Towards Generating Diverse Topologies of Path Tracing Compliant Mechanisms Using A Local Search Based Multi-Objective Genetic Algorithm Procedure

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***Abstract*— A new bi-objective optimization problem is formulated for generating the diverse topologies of compliant mechanisms tracing a user-defined path. Motivation behind the present study is to generate the compliant mechanisms which perform the same task of tracing a prescribed trajectory near minimum-weight solution. Therefore, the constraint are imposed at each precision point representing a prescribed path for accomplishing the tracing task. An additional constraint on stress is also included for the feasible designs. The study starts with a single objective analysis of minimum-weight of compliant mechanism and the obtained topology is referred as the *reference* design. Thereafter, a bi-objective optimization problem is solved by considering the objectives as minimization of weight of structure and maximization of diversity of structure with respect to the *reference* design. Here, the diversity is evaluated by finding the dissimilarity in the bit value at each gene position of the binary strings of the *reference* design and a structure evolved from the GA population.**

A local search based multi-objective genetic algorithm (MOGA) optimization procedure is used in which the NSGA-II is used as a global search and optimization algorithm. A parallel computing is employed in the study for evaluating non-linear geometric FE analysis and also for the NSGA-II operations. After the NSGA-II run, a few solutions are selected from the non-dominated front and the local search is applied on them. With the help of a given optimization procedure, compliant mechanism designs tracing curvilinear and straight line trajectories are evolved and presented in the study. In both examples, compliant mechanisms are designed to have any arbitrary support and loading regions.

I. Introduction

Compliant mechanisms are flexible elastic structures which can deform to transmit the force and/or generating some desired path on the application of applied load. Compliant mechanisms have shown many advantages over pseudo-rigid-body mechanisms as jointless and monolithic structures, involved less friction, wear and noise [1], ease of manufacturing without assembly, light weight devices [2] etc. Applications of compliant mechanisms are in the area of product design, off-shore structures, smart structures, MEMS [3] etc.

Two approaches for designing the compliant mechanisms can be found in the literature. In the first approach, designs are inspired by the traditional kinematic synthesis of rigid-body mechanism [4], [5], whereas the second approach is a continuum mechanics based approach which focuses on the determination of topology, shape and size of compliant mechanisms. Among the methods found in the topology optimization literature, a popular strategy is to initially discretize the allowable design space into finite elements (FE) and then to define the required loading/boundary conditions. The optimization procedure will then be concerned with determining which elements should contain material (and so form the structure) and which elements are void (and thus represent the surrounding empty space).

Based on the continuum mechanics approach, studies have been made by considering homogenization method [6], [7], material density approach [8] and by using contours of a 'shape density' function [9], where the discrete nature of compliant mechanism problem is converted into the continuous design variable problem and a threshold value is assigned for making a decision whether an element is represented by a material or a void. Sometimes, assigning an arbitrary threshold value may lead to non-optimum designs. An approach using the binary (0-1) representation of material for finite element, helps in preserving the discrete nature of compliant mechanism problems [10], [11], [12], [13], but this representation usually results in 'checker board' pattern and 'floating' elements of material which are not the part of structure.

In the past studies of topology optimization of compliant mechanisms, different objectives with several optimization techniques have been considered and incorporated. In this paper, a few important studies incorporating single and multi-objective optimization formulation are presented. An early method used in the literature optimizes the weighted linear combination of deformation at the prescribed output port and strain energy using sequential quadratic programming [14], a continuum type topology optimization method minimizes the least-square errors between prescribed and actual values of geometric and mechanical advantages using sequential linear programming method [15], maximization of mechanical gain subjected to the constraints on volume and input displacement is solved using sequential linear programming [16], a multi-criteria optimization problem which maximizes the ratio of mutual energy to strain energy subjected to equilibrium equation of loading conditions and a constraint on the total material resource is solved using sequential linear programming [17]. Another multi-objective optimization deals with the maximization of mechanical efficiency,

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geometric and mechanical advantages, and minimization of the maximum compressive load and it is optimized using genetic programming [18]. Using an approach of describing load path synthesis for topology and dimensional synthesis of compliant mechanisms, an optimization problem which maximizes the mutual potential energy is solved using genetic algorithm with local search for further improving the GA designs using gradient based algorithm [19]. So far, these studies employ linear elastic finite element models with classical and evolutionary methods of optimization for the synthesis of compliant mechanisms.

