

# Evolutionary Multi-Objective Environmental/Economic Dispatch: Stochastic vs. Deterministic Approaches

Robert T. F. Ah King<sup>1</sup>, Harry C. S. Rughooputh<sup>1</sup> and Kalyanmoy Deb<sup>2</sup>

<sup>1</sup> Department of Electrical and Electronic Engineering, Faculty of Engineering,  
University of Mauritius, Reduit, Mauritius  
{r.ahking, r.rughooputh}@uom.ac.mu

<sup>2</sup> Kanpur Genetic Algorithms Laboratory, Department of Mechanical Engineering,  
Indian Institute of Technology, Kanpur, PIN 208 016, India  
deb@iitk.ac.in

**KanGAL Report Number 2004019**

**Abstract.** Due to the environmental concerns that arise from the emissions produced by fossil-fueled electric power plants, the classical economic dispatch, which operates electric power systems so as to minimize only the total fuel cost, can no longer be considered alone. Thus, by environmental dispatch, emissions can be reduced by dispatch of power generation to minimize emissions. The environmental/economic dispatch problem has been most commonly solved using a deterministic approach. However, power generated, system loads, fuel cost and emission coefficients are subjected to inaccuracies and uncertainties in real-world situations. In this paper, the problem is tackled using both deterministic and stochastic approaches of different complexities. The Nondominated Sorting Genetic Algorithm – II (NSGA-II), an elitist multi-objective evolutionary algorithm capable of finding multiple Pareto-optimal solutions with good diversity in one single run is used for solving the environmental/economic dispatch problem. Simulation results are presented for the standard IEEE 30-bus system.

## 1 Introduction

The classical economic dispatch problem is to operate electric power systems so as to minimize the total fuel cost. This single objective can no longer be considered alone due to the environmental concerns that arise from the emissions produced by fossil-fueled electric power plants. In fact, the Clean Air Act Amendments have been applied to reduce SO<sub>2</sub> and NO<sub>x</sub> emissions from such power plants. Accordingly, emissions can be reduced by dispatch of power generation to minimize emissions instead of or as a supplement to the usual cost objective of economic dispatch. Environmental/economic dispatch is a multi-objective problem with conflicting objectives because pollution is conflicting with minimum cost of generation. Various techniques have been proposed to solve this multi-objective problem whereby most researchers have concentrated on the deterministic problem.

Economic dispatch calculates the cost of generation based on data relating fuel cost and power output. This cost function is approximated by a quadratic equation with

cost coefficients. In conventional economic dispatch the coefficients are assumed to be deterministic, but in real-world situations, these data are subjected to inaccuracies and uncertainties. These deviations are attributed to (i) inaccuracies in the process of measuring and forecasting of input data and (ii) changes of unit performance during the period between measuring and operation [1]. Thus, the operating point in practice will differ from the planned operating point and will thus affect the actual fuel cost. Similarly, emission coefficients may also be subjected to some deviations resulting in definite differences in practical systems.

There has been much research using the deterministic approach to solve the environmental/economic dispatch problem. Gent and Lamont [2] introduced the minimum-emission dispatch concept where they developed a program for on-line steam unit dispatch that results in the minimizing of  $\text{NO}_x$  emission. These authors introduced the mathematical representation of  $\text{NO}_x$  emission of steam generating units and used a Newton-Raphson convergence technique to obtain base points and participation factors. Zahavi and Eisenberg [3] proposed a dispatch procedure for power that meets the demand for energy while accounting for both cost and emission considerations. A tradeoff curve which present the decision maker with all possible courses of action (dispatch policies) for a given demand was introduced. Nanda et al. [4] presented an improved Box complex method for economic dispatch and minimum emission dispatch problems. Dhillon et al. [5] formulated the multiobjective thermal power dispatch using noncommensurable objectives such as operating costs and minimal emission. The epsilon-constraint method was used to generate non-inferior solutions to the multiobjective problem considering the operating cost as the objective and replacing emission objective as a constraint. More recently, multi-objective evolutionary algorithms have been applied to the problem at hand. Abido has pioneered this research by applying NSGA [6], NPGA [7] and SPEA [8] to the standard IEEE 30-bus system. In fact, it has been shown that NSGA-II can obtain minimum cost and minimum emission solutions comparable to Tabu search [9].

