In Search of Optimal Operating Principles for An Overhead Crane Maneuvering Using Multi-Objective Evolutionary Algorithms

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Abstract

While operating a crane for maximum productivity, the time of operation and the required energy are two important conflicting factors faced by a crane operator. In such a case, trying to reach the destination too quickly demands a large energy supply, while a small powered motion requires longer time. In this paper, we consider such a problem for two different pairs of objectives and employ a multi-objective genetic algorithm for the task. Besides finding a set of trade-off optimized solutions (operating conditions), an analysis of these solutions reveals salient operating principles, which would be difficult to achieve by other means. The methodology demonstrated in this paper can be used for other similar engineering design and application problems.

Keywords: Crane maneuvering, Multi-objective GAs, Optimal trade-off, Dynamics of cranes.

1 Introduction

Overhead cranes are used in the industries, workshops, factories, docks, and other places to transport the heavy components from one place to another. In order to increase the productivity, individual such operation must be optimized to find what speed the overhead trolley must be moved so that the supplied energy to the crane and the overall operation time are minimum. The overall operation time has two components: (i) the trolley time which denotes the time needed for the trolley to move from the starting position to the destination point and (ii) the sway time which denotes the time needed by the hanging load to damped out its oscillation to a critical acceptable limit. Although not obvious, these two objectives have a conflicting effect. If an operator tries to reach the destination too fast (by spending too much energy) the trolley time will be saved, but the sudden stopping of the trolley will cause the remaining energy to be transferred to the hanging load, thereby starting a large-amplitude oscillation to the hanging load. On the other hand, a careful and slow motion towards the destination point will, though not impart a large-amplitude oscillation to the hanging load, require a larger trolley time. Thus, it is important to know what and how to move the trolley right from the starting position so that certain goals are achieved.

Although there exist a number of classical multi-objective optimization techniques [14, 1, 12], multi-objective evolutionary algorithms (EMO) [3, 15, 2] have gained tremendous popularity in solving different kinds of engineering problems. In this paper, rather than finding one solution to the problem, we employ a multi-objective genetic algorithm — the elitist non-dominated sorting

GA or NSGA-II [6] — to first find a set of trade-off optimal solutions for two conflicting objectives of operation. Thereafter, the obtained solutions are analyzed to reveal important operating principles for the task. By considering two different pairs of objectives, important information about the optimal crane operations are found. The methodology used in this study can be followed in handling similar other engineering problems.

2 Modeling the Dynamics of Crane Operation

Figure 1 shows a schematic model of the crane used in this study. In this simplified model, the

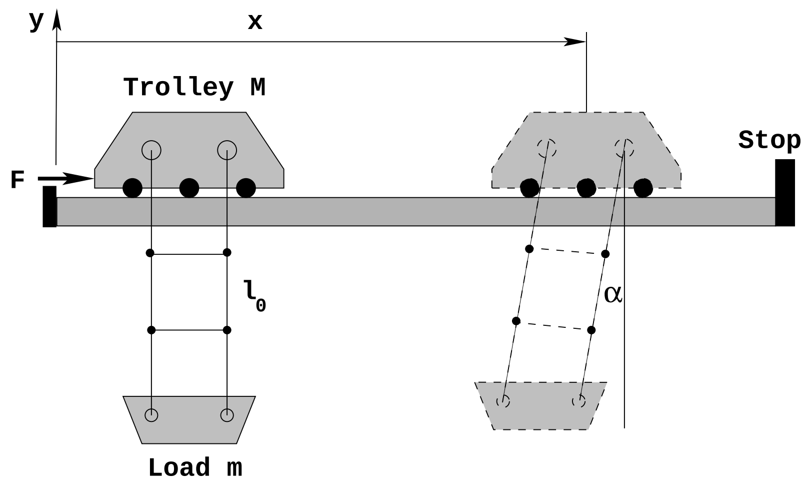

Figure 1: A schematic of the overhead crane consisting of a trolley and a swaying load.

cable connecting the trolley and the hanging load is considered of fixed length, however in practice such cables are varied in length while moving in order to lower or raise the load [11]. The fixed length assumption reduces one degree-of-freedom of the system, however, a similar study without this assumption can also be made. The system has two degrees-of-freedom: (i) x denoting the linear motion of the trolley along x-direction and (ii) α denoting the angular motion of the hanging mass.