But for designing the large displacement and path generating compliant mechanisms, non-linear FE models are required to produce more accurate designs. The studies [20] and [21] deal with the large deformation of structures and the synthesis of large displacement compliant mechanisms in which geometrically nonlinear behavior of the structures are modeled using a total Lagrangian finite element formulation and the equilibrium is found using Newton-Raphson iteration scheme. Study shows a significant gain in output performance of compliant mechanisms using non-linear analysis compared to linear analysis and the problem is solved using the Method of Moving Asymptotes. In the another study, the geometrically non-linear finite element analysis with frame elements is solved using the sequential quadratic programming (SQP) technique to satisfy prescribed force-deflection specification by output point of the compliant mechanisms [22].

These studies also reveal the non-linear nature of path generating and large displacement compliant mechanism's problems. When the discrete optimization problem of compliant mechanisms is converted to continuous variable problem with some threshold value, the classical methods of optimization might not be suitable because these methods can stuck at some local optimal designs. The application of genetic algorithm in solving such type of problems can help in preserving the discrete nature and also eliminate the convergence of a problem to some local optimal designs. Using genetic algorithm, large displacement compliant mechanisms [23] and compliant mechanisms generating given nonlinear output path [24] are designed for their optimal topology, shape and size with geometrically nonlinear analysis. Individual multiple least square objectives of actual and desired output responses for path generating compliant mechanism problem are optimized using NSGA-II [25]. A novel representation scheme using the arrangement of skeleton and flesh to define a structure uses the Bezier parametric curves for designing the structural and compliant mechanism problems [26], [27], [28]. Genetic algorithm is used as a global optimizer for designing and synthesis of path generation compliant mechanism problem which is defined by minimizing the deviation between the distance of desired and corresponding actual trajectories after dividing them into N precision points [29]. Different ways of performing the crossover operator are suggested in the studies using morphological representation scheme of strings. Aforementioned studies of path generation compliant mechanism synthesis can generate the path with minimum error or deviation but these formulations do not guaranteed that these mechanisms always follow the prescribed path. In some solutions, it may not follow the intermediate/end trajectory points adequately.

After the successful implementation of a local search based MOGA procedure for evolving the path generation complaint mechanisms [30], the same optimization procedure is used in the present study with a new bi-objective formulation for generating the diverse topologies of compliant mechanisms tracing a user-defined path. Motivation behind the present study is to generate the compliant mechanisms which perform the same task of tracing a prescribed trajectory near minimum-weight solution. Therefore, the constraints on each precision points representing a prescribed path are imposed to accomplish the tracing task. The present study starts with a single-objective analysis of minimum-weight of compliant mechanism tracing a user-defined path. The topology obtained after the local search is referred as the $reference$ design. Thereafter, a bi-objective optimization problem is dealt by considering the primary objective of minimizing the weight of structures and the secondary objective of maximizing the diversity of structure with respect to the $reference$ design. This set of conflicting objectives will help in generating the 'trade-off' solutions in one run of an optimization procedure. Here, the diversity is evaluated by finding the dissimilarity in the bit value at each gene position of the binary strings of the $reference$ design and a structure evolved from the GA population.

A parallel computing with MOGA is employed in the study for evaluating the non-linear geometric FE analysis and also for NSGA-II operations. After obtaining the non-dominated front of NSGA-II, a few solutions are selected and the local search is exercised on them. With the help of the given procedure, the compliant mechanisms tracing curvilinear and straight line trajectories are evolved and presented in the study. In both examples, the compliant mechanisms are designed to have any arbitrary support and loading regions.