Not long after the introduction of the environmental consideration in the economic dispatch problem, researchers started considering stochastic approaches bearing in mind the uncertainties that are inherent in real-world situations. Viviani and Heydt [10] incorporated the effects of uncertain system parameters into optimal power dispatch. Their method employed the multivariate Gram-Charlier series as means of modeling the probability density function (p.d.f.) which characterizes the uncertain parameters. Parti et al. [1] extended the Lagrange multiplier solution method to solve the economic thermal power dispatch problem using an objective function consisting of the sum of the expected production costs and expected cost of deviations (a penalty term proportional to the expectation of the square of the unsatisfied load because of possible variance of the generator active power). Bunn and Paschentis [11] developed a stochastic model for the economic dispatch of the electric power. These authors used a form of stochastic linear programming method for online scheduling of power generation at 5 minute intervals taking into account the mismatch between dispatched generation and actual load demanded. Experimental results on real data demonstrated the efficiency of the approach compared to conventional deterministic linear programming model. Dhillon et al. [12] have used the weighted minimax technique to obtain trade-off relation between the conflicting objectives and fuzzy set theory is subsequently used to help the operator choose an optimal operating point. In another

attempt, Dhillon et al. [13] solved the multiobjective stochastic economic dispatch problem whereby the weighted sum technique and Newton-Raphson algorithm are used to generate the non-inferior solutions considering expected operating cost and expected risk associated with the possible deviation of the random variables from their expected values. In their study, the random variables are assumed to be normally distributed and statistically dependent on each other, hence the deterministic objective functions have both variance and covariance terms. Recently, Bath et al. [14] presented an interactive fuzzy satisfying method for multi-objective generation scheduling with explicit recognition of statistical uncertainties in system production cost data. However, the multi-objective problem is converted into a scalar optimization problem and solved using weighted sum method. Hooke-Jeeves pattern search, evolutionary optimization and weight simulation methods are used to find the optimal weight combinations and fuzzy sets are used to obtain the 'best' solution from the non-inferior solutions set.

In this paper, both the deterministic and stochastic approaches are addressed. More precisely, the stochastic problem is considered in a unique way due to the nature of the problem when the load flow calculations determine the power generated by the slack bus. Thus, a reliability measure is used to test the power system under different stochastic considerations. The paper is organized as follows. The environmental/economic dispatch problem is defined in Section 2. Section 3 outlines the system parameters considered in this study. The simulation results of the deterministic approach are given in Section 4 while those of the stochastic approach are presented in Section 5. Based on these results, the main findings and some conclusions are outlined in Section 6.

## 2 Environmental/Economic Dispatch

The environmental/economic dispatch involves the simultaneous optimization of fuel cost and emission objectives which are conflicting ones. The deterministic problem is formulated as described below.

### 2.1 Objective Functions

**Fuel Cost Objective.** The classical economic dispatch problem of finding the optimal combination of power generation, which minimizes the total fuel cost while satisfying the total required demand can be mathematically stated as follows [15]:

$$C = \sum_{i=1}^n (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \text{ \$/hr} \quad (1)$$

where

- $C$ : total fuel cost (\\$/hr),
- $a_i, b_i, c_i$ : fuel cost coefficients of generator  $i$ ,
- $P_{Gi}$ : power generated (p.u.) by generator  $i$ , and

#### 4 Ah King, Rughooputh and Deb

$n$ : number of generators.

**NO<sub>x</sub> Emission Objective.** The minimum emission dispatch optimizes the above classical economic dispatch including NO<sub>x</sub> emission objective, which can be modeled using second order polynomial functions [15]:

$$E_{NO_x} = \sum_{i=1}^n (a_{iN} + b_{iN} P_{Gi} + c_{iN} P_{Gi}^2 + d_{iN} \sin(e_{iN} P_{Gi})) \text{ ton/hr} \quad (2)$$

### 2.2 Constraints

The optimization problem is bounded by the following constraints:

**Power balance constraint.** The total power generated must supply the total load demand and the transmission losses.

$$\sum_{i=1}^n P_{Gi} - P_D - P_L = 0 \quad (3)$$

where

$P_D$ : total load demand (p.u.), and  
 $P_L$ : transmission losses (p.u.).

The transmission losses is given by

$$P_L = \sum_{i=1}^N \sum_{j=1}^N \left[ \begin{aligned} &(r_{ij} / V_i V_j) \cos(\delta_i - \delta_j) (P_i P_j + Q_i Q_j) + \\ &(r_{ij} / V_i V_j) \sin(\delta_i - \delta_j) (Q_i P_j - P_i Q_j) \end{aligned} \right] \quad (4)$$

where

$N$ : number of buses  
 $r_{ij}$ : series resistance connecting buses  $i$  and  $j$   
 $V_i$ : voltage magnitude at bus  $i$   
 $\delta_i$ : voltage angle at bus  $i$   
 $P_i$ : real power injection at bus  $i$   
 $Q_i$ : reactive power injection at bus  $i$

**Maximum and minimum limits of power generation.** The power generated  $P_{Gi}$  by each generator is constrained between its minimum and maximum limits, i.e.,

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad (5)$$

where

$P_{Gimin}$ : minimum power generated, and  
 $P_{Gimax}$ : maximum power generated.

