International Conference on Trends in Product Life Cycle,
Modeling, Simulation and Synthesis
PLMSS-2006.

Evolving Path Generation Compliant Mechanisms (PGCM) using Local-search based Multi-objective Genetic Algorithm

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Abstract

Path generation compliant mechanisms (PGCM) are flexible structures which generate some desired path and/or transmit force by undergoing elastic deformation (under some applied load) instead of through rigid linkages/joints as in rigid body mechanism.

In the present work, multi-objective problem is posed for evolving of PGCM using local-search based genetic algorithm. Minimization of weight of structure and minimization of input energy to the structure have been considered as two conflicting objective functions subjected to maximum of 10% of deviation between five precision points of prescribed path and corresponding five points obtained on the actual path of a specified point on structure after FEM analysis and a constraint on stress. Geometrical non-linear finite element model is used for the synthesis of PGCM. On the basis of the calculated function and constraint values, an evolutionary algorithm (NSGA-II) is used to find the optimal solution.

A local search based multi-objective GA is used to reduce the computational time and improve the quality of GA solutions. The Pareto-optimal front obtained shows different trade-off solutions from minimum weight to the maximum weight of structure. The minimum weight solution is corresponding to the maximum input energy to the structure and vice-versa.

Keywords: Compliant mechanisms, Topology optimization, Multi-objective genetic algorithm, Large displacements, Local search method.

1. Introduction

Compliant mechanisms are flexible structures which generate some desired path and / or transmit force by going elastic deformation (under some applied load) instead of through rigid linkages / joints as in rigid body mechanism. Because of jointless and monolithic nature of compliant mechanism, some of many advantages are less friction and wear [1], ease of manufacturing without assembly [2] etc. Compliant mechanisms are used in product design, off-shore structures, smart structures, MEMS [19] etc.

The systematic design of compliant mechanism is analyzed by two approaches. In first approach, designs are inspired by traditional kinematic synthesis of rigid body mechanism. The rigid body mechanism is converted to a pseudo-rigid-body mechanism by substituting hinges with elastic hinges. In the next step the pseudo-rigid-body mechanism is converted to fully compliance mechanism [3, 4, 21, 22]. These models accurately represent the nonlinear behavior in the large deformation of uniform beams with the end loads.

The second approach is continuum based approach which focuses on the determination of the topology, shape and size of the mechanism. Among the methods found in the topology optimization literature, a popular strategy is to initially discretize the allowable design space into finite elements (FE) and defines the required loading/boundary conditions. The optimization procedure will then be concerned with determining which elements should contain material (and so form the structure) and which elements are void (and thus represent the surrounding empty space). Based on this strategy, Bendsoe and Kikuchi [5] defined the problem with composite material represented by each element having material plus a void (hole) inside. The material properties of each element are then dependent on the size and orientation of the void within the element according to the homogenization method. An alternative but conceptually similar approach is directly used the material density of each element (instead of voids) as the design variable. An empirical formula is then applied to relate this density with the elastic modulus, without the need of a homogenization method [6]. In another approach, a design space is discretized into finite element meshes with the whole set of elements taken together (row after row) to form a one dimensional (0-1) binary coded bit string chromosome and genetic algorithms (GA) can be applied to yield a population of improved (optimum) designs[7, 8, 9].

An early method used for topology optimization of compliant mechanism was by Anathasuresh et al. [23] who posed the design objective as maximizing the stiffness (or minimizing the strain energy). Frecker et al. [10] and Nishiwaki et al. [11] posed multi-criteria objective as maximizing the ratio of mutual potential energy (or output displacement) and strain energy. Sigmund [12] proposed a similar method of maximizing the mechanical advantage (a ratio between the input and output forces) subjected to volume and input displacement constraints. Larsen et al.

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[13] presented the synthesis and fabrication of compliant micro mechanism and structures with negative Poisson ratio with multiple input and output ports.