II. Bi-objective Problem formulation

Consider a design domain of 50 mm by 50 mm in Figure 1 which is divided into three regions of interest. The first region is called support region where the structure is supported (restrained, with zero displacement) where-in the second region (loading region) some specified load (input displacement) is applied. The output region is the third region of interest, that is, the point on the structure which traces out the desired path defined by user. Thus, the task of tracing a user-defined path is checked by imposing the constraints on the precision points. Therefore, a prescribed path is divided into N number of precision points as shown in Figure 2. A fixed percent of deviation (η) is allowed between the precision points (i) of prescribed path and corresponding points (ia) obtained on the actual path after FE analysis of a structure. Hence, the problem is subject to N number of constraints on the basis of the number of precision points

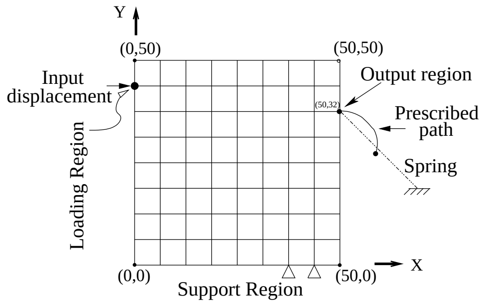


Fig. 1. Design space with loading, output and support regions.

representing the prescribed path, along with a constraint on stress as given in Equation 1. In the implied stress constraint, maximum stress developed in the structure should be less the flexural yield stress of the material. For providing some resistance at the output region and for some work meant to be done, a spring of constant stiffness (κ) is attached [30].

In the present work, first a single objective analysis of minimizing the weight of structure subjected to the constraints given in Equation 1 is performed. The minimum-weight topology after the local search is referred as the *reference* design. Thereafter, a bi-objective optimization problem given in Equation 1 is dealt which comprises of the primary objective of minimizing the weight and the secondary objective of maximizing the diversity of GA evolved structures with respect to the *reference* design. Such a pair of conflicting objectives will help in maintaining a diverse set of solutions in the GA population. After the NSGA-II run, a few representative solutions are chosen from the non-dominated front and the local search is applied on them. Afterward, the non-dominated topologies near to minimum-weight solution are picked for further analysis.

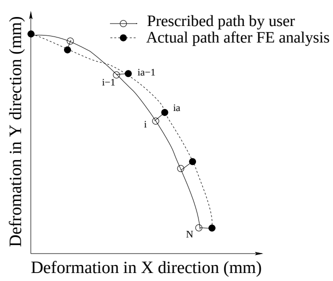


Fig. 2. Prescribed path by user and actual path after FEM analysis for a hypothetical case.

Minimize: Weight of structure,
Maximize: Diversity of structure,
subject to:

$$1-\frac{\sqrt{(x_{ia}-x_i)^2+(y_{ia}-y_i)^2}}{\eta\times\sqrt{(x_i-x_{i-1})^2+(y_i-y_{i-1})^2}}\geq 0, i=1,2,...,N \quad (1)$$

$$\sigma_{flexural}-\sigma\geq 0,$$

where $\eta = 15\%$ is allowed deviation, and $\sigma_{flexural}$ and σ are flexural yield strength of material and maximum stress developed in the structure, respectively.

III. A Local Search Based Multi-Objective Genetic Algorithm Procedure

Among different multi-objective optimization algorithms, the elitist non-dominated sorting genetic algorithm (known as NSGA-II) is popularly used. For two objective problems, it is shown to have a good convergence property to the global Pareto-optimal front as well as to maintain the diversity of population on the Pareto-optimal front [25]. As topology optimization of compliant mechanism problem is non-linear and discrete in nature, NSGA-II with local search procedure is used in the present study. In the present work, a population of 240, crossover probability of 0.95 and mutation probability of $(1/\text{string length})$ are assigned and NSGA-II is run for a maximum of 100 generations. For each NSGA-II population member, a binary string length of 633 bits is used in which first 625 bits are used to represent a structure (representing 25×25 grids) and additional eight bits are decoded to determine the optimum regions. As shown in Figure 3, these eight bits are further divided into the two sets of five and three bits for indicating the support and loading region's elements respectively.

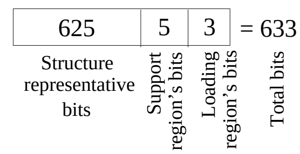


Fig. 3. A binary string representation.