### 2.3 Multiobjective Formulation

The multiobjective deterministic environmental/economic dispatch optimization problem is therefore formulated as:

$$\text{Minimize } [C, E_{NO_x}] \quad (6)$$

$$\text{subject to: } \sum_{i=1}^n P_{Gi} - P_D - P_L = 0 \quad (\text{power balance}), \text{ and}$$

$$P_{Gimin} \leq P_{Gi} \leq P_{Gimax} \quad (\text{generation limits})$$

### 3 System Parameters

Simulations were performed on the standard IEEE 30-bus 6-generator test system (Fig. 1) using the Elitist Nondominated Sorting Genetic Algorithm (NSGA-II) for both deterministic and stochastic approaches. Details of the algorithm of NSGA-II can be found in [16].

The power system is interconnected by 41 transmission lines and the total system demand for the 21 load buses is 2.834 p.u. Fuel cost and  $NO_x$  emission coefficients for this system are given in Tables 1 and 2 respectively.

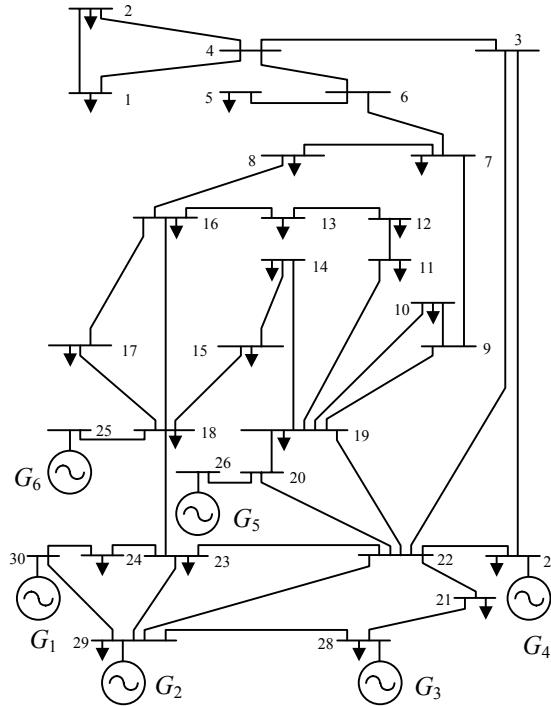


Fig. 1. Single-line diagram of IEEE 30-bus test system [8]

**Table 1.** Fuel Cost coefficients

Unit $i$	$a_i$	$b_i$	$c_i$	$P_{Gimin}$	$P_{Gimax}$
1	10	200	100	0.05	0.50
2	10	150	120	0.05	0.60
3	20	180	40	0.05	1.00
4	10	100	60	0.05	1.20
5	20	180	40	0.05	1.00
6	10	150	100	0.05	0.60

**Table 2.** NO<sub>x</sub> Emission coefficients

Unit $i$	$a_{iN}$	$b_{iN}$	$c_{iN}$	$d_{iN}$	$e_{iN}$
1	4.091e-2	-5.554e-2	6.490e-2	2.0e-4	2.857
2	2.543e-2	-6.047e-2	5.638e-2	5.0e-4	3.333
3	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.000
4	5.326e-2	-3.550e-2	3.380e-2	2.0e-3	2.000
5	4.258e-2	-5.094e-2	4.586e-2	1.0e-6	8.000
6	6.131e-2	-5.555e-2	5.151e-2	1.0e-5	6.667

In all simulations, the following parameters were used:

- population size = 50
- crossover probability = 0.9
- mutation probability = 0.2
- distribution index for crossover = 10
- distribution index for mutation = 20

The simulations were run for five different cases:

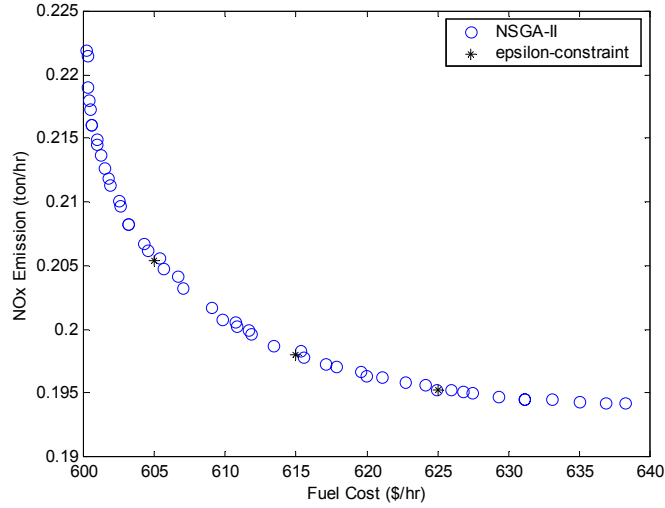
- Case D1: Deterministic - System is considered as lossless
- Case D2: Deterministic - Transmission losses are considered
- Case S1: Stochastic power generated
- Case S2: Stochastic power generated and system loads
- Case S3: Stochastic power generated, system loads, fuel cost and emission coefficients

## 4 Deterministic approach

Using the deterministic parameters as given in Tables 1 and 2, the simulation results obtained are presented.