Aforementioned studies of topology optimization employed a single objective problem for the synthesis of compliant mechanism or changing the multi objective problem into single objective by weighted residual method or by considering a ratio of two conflicting objectives with linear finite element models. Studies used classical optimization techniques by transforming a discrete problem into a continuous one by using continuous variable like void size, material density or cross-sectional parameters, etc. Hence, when the final optimum variables take on intermediate values, the geometry of structure has to be interpreted from these values by prescribing a threshold point. This can be difficult and arbitrary interpretation may lead to non-optimum design.

When an input force or displacement boundary condition is prescribed, structure deforms such that another part is displaced along some desired path which may be a curvilinear or straight line trajectory. These structures are called “Path generation complaint mechanism”. Design structure is divided into three regions: first region is support region (restrained, with zero displacement boundary condition) while the loading region where specified load (force or displacement) is applied to deform the structure. The output region is the region of interest, i.e., the point on the structure where the desired output is attained which is specified by designer. Tai at el. [14, 24, 26] described the design synthesis of path generation Compliant mechanism using evolutionary optimization. A desired trajectory is divided into *N* numbers of segments and summation of average distance between corresponding points of the desired and actual trajectories is minimized subjected to the constraint that the force required to exert that input displacement does not exceed some specified value. A morphological geometric representation scheme was coupled with evolutionary algorithm to synthesize the mechanism. Pederson at el. [15, 16] described the topology synthesis of large displacement compliance mechanism together with a formation for synthesis of path generation compliant mechanism. Geometrical nonlinear planes stress models were employed for large deflection for compliant mechanism and topology optimization problem was posed to maximize the output displacement subjected to constraint on input displacement, element volume and bounds on density. Path generation mechanism problem was formulated to minimize the summation of error between the *M* precision points of actual and prescribed output displacements with a constraint on input reaction force for each precision point, element volume and bounds on density. Saxena and Anathasuresh [17] also used geometric nonlinear finite element models for the synthesis of compliant mechanism for nonlinear force-deflection and curved path specification using frame elements.

Saxena [18] employed genetic algorithms for synthesis of path generation complaint mechanism. NSGA-II [25] was implemented to solve multi objective problem for path generation complaint mechanism. After dividing the actual and prescribed trajectory into *M* precision points, minimizing the error between an actual and prescribed precision point considered as an objective and hence for *M* precision points, it constituted *M* objectives.

In this paper, multi-objective problem is posed for synthesis of path generation compliant mechanism using local-search based genetic algorithm. In this approach, design domain is discretized by 4-noded rectangular finite elements having 2 degree of freedom per node. The discretized structure is represented by one dimensional (0-1) binary coded bit string chromosome where an element with zero represents empty space while an element with value of one represents material and so forms part of the structure. Geometrical non-linear finite element model is used for path generation compliant mechanism. On the basis of the calculated function and constraint values, an evolutionary algorithm (NSGA-II) is used to find the optimal solution.

As the design of path generation compliance mechanism is a computationally intensive problem and the search power of an evolutionary algorithm is directly proportional to the number of function evaluations needed to find the optimum. A parallel multi-objective GA is used to reduce the computation time. In this parallelization process, the root processor initializes a random population for the optimization process. Then entire population is subdivided into different subpopulation equal to number of processors available. After this, each subpopulation is sent to different processors from root processor for function evaluations. Slave processors calculate the values of objective function and constraints and send them to the root for the further processing. On receiving the function values, the root applies different optimization operators, like selection, crossover and mutation operators, Pareto-optimal front ranking etc. on the population and replaces it with good individuals. The above process is repeated for a few generations.

To improve the quality of GA solutions and also reduce the computational time, a local search based MOGA is used. In this approach, the Pareto-optimal front evolved by the MOGA is divided into different regions and an appropriate solution is taken from each region to represent that part of Pareto-optimal front. These representatives are processed future by a local search algorithm.

2. Objective functions and constraints for topology optimization of PGCM

The problem is to design a structure which deforms, when some part of it is given a prescribed displacement or force such that another part is displaced along some desired path. Consider a general design domain in Fig. 1. The design domain is divided into three region of interest. First region is called support region where the structure is supported (restrained, with zero displacement) where in the second region (loading part) some specified load (input force) is applied. The output region is the third region of interest, that is, the point on the structure where the desired output is attained. For a PGCM, this output should be some desired (trajectory) curve specified by user/designer. For providing some resistance at the output region and for some work meant to be done, a spring of constant stiffness is attached at the output point.