A. Representation Scheme

A continuum structure is discretized by 4 node rectangular elements and each element is represented either by 0 or 1, where 1 signifies the presence of material and 0 represents the void. This makes a binary string which is copied to two dimensional array as per the sequence shown in Figure 4. In the present study, one bit of the binary string represents four elements for FE analysis with same gene value as shown in Figure 4.

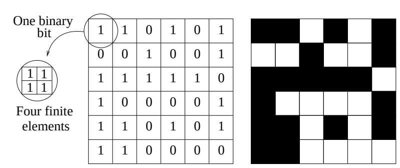


Fig. 4. Representation of structure using binary string and material connectivity.

B. Connectivity Analysis

In the representation scheme, a binary (0-1) value of the finite element is assigned at random. This ensures the material present in the search domain does not follow any particular pattern. To make the designs meaningful, the first task is to find clusters of material at all three regions (support region, loading region and output region) and check the connectivity among them. When, all three regions are connected either directly or indirectly to each other, the given procedure fills those elements with the material which are void and surrounded by the neighboring eight elements of material. If two elements generate a point connection, then the given procedure puts one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity. If any cluster of material which is not a part of any clusters of three regions of interest as mentioned above, then it is deleted from the structure (assigned '0' to each element of this cluster). Hence, the type of representation and connectivity eliminates the problems of 'checker-board' pattern and 'floating' elements of material.

C. Parallel Computing

A parallel MOGA is used to reduce the computational time of designing the compliant mechanisms. In this parallelization process, the root processor first initializes a random population. Then, it divides the entire population into different sub-populations in proportion to the number of processors available. After this, each sub-population is sent to different slave processors. These processors further evaluate the objective functions and constraints values, and send them to the root processor. Thereafter, root processor performs the GA operators, like selection, crossover and mutation operators, non-dominated front ranking etc. on the population and replaces it with good individuals. The above process is repeated till the termination criterion of NSGA-II is met. The parallel implementation of NSGA-II is done in the context of FE analysis through $ANSYS$ FE package which consumes the maximum time of the optimization procedure [30]. A MPI based Linux cluster with 24 processors is used in the present study. A detailed specification and configuration of the Linux cluster is given at *http://www.iitk.ac.in/kangal/facilities.shtml*.

D. Crossover and Mutation operator

A two-dimensional crossover is used here which has shown a successfully implementation in shape optimization [31], [32]. In the present recombination operator, two parent solutions are selected and a coin is flipped to decide either to go row-wise or column-wise crossover. If a row crossover is done, a row is chosen with an equal probability of (P_{xover}/no. of rows) for swapping. The same is done if a column-wise crossover has to be done. During crossover, a random number is generated to identify the number of rows (columns) to be swapped and then, another generated random number helps in getting the first row (column) number of patch. A range of row (column) index is calculated and swapped with other parent. Mutation is done with a low probability on the each bit of a string to change from a void to a filled or from a filled to a void element. The detailed discussion of the crossover and mutation operators are given in these literature [33], [34].

E. Clustering Procedure

For an adequate convergence near to the global 'Pareto-optimal' front, a MOGA needs a fairly large number of population members and generations depending upon the problem complexity. Thus, the number of feasible solutions after the MOGA run are usually close to the population size. It is not advisable to represent so many solutions to the end user for a subsequent decision-making task. Therefore, the clustering procedure is employed in the study in which the neighboring solutions are grouped together and solutions from each group representing that region of the non-dominated front are chosen as representative solutions [35]. Figure 5 shows the procedure pictorially.

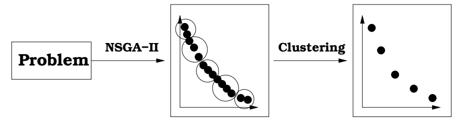


Fig. 5. Clustering Procedure.