#### 4.1 Case D1: Deterministic without Transmission Losses

Fig. 2 shows a good diversity in the nondominated solutions obtained by NSGA-II after 200 generations.



**Fig. 2.** Nondominated solutions for Case D1

Table 3 and 4 show the best fuel cost and best  $\text{NO}_x$  emission obtained by NSGA-II as compared to Linear Programming (LP) [15], Multi-Objective Stochastic Search Technique (MOSST) [17], Nondominated Sorting Genetic Algorithm (NSGA) [6], Niche Pareto Genetic Algorithm (NPGA) [7] and Strength Pareto Evolutionary Algorithm (SPEA) [8]. It can be deduced that NSGA-II finds comparable minimum fuel cost and comparable minimum  $\text{NO}_x$  emission to the last three evolutionary algorithms. To confirm that NSGA-II is able to obtain the Pareto front for the problem, the epsilon-constraint method [18] has been used as shown on the plot of Fig. 2. Genetic algorithm was used to solve the resulting single-objective problem.

**Table 3.** Best fuel cost

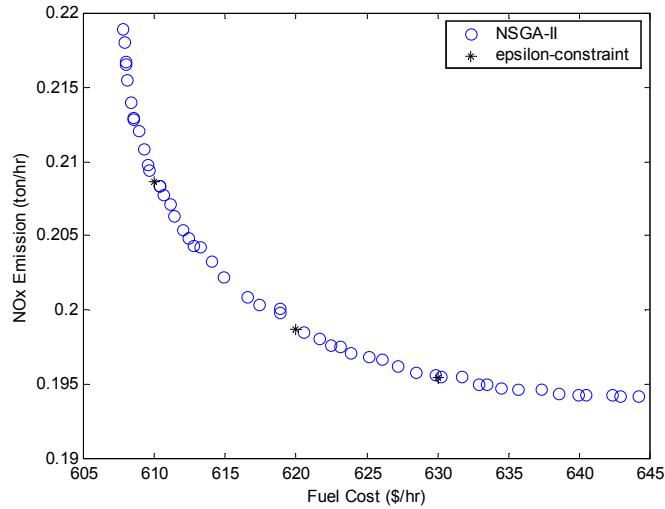
	LP [15]	MOSST [17]	NSGA [6]	NPGA [7]	SPEA [8]	NSGA-II
$P_{G1}$	0.1500	0.1125	0.1567	0.1080	0.1062	0.1059
$P_{G2}$	0.3000	0.3020	0.2870	0.3284	0.2897	0.3177
$P_{G3}$	0.5500	0.5311	0.4671	0.5386	0.5289	0.5216
$P_{G4}$	1.0500	1.0208	1.0467	1.0067	1.0025	1.0146
$P_{G5}$	0.4600	0.5311	0.5037	0.4949	0.5402	0.5159
$P_{G6}$	0.3500	0.3625	0.3729	0.3574	0.3664	0.3583
Best cost	<b>606.314</b>	<b>605.889</b>	<b>600.572</b>	<b>600.259</b>	<b>600.15</b>	<b>600.155</b>
Corresp. emission	0.22330	0.22220	0.22282	0.22116	0.2215	0.22188

**Table 4.** Best NO<sub>x</sub> emission

	LP [15]	MOSST [17]	NSGA [6]	NPGA [7]	SPEA [8]	NSGA-II
$P_{G1}$	0.4000	0.4095	0.4394	0.4002	0.4116	0.4074
$P_{G2}$	0.4500	0.4626	0.4511	0.4474	0.4532	0.4577
$P_{G3}$	0.5500	0.5426	0.5105	0.5166	0.5329	0.5389
$P_{G4}$	0.4000	0.3884	0.3871	0.3688	0.3832	0.3837
$P_{G5}$	0.5500	0.5427	0.5553	0.5751	0.5383	0.5352
$P_{G6}$	0.5000	0.5142	0.4905	0.5259	0.5148	0.5110
Best emission	<b>0.19424</b>	<b>0.19418</b>	<b>0.19436</b>	<b>0.19433</b>	<b>0.1942</b>	<b>0.19420</b>
Corresp. cost	639.600	644.112	639.231	639.182	638.51	638.269

#### 4.2 Case D2: Deterministic with Transmission Losses Considered

In this case, the transmission losses are considered and the NSGA-II algorithm was run for 200 generations. Fig. 3 shows the nondominated solutions obtained by NSGA-II for Case D2 where a good distribution of the solutions is observed.

**Fig. 3.** Nondominated solutions for Case D2

The best fuel cost and best NO<sub>x</sub> emission obtained by NSGA-II as compared to NSGA, NPGA and SPEA are given in Table 5 and 6. It is observed that NSGA-II again finds better minimum fuel cost and emission level than the other evolutionary algorithms.