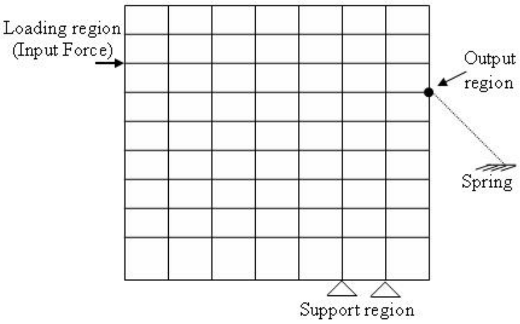


Fig. 1 Design space with input/ output and support regions

For a given structure, its deformation at the output point can be analyzed through a geometric nonlinear (large displacement) FE analysis. The analysis results obtained are the incremental force steps which are traced out the trajectory realized by the output point. To evaluate how closely a given structure design traces out the desired trajectory, the error between the desired and actual trajectory is computed. For the same, the desired trajectory is divided into *N* number of precision points and allows maximum of 10% of deviation between the precision points of prescribed path and corresponding points obtained on the actual path of a specified point on structure after FEM analysis. Hence the problem is subject to *N* number of constraint on the basis of error or deviation between the desired and actual precision point. Stress constraint is also implied in the study for getting the practical models wherein maximum stress developed in the structure should be less the flexural yield stress. Minimization of weight of structure and minimization of input energy to the structure have been considered as two conflicting objective functions.

3. Multi-objective optimization

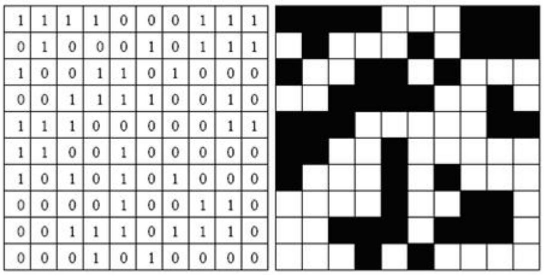
Real-world optimizations deal with the simultaneously optimization of multiple objectives. In single objective optimization, the objective is to find out the best feasible design solution, which corresponding to the maximum or minimum values of the objective function. But in multi-objective optimization with conflicting objectives, there is no single optimal solution. The outcome of such an optimization problem is a set of compromised solutions of different objectives. This set is known as “Pareto-optimal” solutions. The solutions belonging to the Pareto-optimal solution set are superior to the rest of the solutions in the search space, when all objectives are considered but are inferior to other solutions in the set in one or more objectives. These solutions are also known as non-dominated solutions.

Among the different multi-objective algorithms, it is observed that an elitist non-dominated sorting genetic algorithm (known as NSGA-II) has the features required for a good multi-objective genetic algorithm (MOGA). It is shown that it can converge to the global Pareto-optimal front as well as maintain the diversity of population on the Pareto-optimal front [25].
Topology optimization problems are highly non-linear and discrete in nature. To circumvent the problem of non-convergence of classical optimization techniques for non-linear problems and for analyzing the discrete nature of problem, genetic algorithm is used to find the optimum solutions (NSGA-II).

4. Representation of structural geometry and parallel computing

4.1 Representation scheme

Continuum structure is discretized by 4 node rectangular elements and each element is represented by either by “0” or “1” where “1” signifies that material is present and “0” represents the void. This makes a string of binary string which is copied to two dimensional array as per the sequence shown in Fig. 2.



