of Y-type lamination, 23 turns per coil and Gauge 17 seem to be common to all solutions, and for region IV, the combination of Z-type lamination, 24 turns per coil and Gauge 16.5 seem to be common.

V. Results on a Modified Problem

Figure 9 has shown that to achieve near minimum-cost solutions different combinations of variables are needed. If an optimal solution has certain irregularities (isolation, constraint or variable boundaries) in the neighborhood, such optimal solutions must be avoided, if possible. In this problem, the irregularity near the minimum-cost solution is caused due to the chosen lower bounds on variable n_l and N. In this section, we redo the optimization task by allowing variable n_l to vary in $[20, 200]$, and defer the extension of lower bound of N. Rest of the problem formulation is identical to that of the original study. We employ the same local search based NSGA-II and obtain a new Pareto-optimal front, which is shown in Figure 12.

The figure clearly shows that the irregularity in the front near the minimum-cost region are now absent and a more smooth front is obtained. Now, the front ranges from cost values in \$ [29.898, 63.278] and peak-torques in [0.875, 5.254] N-m. Figure 13 shows that the entire Pareto-optimal front can be divided into two regions:

1) Region I ($T_p \leq 2.565$ N-m): Wire of gauge 16.5 and 20 turns per coil are needed and to obtain a trade-off among cost and torque, the number of laminations must be increased linearly within [29, 85].

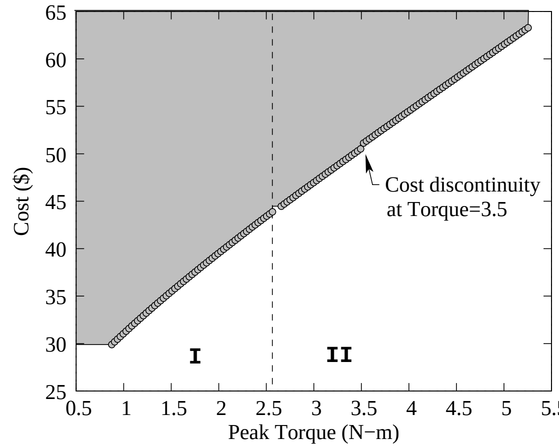

Fig. 12. Pareto-optimal front for the modified problem.

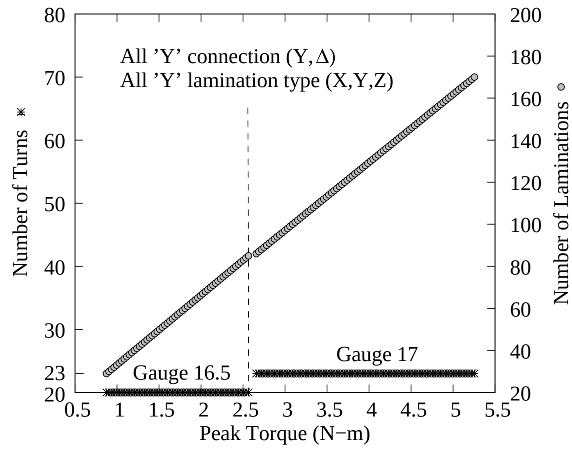

Fig. 13. Variation of variables for the modified problem.

2) Region II ($T_p \geq 2.658$ N-m): Wire of gauge 17 and 23 turns per coil are now needed and to obtain a trade-off among cost and torque, the number of laminations must be increased linearly within [86, 170].

Interestingly, all optimal solutions now require the Y-type electric connection and type Y-type lamination. Once again, the generic variation of optimal solutions for increased peak torque results from a linear variation of the number of laminations (thereby having an effect linear variation of length of the motor).

VI. Results on a Further Modification

Realizing that the variable N hits it lower bound in some Pareto-optimal solutions in the above modified problem, we make one more modification in which N is varied in [10, 80]. All other parameters are the same as before. Figure 14 shows the new Pareto-optimal front. An investigation of all variables now reveals that all Pareto-optimal solutions vary by changing n_l (number of laminations) linearly in the range [28, 172]. Other four variables remain identical in all solutions to the following values: (i) Y-type electric connection (ii) Y-type lamination, (iii) the number of turns per coil is 18, and (iv) a constant gauge size of 16. It is interesting that although a lower value of N and a broader range of n_l are allowed, the Pareto-optimal solutions take intermediate values, thereby indicating that these optimal values are properties of the objectives and constraints and are not artificial manifestation of variable bounds. With the chosen 16 gauge sizes, it seems that the minimum-cost solution to the BDCPM motor design problem is $n_l = 28$, $N = 18$, $L_{\rm type} = \texttt{Y}$, $M_{ph} = \texttt{Y}$, and $A_{\rm gauge} = 16$, having a cost value of \$ 25.486 and peak-torque is 0.854 N-m. On the other hand, the maximum peak-torque solution is $n_l = 172$, $N = 18$, $L_{\rm type} = \texttt{Y}$, $M_{ph} = \texttt{Y}$, and $A_{\rm gauge} = 16$, having a cost value of \$ 62.940 and peak-torque is 5.246 N-m.

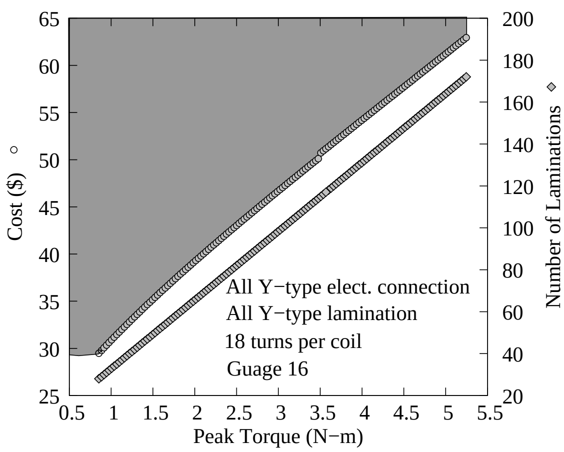

Fig. 14. Pareto-optimal front and variation of variables for the further modified problem.

Starting from the original problem portraying many piece-wise design properties occurring due to a consideration of artificial variable bounds and existence of marginally non-dominated solutions, we have systematically modified the motor design problem into a more generic one by extending bounds of a couple of variables. When the optimal solutions are not affected by the chosen variable bounds, the Pareto-optimal solutions having different peak-torques (and costs) are found to depend on the choice of different values of only one of the variables – the number of laminations. This has a direct implication in producing different powered motors by simply changing the length of the motor. However, other four variables must be fixed to certain specific values for them to be Pareto-optimal.

VII. Conclusions

In this paper, we have investigated a brushless DC permanent magnet (BDCPM) motor design from the point of two conflicting objectives (cost of manufacturing and achievable peak-torque). Five design variables control the electric connections, lamination type, number of laminations, number of turns per coil, and gauge number of the wire. Due to the discreteness of all five design variables, the search space and resulting objective space are discrete. We have suggested a local search based NSGA-II procedure for finding multiple discrete Pareto-optimal solutions. The resulting trade-off solutions have revealed a number of interesting and important information about the properties of the Pareto-optimal solutions. Overall, it has been observed that of the five variables the trade-off among objectives is largely contributed by the variation of the number of laminations. Interestingly, for all optimal solutions the Y-type electric connection has been found to be better than a Δ-type connection.

Such innovative design principles are useful as ’thumb-rules’ or ’recipes’ for evolving a high-performing design without performing another detailed optimization study. Such a study, performed once, should provide valuable information about a real-world design task, which has far-reaching consequences in providing useful ‘knowledge’ about the problem, which may not be possible to obtain by other means.

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