**Table 5.** Best fuel cost

	NSGA [6]	NPGA [7]	SPEA [8]	NSGA-II
$P_{G1}$	0.1168	0.1245	0.1086	0.1182
$P_{G2}$	0.3165	0.2792	0.3056	0.3148
$P_{G3}$	0.5441	0.6284	0.5818	0.5910
$P_{G4}$	0.9447	1.0264	0.9846	0.9710
$P_{G5}$	0.5498	0.4693	0.5288	0.5172
$P_{G6}$	0.3964	0.3993	0.3584	0.3548
Best cost	<b>608.245</b>	<b>608.147</b>	<b>607.807</b>	<b>607.801</b>
Corresp. emission	0.21664	0.22364	0.22015	0.21891

**Table 6.** Best NO<sub>x</sub> emission

	NSGA [6]	NPGA [7]	SPEA [8]	NSGA-II
$P_{G1}$	0.4113	0.3923	0.4043	0.4141
$P_{G2}$	0.4591	0.4700	0.4525	0.4602
$P_{G3}$	0.5117	0.5565	0.5525	0.5429
$P_{G4}$	0.3724	0.3695	0.4079	0.4011
$P_{G5}$	0.5810	0.5599	0.5468	0.5422
$P_{G6}$	0.5304	0.5163	0.5005	0.5045
Best emission	<b>0.19432</b>	<b>0.19424</b>	<b>0.19422</b>	<b>0.19419</b>
Corresp. cost	647.251	645.984	642.603	644.133

Again, it can be deduced that the algorithm is capable of obtaining the Pareto front for the given problem as verified by the minimum of each objective and points obtained by the epsilon-constraint method in Fig. 3.

It has been shown that NSGA-II can obtain the Pareto front of the problem and it is therefore ideal for solving the multiobjective environmental/economic dispatch optimization problem which has conflicting objectives from the fact that the multiobjective approach yields multiple Pareto-optimal solutions in a single simulation run whereas multiple runs are required for the single objective approach with weighted objectives.

## 5 Stochastic approach

Previous stochastic approaches involved the inclusion of deviational (recourse) costs to account for mismatch between scheduled output and actual demand in the formulation of the objective function [11], and conversion of stochastic models into their deterministic equivalents by taking their expected values and formulating the problem as the minimization of cost and emission plus additional objective for the expected deviation between generator outputs and load demand (unsatisfied load

demand) [12, 13, 14]. The approach adopted in this paper is based on the reliability concept and simulations are performed to test the reliability of the stochastic system under different problem formulations. Decision variables  $P_{Gi}$  ( $i = 1, \dots, 6$ ) are assumed to be normally distributed with Mean  $P_{Gi}$  and Standard Deviation (SD)  $\sigma_i = 0.1P_{Gi}$ . For each solution  $P_{Gi}$  ( $i = 2, \dots, 6$ ), 100 random instantiates having Mean  $P_{Gi}$  and SD  $\sigma_i$  are created within  $2\sigma_i$ . A good measure of system performance in the case of stochastic systems is its reliability [19]. We define reliability  $R$  as:

$$R = \frac{n}{m} \quad (7)$$

and an additional constraint is included in the optimization problem:

$$R \geq R^{cr} \quad (8)$$

where  $R^{cr}$  is the required reliability which is 95.6% for which  $Pr\{\mu_1 - 2\sigma_1 < P_1 < \mu_1 + 2\sigma_1\}$ . Thus, Reliability  $R$  is calculated according to the number of cases for which  $P_1$  is found to be within  $2\sigma_1$ .

In the stochastic approach, the objective functions are now reformulated as follows:

$$\text{Min. } \overline{Cost} + 2\sigma_{Cost} \quad (9)$$

$$\text{Min. } \overline{NO_x} + 2\sigma_{NO_x}$$

subject to the following constraints:

$$\begin{aligned} \sum_{i=1}^n P_{Gi} - P_D - P_L &= 0 \\ P_{Gimin} &\leq P_{Gi} \leq P_{Gimax} \\ R &\geq R^{cr} \end{aligned}$$

where  $\overline{Cost}$ ,  $\overline{NO_x}$ ,  $\sigma_{Cost}$  and  $\sigma_{NO_x}$  are the Expected Cost and Expected  $NO_x$  emission and SD of Expected Fuel Cost and SD of Expected  $NO_x$  emission respectively.

Note that  $P_{Gi}$  is calculated from the loadflow program and this satisfies implicitly the power balance constraint (equation 3).

The procedure used in this stochastic method is described as follows:

For each feasible solution  $\overline{P}_j$  ( $j=2, \dots, n$ ) obtained by NSGA-II,

Create  $m$  instantiates  $P_j^{(i)}$  ( $j=2, \dots, n$ ) by perturbing each  $P_j$  as  $N(\overline{P}_j, \sigma_j)$  where  $m = 100$  and  $\sigma_j = 0.1\overline{P}_j$ .