1	1	1	1	0	0	0	1	1	1
0	1	0	0	0	1	0	1	1	1
1	0	0	1	1	0	1	0	0	0
0	0	1	1	1	1	0	0	1	0
1	1	1	0	0	0	0	0	1	1
1	1	0	0	1	0	0	0	0	0
1	0	1	0	1	0	1	0	0	0
0	0	0	0	1	0	0	1	1	0
0	0	1	1	1	0	1	1	1	0
0	0	0	1	0	1	0	0	0	0

Fig. 2 Representation of structure using numbers and material

4.2 Connectivity analysis

In the above representation scheme, values (0-1) of the finite elements are assigned at random. This ensures the material present in the search domain does not follow any particular pattern. To make solution feasible, the first task to find clusters of material of all three regions (support region, loading region and output region) and check the connectivity amongst them. If two elements are connected point by point, then put one extra material at the nearby element (according to the nature of connectivity) to eliminate the problem of high stress at the point connectivity. And if any cluster of material which is not a part of any clusters of three regions of interest as mentioned above, will be deleted from the structure (assigned zero value to each element of this cluster). The topology of structure formed after the connectivity analysis is ready for FEM analysis (non linear geometrical FEM) with the help of FEM package ‘ANSYS’.

4.3 Parallel computing

A parallel multi-objective GA is used to reduce the computation time because the design of PGCM is a computationally intensive problem and the search power of an evolutionary algorithm is directly proportional to the number of function evaluations needed to find the optimum. In this parallelization process, the root processor initializes a random population for the optimization process. Then entire population is subdivided into different subpopulation equal to number of processors available. After this, each subpopulation is sent to different processors from root processor for function evaluations. Slave processors calculate the values of objective function and constraints and send them to the root for the further processing. On receiving the function values, the root applies different optimization operators, like selection, crossover and mutation operators, Pareto-optimal front ranking etc. on the population and replaces it with good individuals.

Hence before FEM analysis, a binary string is transformed to two dimensional array and for each array, a macro is generated in each processor for ANSYS as an input file. After executing an input file, desired outputs are saved into files and read again for objective function evaluations and constraints. Outputs are sent to root computer for further analysis like selection, crossover and mutation, Pareto-optimal front ranking etc. on the population and replaces it with good individuals. The above process is repeated for a few generations.

4.4 Clustering

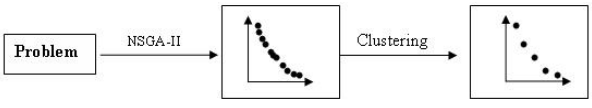


Fig. 3 Clustering algorithm

The number of feasible solutions after the MOGA run is equal to or less than the population size. But actually at the end of MOGA run, there are not many distinct solutions. So it is not advisable to represent all the solution to the end users. But to convergence at the global Pareto-optimal front, GA needs a fairly large amount of population and generations depending on the problem complexity. To get a meaningful idea of the type of solution at the cluster algorithm is done. The neighboring solutions are grouped together and solutions from each group represented that zone of the Pareto-optimal front [27]. Fig. 3 shows the algorithm pictorially.

4.5 Local search method

The local search method used for topology optimization is a combination of evolutionary method and a classical method [28, 29]. In this approach, first a multi-objective genetic algorithm is applied on a set of random solutions. After some generation (so that MOGA can reach near to the global Pareto-optimal front) a local search is applied to the representative solutions of clustering algorithm. This local search algorithm is a variant of classical hill climbing process. As a single objective function is needed for the hill climbing, the multi-objective problem is reduced to a single objective problem. This is done by taking a weighted sum of different objectives. This scaled single objective function is minimized or maximized according to the problem. The scaled function looks like [20]:

$$F(x)=\sum_{j=1}^{M}\frac{\overline{w}_j\left(f_j^{(x)}{}_{\max}-f_j^{(x)}\right)}{\left(f_j^{(x)}{}_{\max}-f_j^{(x)}{}_{\min}\right)} \qquad (1)$$