F. Local Search Method

The local search method used here is a combination of evolutionary and classical methods. It is a variant of classical hill climbing process. As a single objective function is needed for the hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. The scaled single objective function is minimized in the present study and it is given in Equation 2.

$$F(x) = \sum_{j=1}^{n} \frac{\overline{w}_j^x (f_{j_{max}}^x - f_j^x)}{f_{j_{max}}^x - f_{j_{min}}^x}, \tag{2}$$

where, f_j^x is j^{th} objective function, $f_{j_{min}}^x$ and $f_{j_{max}}^x$ are minimum and maximum values of j^{th} objective function in the population respectively, n is number of objectives and $\overline{w}_j^x$ is the corresponding weight to the j^{th} objective function which is computed as:

$$\overline{w}_j^x = \frac{(f_{j_{max}}^x - f_j^x) \setminus (f_{j_{max}}^x - f_{j_{min}}^x)}{\sum_{k=0}^{M} (f_{k_{max}}^x - f_k^x) \setminus (f_{k_{max}}^x - f_{k_{min}}^x)}, \tag{3}$$

where M is the number of representative solutions after the clustering procedure.

In Equation 2, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space [30], [34], [36]. First the weighted sum of the scaled fitness of a selected representative solution after the clustering procedure is executed as given in Equation 2. One bit of representative solution is mutated at a time and the design is extracted from the new string. This new string is now ready for FE analysis and after the *ANSYS* simulation, objective and constraint functions are evaluated. If the new design does not satisfy the constraints, then the change in the new string is discarded and old values are restored. Otherwise, the weighted sum of scaled fitness of new string is calculated and compared with the old string. In case of mutating a '0' to '1' , a change is only accepted when the weighted sum of scaled fitness of new string is strictly better than that in the old string, else it is rejected. For the case of mutating '1' to '0', if the weighted sum of scaled fitness of new string is better than or equal to the old strings weighted sum value, then it is accepted else discarded the change. In the case of rejection, the previous bit values are restored.

Before mutating any bit, a binary string is converted into a two-dimensional array and checked for the element having a material. Then, one by one, all nine neighboring bits including its own bit value are mutated. If a change brings an improvement in scaled fitness, then the changed is accepted. This process is repeated till all bits are mutated once. If there is no change in the values of weighted sum of scaled fitness, the local search is terminated. In the same way, all representative solutions are mutated to achieve a local search. As we discussed in Section III-A, that one binary bit represents four elements for FE analysis, therefore the local search is performed on these elements.

IV. Diverse Topologies

The diverse topologies of compliant mechanisms tracing curvilinear and straight line trajectories are generated using a local search based NSGA-II procedure. The design domain of compliant mechanism is discretized with 50 by 50 rectangular elements in x and y directions respectively. A material with Young's modulus of 3.3 GPa, flexural yield stress of 6.9 MPa, density of 1.114 gm/cm^3 and Poisson ratio of 0.40, is assumed for the synthesis of compliant mechanisms. Input displacement of 8.15 mm is applied in the sub-step of five (1.63 mm in each sub-step) at the loading region in x direction. Here, a prescribed path is divided into the five precision points and the trajectory traced by an output point of a structure is evaluated through a geometric nonlinear FE analysis. During the FE analysis, a small region near the support position is declared as plastic zone and is not considered for stress constraint evaluation. A spring of constant stiffness ($\kappa = 0.4$ KN/m) is attached to the output port for providing some resistance to simulate a real application. After the NSGA-II run, six representative solutions are chosen from the non-dominated front with the help of a clustering procedure.

As discussed in Section III, the additional eight bits are used for indicating the support and loading region's elements which help in designing the structures at any support and loading regions. Therefore, the NSGA-II operations on these eight bits assist in developing the topologies with their optimum regions. In both examples, the output region is fixed with the fixed input displacement magnitude and a prescribed (curvilinear or straight line) path. Figure 1 shows the coordinates of output point $(50, 32)$ which traces out the prescribed trajectory. It also shows that the structures are designed to be supported and loaded at the bottom and left-hand side of the structures, respectively.

Fig. 6. A *reference* design tracing curvilinear path.

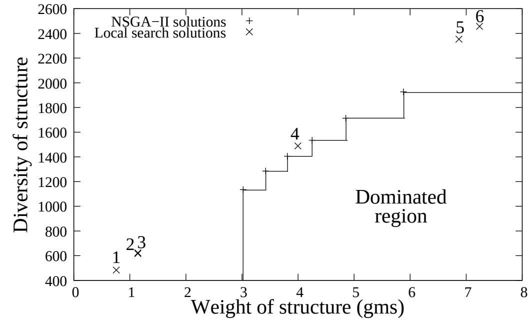


Fig. 7. NSGA-II front and local search solutions.