Count the number of instantiates  $n$  for which  
 $P_1^{(i)} \in [P_1^{\min}, P_1^{\max}]$

$$\text{where } P_1^{\min} = \bar{P}_1 - 2\sigma_j \bar{P}_1 = 0.8\bar{P}_1$$

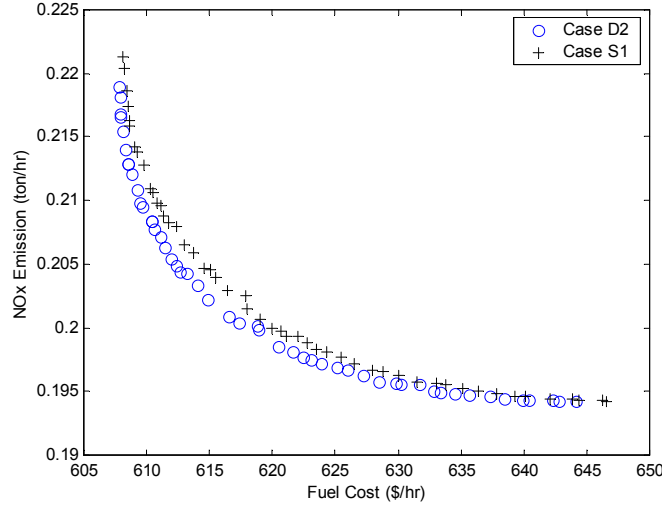
$$\text{and } P_1^{\max} = \bar{P}_1 + 2\sigma_j \bar{P}_1 = 1.2\bar{P}_1$$

$$\text{Calculate } R = \frac{n}{m}$$

Calculate Expected Cost  $\overline{Cost}$  and Expected  $NO_x$   $\overline{NO_x}$  and  
 SD of Cost  $\sigma_{Cost}$  and SD of  $NO_x$   $\sigma_{NO_x}$

### 5.1 Case S1: Stochastic power generated with fixed system load

The multi-objective optimization problem is formulated as above with fixed total system load  $P_D = 2.834$  p.u. Thus, power generated  $P_{Gi}$  are random variables. Fig. 4 shows the nondominated solutions for the stochastic case with fixed system load obtained as compared to the deterministic case.



**Fig. 4.** Nondominated solutions obtained for stochastic power generated with fixed demand (Case S1) as compared to deterministic case (Case D2)

It can be inferred that for solutions excluding the minimum fuel cost and minimum  $NO_x$  emission (i.e. the two extreme points on the curve, optimum values of the two

objectives would be generally worse than the deterministic case. In other words, for pseudo weights excluding (1, 0) and (0, 1), deterministic solutions always dominate stochastic ones.

## 5.2 Case S2: Stochastic power generated and system loads

The multi-objective optimization problem is formulated as above but the individual loads on the system are treated as stochastic variables. Thus, power generated and system loads are random variables. Each of 21 loads is normally distributed with mean  $P_{Li}$  and  $\sigma_i = 0.1P_{Li}$ . Power factor for each load is maintained as at the base load, i.e. ratio  $P_{Li}$  to  $Q_{Li}$  is constant.

Fig. 5 shows the nondominated solutions obtained for the three cases: deterministic (Case D2), stochastic power generated with fixed system load (Case S1), stochastic power generated and system loads (Case S2). It can be observed that the deterministic case shows that the minimum fuel cost obtained is no longer optimal when the decision variables are taken as stochastic. An interesting observation reveals that the minimum  $\text{NO}_x$  emission is not affected by stochastic considerations.

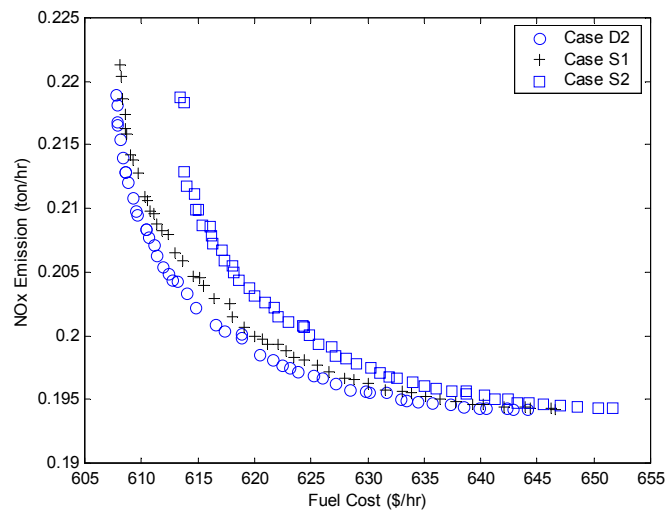


Fig. 5. Nondominated solutions for Cases D2, S1 and S2 on same plot

Fig. 6 shows the variation of average reliability with pseudo weights (1, 0), (0.5, 0.5) and (0, 1) of the two objectives. The average reliability was calculated over 100 runs for each set of decision variables (defining an operating point). This figure clearly verifies the statement above regarding the non-dependability of the  $\text{NO}_x$  emission on the nature of the decision variables for minimum emission level.