In the above equation, the values of the objective functions are normalized to avoid bias towards any objective function. In this approach, the weight vector decides the importance of different objectives, in other words it gives the direction of local search in the objective space. Any setting of weight vector will lead to different optima, Moreover if the weight is not chosen properly, it may mislead the search. Classical theories have different ways of selecting weights, but all of them require prior problem information. It is shown that the weights can be assigned without the knowledge of the actual Pareto-optimal front [20, 28, 29]. In that method the weight vector for a particular solution at the Pareto-optimal front, is assigned on the position of that solution space in the objective space. When a solution is near to minimum value of an objective function, the weight of that function will get a larger value. Higher weight associated with a particular objective, signifies higher priority of that objective. The formula used for weighted calculation is given below:

$$\overline{w}_j=\frac{\left(f_j^{\max}-f_j^{(x)}\right)\Big/\left(f_j^{\max}-f_j^{\min}\right)}{\sum_k^M\left(f_k^{\max}-f_k^{(x)}\right)\Big/\left(f_k^{\max}-f_k^{\min}\right)} \qquad (2)$$

Local search starts using the above calculated weighted vectors. Before starting it needs an initial guess and that is provided by GA module. This local search method is similar to the classical steepest descent search algorithm, trying to find the solution in global basin, when it is near to the global optimum. First a representative solution is selected and the weighted sum of the scaled fitness is calculated for the solution. One bit of representative solution is mutated at a time and the design is extracted from the new string. This new string is now ready for FEM analysis and after ANSYS run, objective functions and constraint functions are evaluated. The weighted sum of scaled fitness is calculated and if the new design does not satisfy the constraints, then the change in the new string is discarded and restore the old values. The weighted sum of scaled fitness of new string and old string is compared. In case of mutating “0” bit to “1” bit, a change is only accepted when weighted sum of scaled fitness of new string is strictly better than the old string, else it is rejected. For the case of mutating “1” bit to “0” bit, if weighted sum of scaled fitness of new string is better than and equal to old string’s weighted sum, it is accepted else discarded the change.

In case of rejection, the previous bit value is restored. After converting a binary string into a two dimensional array, first check the element having a material and then, one by one all eight neighboring element’s bit and including element’s bit value is mutated. This process is repeated till all the bits are mutated once. If there is no bit change and there is no change in values of weighted sum of scaled fitness, the local search is terminated. In same way all representative solutions have undergone to the local search in each slave processor. Fig. 4 shows the local search direction of the solutions on Pareto-optimal front after the application of GA. It also shows the Pareto-optimal before and after local search for a hypothetical case.

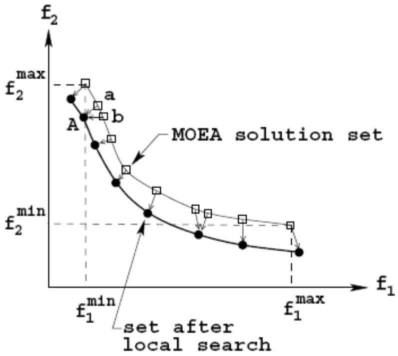


Fig. 4 Optimal front after Local search

5. Examples

Two PGCM problems are solved, each with its own desired trajectory but with same material parameters and design space definition (50mm by 50mm). In both of the examples, design domain is discretized by 4 node rectangular element. The material young’s modulus (Nylon) for the structure is 3.3 GPa and Poisson ratio of 0.4. Input force is applied in the sub-step of five at the input region. The output point has to follow some prescribed path or trajectory. Here a prescribed path or trajectory is divided into five precision points and the trajectory traced by output point of any design is evaluated through a geometric nonlinear FE analysis using ANSYS. A spring of constant stiffness (0.4KN/m) is attached to the output port. In both the problem, the design domain is discretized into 25 by 25 elements.

NSGA-II is used to find the optimum structures for 200 generation and population size of 192. Crossover and mutation probability of 0.95 and 1/ (string length) are used for both of the examples respectively. After getting feasible solutions, six representative solutions have been chosen from Pareto-optimal front and local search method is employed to get optimum structure.

For the first problem with desired trajectory #1 (shown in Fig. 5), the magnitude of inward force applied at the input region is 2.25 N which is applied in sub-step of five. For the desired trajectory #2 (shown in Fig. 8) of second problem, the input force is 2.6 N. In both of the problems, five constraints on 10% deviation at precision points and one constraint on stress have been considered for evolving the optimal structure.