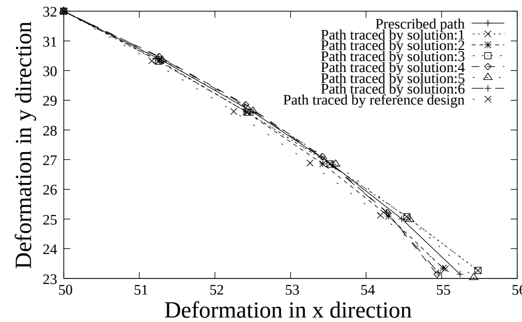


Fig. 8. Prescribed path and path traced by the *reference* design and local search solutions show good adherence to prescribed path.

A. Compliant mechanism topologies tracing curvilinear path

Topologies of compliant mechanisms tracing curvilinear path are evolved by solving the proposed bi-objective optimization problem. For the same, a single objective analysis of minimum-weight structure tracing curvilinear path is performed first, to generate the *reference* design which helps in finding the diversity of structures evolved from GA population for bi-objective analysis. As discussed in Section III-A that one binary bit represents four finite element, therefore diversity of the structures can vary from 0 to 2500 depending on dissimilarity in the bit value of the finite elements.

1) Reference design: For generating the *reference* design, a single-objective analysis of minimum-weight topology subjected to the constraints given in Equation 1 is performed using a local search based NSGA-II procedure. After local search, the *reference* design of 1.053 gms is evolved and its deformed and undeformed topologies are shown in Figure 6. The *reference* design consists of quite a few loops for generating the prescribed path. Here, the design is supported at fourth element and loaded at 44^{th} element in their respective regions.

2) Topologies tracing curvilinear path: After generating the *reference* design, a bi-objective optimization problem given in Equation 1 is solved. The positions of NSGA-II and local search solutions tracing curvilinear prescribed trajectory are shown in Figure 7. It shows that the NSGA-II solutions are distributed in the two-objective space but after a local search, these solutions are grouped in different regions. In one region, solutions 1, 2 and 3 show the minimum-weight structures with minimum diversity with respect to the *reference* design, whereas the solutions 5 and 6 show maximum diversity with respect to the *reference* design but appears as the heavy structures.

Accomplishment of the task of tracing the prescribed curvilinear path by the local search solutions and *reference* design is shown in Figure 8. All these structures are deformed to 10.49% in x direction and 17.72% in y direction at the output point. The times taken by the given optimization procedure for solving the single and bi-objective optimization problems are shown in Table I. In the parallel computing, the maximum time is consumed in the function evaluations of an optimization procedure whereas, the communication among the procesors takes a smaller time. Therefore, the parallel implementation of NSGA-II helps in reducing the compuational time almost in proportion to the number of processors available, that is, 24 in the present study. The local search is performed individually in different processors which take a considerable amount of time of a given procedure to improve the representative NSGA-II solutions.

Although the present study finds a diverse set of solutions from light to heavy weight structures (refer Figure 7), the emphasis here is to generate the 'trade-off' solutions near minimum-weight solutions as discussed in Section II. Therefore, solutions 1, 2 and 3 signifying near minimum-weight topologies are presented in Figure 9. These solutions can also be used for further analysis. For example, solution 1 of 0.7586 gms has a smaller weight compared to that of *reference* design. This also demonstrates the superiority of the proposed bi-objective result over the usual single-objective study.

B. Compliant mechanism topologies tracing straight line path

The non-intuitive designs of complaint mechanism tracing straight line trajectory and difficult to solve such optimization problem is dealt in this section. Here also, a single-objective analysis of minimum-weight structure tracing a straight line path is performed first and the topology is referred as the *reference* design. Thereafter, a bi-objective optimization problem is solved with respect to the corresponding *reference* design.

TABLE I

TIME TAKEN BY GIVEN OPTIMIZATION PROCEDURE.