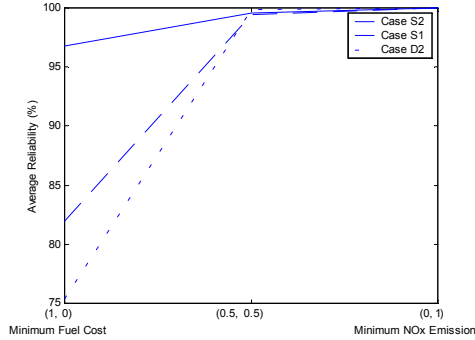


Fig. 6. Average reliability for different pseudo weights for Cases D2, S1 and S2

**5.3 Case S3: Stochastic power generated, system loads, fuel cost and emission coefficients**

This case is similar to Case S2 but in addition the fuel cost and NO<sub>x</sub> emission coefficients are considered as stochastic variables with mean as given in Tables 1 and 2 respectively and standard deviation as 0.1 of their respective means. Fig. 7 shows the nondominated solutions obtained for the stochastic case considering power generated, system loads, fuel cost and emission coefficients considered as random variables compared to the deterministic case as in Case D2.

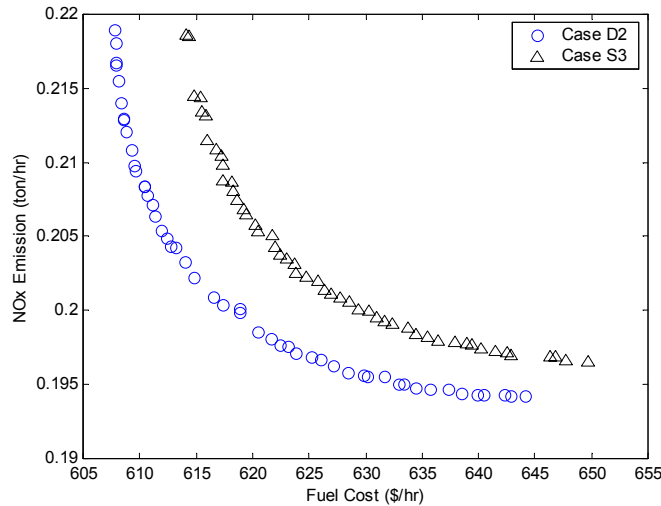


Fig. 7. Nondominated solutions for stochastic power generated, system loads, fuel cost and NO<sub>x</sub> emission coefficients (Case S3) as compared to deterministic case (Case D2)

It can be observed that in this case the nondominated solutions obtained are shifted away from those of the deterministic case, that is, the solutions obtained when power

generated, system loads, cost and emission coefficients are stochastic variables, are all dominated by the deterministic solutions. Therefore, higher cost and emission are expected when the power system is operated in real-world situation. The expected increase in minimum cost and minimum emission are about 6 \$/hr and 0.002 ton/hr. Thus, a 1% increase has been obtained in both objectives when the standard deviation was taken as 10% of the mean value of the variables. It is to be noted that these figures are not negligible when the power system is operated over a year, corresponding figures would be \$52,560 and 17.52 tons respectively.

## **6 Conclusions**

In this paper, the multi-objective environmental/economic dispatch problem has been solved using the elitist Nondominated Sorting Genetic Algorithm. The algorithm has been run on the standard IEEE 30-bus system. Both the deterministic and stochastic approaches have been addressed.

In the deterministic problem, two cases have been studied: (i) the lossless system and (ii) when transmission losses are taken into consideration and given by the load flow solution. In the first case, the minimum cost and minimum emission solutions found by NSGA-II are better than those found by the conventional Linear Programming method. Moreover, these solutions are comparable, if not better than MOSST, NSGA, NPGA and SPEA reported from earlier studies. Considering the transmission losses, similar results were obtained thus confirming the superiority of NSGA-II as a fast evolutionary multi-objective algorithm.