5.1 Results of trajectory #1

Trajectory #1 is straight line for which a PGCM output port has to move in a straight line. Fig. 5 shows predefined path described by the used/designer and actual paths of optimized structures after FEM analysis and Fig. 7(a) shows three regions. After the application of input force, structures deforms 5.12% (approx.) of size of design domain in X direction. Fig. 6 shows the Pareto-optimal front of NSGA-II solutions and the solutions after local search.

Fig.6 shows that all 6 representative solutions after employing local search method are non-dominated solutions and Fig 7 shows optimal designs after local search. Among the six representation solutions, four of them are showing the similar design of structures for PGCM and have a small difference in the values of both objective functions. These solutions are called as knee solutions and any one of the design among the four knee solutions can represent one solution of PGCM design.

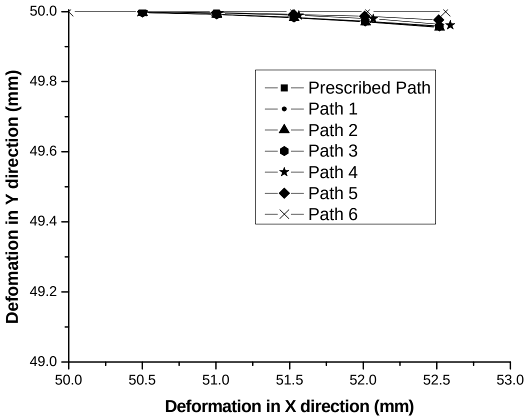


Fig.5 Prescribed path by user/designer and path traced by optimum structures

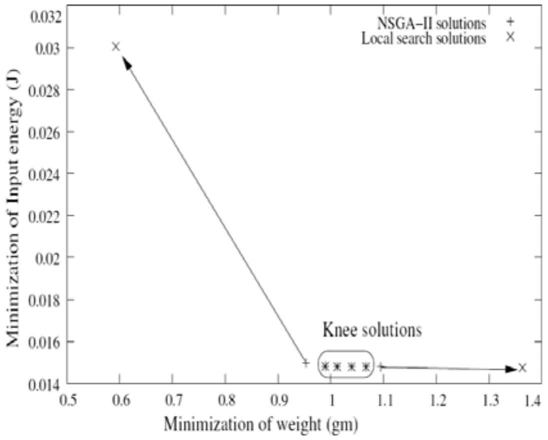


Fig. 6 NSGA-II solutions front and local search solution front.

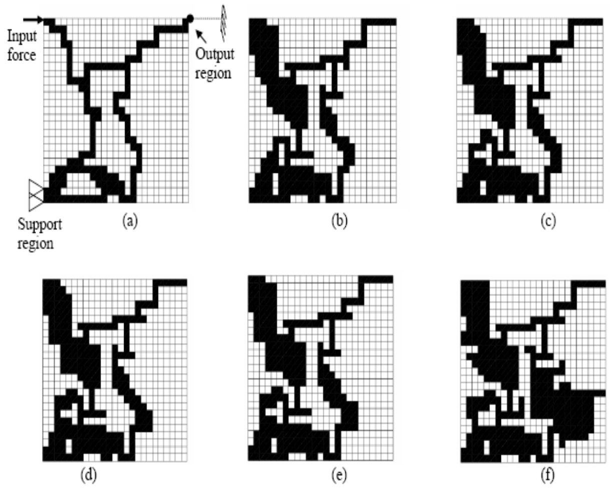


Fig. 7 Non-dominated optimal structure designs after local search method in the increasing order of minimization of weight from (a) to (f)

5.2 Results of trajectory #2

Trajectory # 2 (Fig. 8) is a curvilinear path traced by PGCM and Fig. 10(a) shows three regions. The difference/deviation of prescribed path by user/designer and actual path of optimized structure after FEM analysis is showing within 10 % and also satisfying the stress constraint. Deformation observed in the given example is 8.86% in X direction and 3.62% in Y direction of the size of design domains.

Even though the designs of structures look like same but they are different in the linkage of stiffeners and with showing trade-off between both of the objective functions. In both the problems, the optimal solutions obtained after local search are further post-processed by morphing technique as shown in Fig. 7 and Fig. 10. As the search power of evolutionary algorithms depends on the function evaluation and PGCM is a non-linear problem, a large number of generations is required to get the solutions near to global basin where as the local search method helps in reducing the time of evolving the optimal structures.