Problem	NSGA-II Time (hrs)	Local search Time (hrs)
Single-objective	8.34	22.93
Bi-objective optimization problem	8.19	Solution 1: 16.61 Solution 2: 23.49 Solution 3: 26.91 Solution 4: 32.07 Solution 5: 32.15 Solution 6: 32.36

TABLE II

TIME TAKEN BY GIVEN OPTIMIZATION PROCEDURE.

Problem	NSGA-II Time (hrs)	Local search Time (hrs)
Single-objective	8.01	36.56
Bi-objective optimization problem	7.33	Solution 1: 25.28 Solution 2: 31.96 Solution 3: 33.61 Solution 4: 34.04 Solution 5: 35.00 Solution 6: 35.37

1) Reference design: The *reference* design of 0.9558 gms with its unique topology is shown in Figure 10. In this example, the output point and input displacement boundary condition to the structures are the same as the previous example, but instead of tracing a curvilinear path, the compliant mechanism now traces out the straight line trajectory. Here, the design is supported at 48^{th} element and loaded at 48^{th} element in their respective regions.

2) Topologies tracing straight line path: The positions of the NSGA-II and local search solutions in this case are shown in Figure 11. Here also, the local search solutions are collected in the different regions signifying the trade-off between the primary and secondary objectives. The fulfillment of the task of tracing a user-defined path by these solutions is shown in Figure 12 which indicates the prescribed path, path traced by the local search solutions and the *reference* design. All these structures are deformed to 10.00% in x direction and 0.0% in y direction at the output point. Table II shows the times taken by the given procedure for solving the single and bi-objective optimization problems. Here also, the parallel MOGA helps in reducing the compuational time as in the previous example by a maximum factor of 24. In this case also, the local search consumes an ample amount of time of the given procedure. As discussed in Section II, the preference is given to those solutions which signify the trade-off near to the primary objective of minimum-weight structures. Therefore, solutions 1 and 2 are chosen from Figure 11 and their topologies are shown in Figure 13.

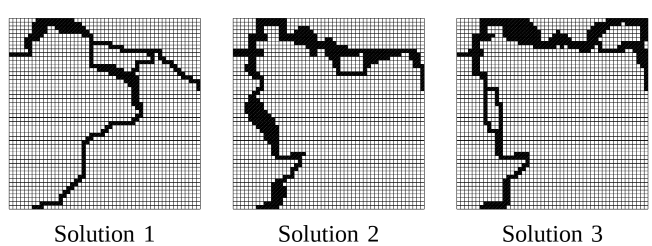


Fig. 9. Near minimum-weight topologies of compliant mechanisms tracing curvilinear path.

Undeformed topology   Deformed topology

Fig. 10. A *reference* design tracing straight line path.

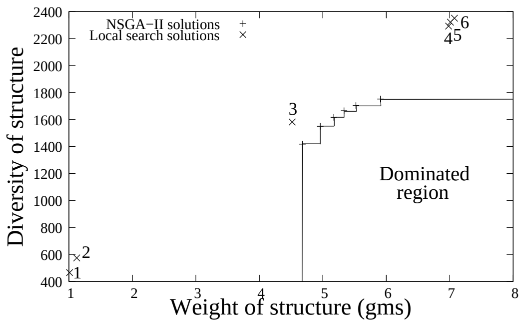


Fig. 11. NSGA-II front and local search solutions.

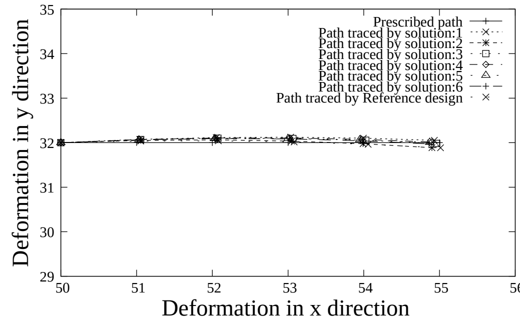
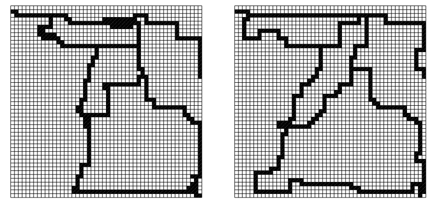


Fig. 12. Prescribed path and path traced by *reference* design and local search solutions.