For the stochastic problem, three cases with different complexities have been analyzed, all taking transmission losses into consideration with the following stochastic variables: (i) power generated, (ii) power generated and system loads, and (iii) power generated, system loads and cost and emission coefficients. The following interesting findings can be stated after comparison with the deterministic non-dominated solutions obtained. In the first case, the stochastic solutions obtained are dominated by the deterministic ones except for the two extreme solutions. This means that in practice, real-world operation cost and emission would be always higher except if the power system is operated at either its minimum fuel cost (economic dispatch) or minimum emission (environmental dispatch). In the second case, the minimum emission solution is not affected by stochastic considerations but all other solutions have higher cost for the same emission level. The minimum cost solution being higher than the deterministic one by about 6 \$/hr. The reliability measure used in this study confirms the non-dependence of the emission level from comparison of operating points based on different pseudo-weights of the two objectives. The third case shows that nondominated solutions obtained are shifted away from those of the deterministic case, the minimum cost and minimum emission solutions being higher by about 6 \$/hr and 0.002 ton/hr, respectively. Thus, in real-world situations, the power system would be operated at an operating point, which would have higher fuel cost and higher emission level than the calculated and planned operating point. In other words, the real-world (stochastic) operating point would always be dominated by the deterministic one.

## References

1. Parti, S. C., Kothari, D. P., Gupta, P. V.: Economic Thermal Power Dispatch. Institution of Engineers (India) Journal-EL, Vol. 64 (1983) 126-132
2. Gent, M. R., Lamont, J. W.: Minimum-Emission Dispatch. IEEE Transactions on Power Apparatus and Systems, Vol. PAS-90, No. 6 (1971) 2650-2660
3. Zahavi, J., Eisenberg, L.: Economic-Environmental Power Dispatch. IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-5, No. 5 (1975) 485-489
4. Nanda, J., Kothari, D. P., Lingamurthy, K. S.: Economic-Emission Load Dispatch through Goal Programming Techniques. IEEE Transactions on Energy Conversion, Vol. 3, No.1 (1988) 26-32
5. Dhillon, J. S., Parti, S. C., Kothari, D. P.: Multiobjective Optimal Thermal Power Dispatch. Electrical Power and Energy Systems, Vol. 16, No. 6 (1994) 383-389
6. Abido, M. A.: A Novel Multiobjective Evolutionary Algorithm for Environmental/Economic Power Dispatch. Electric Power Systems Research, Vol. 65 (2003) 71-81
7. Abido, M. A.: A Niche Pareto Genetic Algorithm for Multiobjective Environmental/Economic Dispatch. Electrical Power and Energy Systems, Vol. 25, No. 2 (2003) 97-105
8. Abido, M. A.: Environmental/Economic Power Dispatch using Multiobjective Evolutionary Algorithms. IEEE Transactions on Power Systems, Vol. 18, No. 4 (2003) 1529-1537
9. Ah King, R. T. F., Rughooputh, H. C. S.: Elitist Multiobjective Evolutionary Algorithm for Environmental/Economic Dispatch. IEEE Congress on Evolutionary Computation, Canberra, Australia, Vol. 2 (2003) 1108-1114
10. Viviani, G. L., Heydt, G. T.: Stochastic Optimal Energy Dispatch. IEEE Transactions on Power Apparatus and Systems, Vol. PAS-100, No. 7 (1981) 3221-3228
11. Bunn, D. W., Paschentis, S. N.: Development of a Stochastic Model for the Economic Dispatch of Electric Power. European Journal of Operational Research 27 (1986) 179-191
12. Dhillon, J. S., Parti, S. C., Kothari, D. P.: Stochastic Economic Emission Load Dispatch. Electric Power Systems Research, 26 (1993) 179-186
13. Dhillon, J. S., Parti, S. C., Kothari, D. P.: Multiobjective Decision Making in Stochastic Economic Dispatch. Electric Machines and Power Systems, 23 (1995) 289-301
14. Bath, S. K., Dhillon, J. S., Kothari, D. P.: Fuzzy Satisfying Stochastic Multi-Objective Generation Scheduling by Weightage Pattern Search Methods. Electric Power Systems Research 69 (2004) 311-320
15. Yokoyama, R., Bae, S. H., Morita, T., Sasaki, H.: Multiobjective Optimal Generation Dispatch based on Probability Security Criteria. IEEE Transactions on Power Systems, Vol. 3, No. 1 (1988) 317-324
16. Deb, K., Pratap, A., Agrawal, S., Meyarivan, T.: A Fast and Elitist Multiobjective Genetic Algorithm: NSGA-II. IEEE Transactions on Evolutionary Computation, Vol. 6, No. 2 (2002) 182-197
17. Das, D. B., Patvardhan, C.: New Multi-Objective Stochastic Search Technique for Economic Load Dispatch. IEE Proceedings. C, Generation, Transmission, and Distribution, Vol. 145, No. 6 (1998) 747-752
18. Haimes, Y. Y., Lasdon, L. S., Wismer, D. A.: On a Bicriterion Formulation of the Problems of Integrated System Identification and System Optimization. IEEE Transactions on Systems, Man, and Cybernetics 1 (3) (1971) 296-297
19. Deb, K., Chakroborty, P.: Time Scheduling of Transit Systems With Transfer Considerations Using Genetic Algorithms. Evolutionary Computation 6 (1) (1998) 1-24