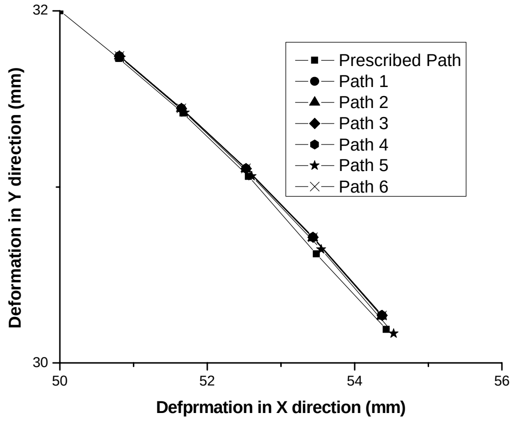


Fig.8 Prescribed path by user/designer and path traced by optimum structures

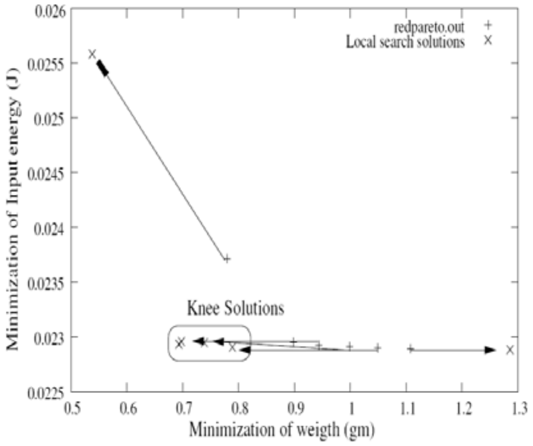


Fig. 9 NSGA-II solutions front and local search solution front

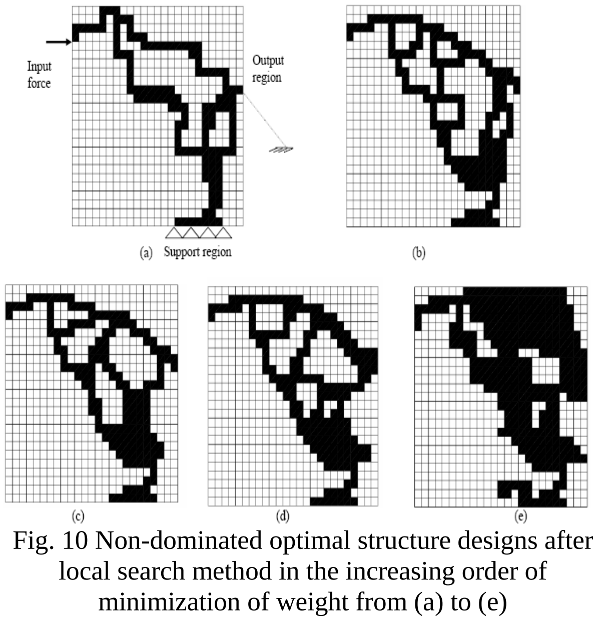


Fig. 10 Non-dominated optimal structure designs after local search method in the increasing order of minimization of weight from (a) to (e)

6. Conclusions

A local search based multi-objective genetic algorithm is successfully employed to evolve the optimal structures of PGCM. The main observations are:

1. Posed multi-objective problem shows a conflicting nature of both objectives for designing PGCM.
2. NSGA-II algorithm is coupled with geometrical finite element analysis show a trade off between the proposed objective functions.
3. Local search method helps in getting the solution nearer to the global basin and reduces the time of evolving the optimal structures.
4. PGCM problems are computationally intensive and parallel implementation makes the entire process fast and linearly reduces the computation time.
5. Local search solutions show knee solutions which are similar in design but they are different in terms of presence of stiffeners.

In future work, new and better conflicting objective functions will help to get better designs of PGCM. Refining the mesh of structure and better morphing techniques will provide a smooth design for improving the results for fabrication purpose.

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