Solution 1   Solution 2

Fig. 13. Near minimum-weight topologies of compliant mechanisms tracing straight line path.

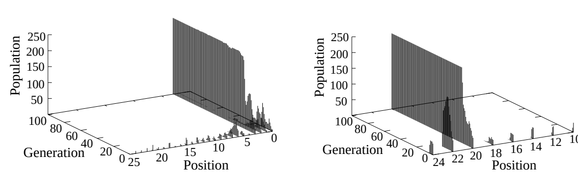
C. Progress and feasibility of complaint mechanism designs

In this section, progress of the GA population and growth of the non-dominated feasible solutions during the NSGA-II run are discussed for more insight into the behavior of an optimization procedure. As the additional eight bits are used for indicating the support and loading regions, the NSGA-II operators on these bits help in generating the non-dominated topologies. Figures 14 shows the progress of NSGA-II run for the compliant mechanisms tracing curvilinear path in which the population members are available at each support and loading region's element. As the NSGA-II generation counter increases, a few of the designs with their respective support and loading regions start dominating the others. Finally, it results in the domination of the designs with one particular support and loading regions. Figure 15 shows the growth of feasible non-dominated NSGA-II solutions tracing curvilinear path which appear at the second iteration. Here, these structures are supported at sixth element and loaded at 40^{th} element in their respective regions. It is clear from the figures that although any support and loading element were possible to be chosen. There does not seen to exist too many options leading to feasible and near-optimal structures from this problem. The designs tracing straight line trajectory show the similar progress and growth of the NSGA-II solutions. In this example, first few feasible non-dominated solutions appear in 35^{th} iteration of NSGA-II and structures are supported at 48^{th} element and loaded at 48^{th} element in their respective regions. Interestingly, in the curvilinear path tracing case, the support location is found to be near the bottom-left corner, whereas in the straight line case, it is near the bottom-right corner.

An important thing to ponder here that the NSGA-II evolves the feasible solutions in its early iteration for curvilinear path tracing compliant mechanisms as compared to the straight line tracing designs due to the fact that a single support is used against a single load which can cause a curvilinear path easily than generating a straight line path. Because of the early appearance of feasible solutions tracing curvilinear path during the NSGA-II iterations, structures after the NSGA-II run are found to have more refined and light weight topologies. Hence, time taken by the local search of light weight solutions tracing curvilinear path is comparatively less compared to that of the solutions tracing straight line trajectory. But heavier structures in both examples consume almost similar amount of time during the local search.

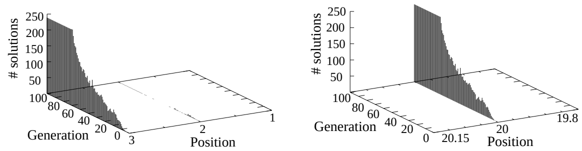
V. Conclusions

A new bi-objective optimization problem of the compliant mechanisms tracing a user-defined path has been formulated and solved using a local search based MOGA. Due to the inclusion of secondary objective providing a conflict scenario with the minimum-weight designs, the proposed formulation is found to be capable of finding diverse topologies which perform the same task of tracing the prescribed path. The



(1) Support region progress   (2) Loading region progress

Fig. 14. Progress of support and loading regions during NSGA-II run.



(1) Support region growth (2) Loading region growth

Fig. 15. Support and loading regions of feasible and non-dominated NSGA-II solutions.

given optimization procedure is found to be competent for generating topologies from a random initial population having no feasible solutions. It has evolved the mechanisms having optimum support and loading locations. The study has also shown that the parallel implementation of NSGA-II helps in reducing the computational time more or less in proportion to the number of processors used, whereas the local search has significantly improved the NSGA-II solutions by taking considerable amount of time. As the posed bi-objective formulation has generated the compliant mechanisms from light to heavy structures, a constraint on weight of structure can be imposed further in a future study for obtaining diverse and light weight structures.

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