In the model, we assume that there is a time-varying force $F(t)$ applied to the trolley in the direction of the destination (along positive x-direction). The trolley experiences a friction force $f = \mu N$ (μ being the coefficient of friction between the trolley and the guide and N is the normal force) opposite to its motion. By considering the force balances in x and y directions of all forces acted on the trolley, we obtain the following two equations:

$$\begin{aligned} M\ddot{x} &= F + 2T\sin(\alpha) - \mu N, \\ N &= Mg + 2T\cos(\alpha), \end{aligned}$$

where, T is twice the tension in each cable, M is the mass (in kg) of the trolley, and $\ddot{x}$ is the acceleration of the trolley in the x-direction. Performing a similar task for the hanging load (of mass m kg), we have the following two equations:

$$\begin{aligned} -2T\sin(\alpha) - c\dot{x}_1 &= m\ddot{x}_1, \\ 2T\cos(\alpha) - c\dot{y}_1 - mg &= m\ddot{y}_1, \end{aligned}$$

where, c is the coefficient of damping arising due to several factors on to the cable and the hanging mass. The variables x_1 and y_1 are the displacement of the hanging load in the x and y directions, respectively.

In addition, the following relationships between trolley and the hanging load motions can be written with variables along x and y directions:

$$\begin{aligned} x_1 &= x + l_o \sin(\alpha), & y_1 &= -l_o \cos(\alpha), \\ \dot{x}_1 &= \dot{x} + l_o \dot{\alpha} \cos(\alpha), & \dot{y}_1 &= l_o \dot{\alpha} \sin(\alpha), \\ \ddot{x}_1 &= \ddot{x} + l_o \ddot{\alpha} \cos(\alpha) - l_o \dot{\alpha}^2 \sin(\alpha). & \ddot{y}_1 &= l_o \ddot{\alpha} \sin(\alpha) + l_o \dot{\alpha}^2 \cos(\alpha). \end{aligned}$$

Here, l_o is the length of the cable, $\dot{\alpha}$ $\ddot{\alpha}$ are the angular velocity and acceleration of the cable. By eliminating T, x_1 and y_1 from the above expressions, we get the following two equations of motion of the trolley and the hanging mass:

$$\begin{aligned} \ddot{x} &= \left[F - c\dot{x}\sin^2(\alpha) + ml_o \sin(\alpha)\dot{\alpha}^2 + mg\sin(\alpha)\cos(\alpha) - f(ml_o\cos(\alpha)\dot{\alpha}^2 \right. \\ &\quad \left. - c\dot{x}\sin(\alpha)\cos(\alpha) - mg\sin^2(\alpha))\right] / (M + m\sin^2(\alpha) - fm\sin(\alpha)\cos(\alpha)), \quad (1) \\ \ddot{\alpha} &= -\left(\ddot{x} + r\dot{x} + g\tan(\alpha)\right)\frac{\cos(\alpha)}{l_o} - r\dot{\alpha}, \quad (2) \end{aligned}$$

where, r is the ratio of c to m. These two equations can be solved using a numerical integration technique and the variation of x and α with time can be found. Here, we use an adaptive scheme for stable solutions to the above equations.

3 Energy and Time Minimizations

A little thought over the problem makes it clear that the two objectives (i) total energy supplied to the system and (ii) the total time for the block-load system to reach at the desired position and stabilize are the two conflicting objectives. The supplied energy will be minimum for the case of moving ever slowly towards the destination. But such a solution will require quite a long time to complete the task. On the other hand, reaching the destination with a large velocity and suddenly stopping at the destination would be a quick way to reach the destination, however some time needs to be spent for the sway of the load to diminish. Although such a solution may not be the quickest overall time solution, there would exist a solution with a reasonable velocity which would minimize the overall time.

In this paper, we use the following parameters for the crane system:

$$\begin{array}{lll} M = 20,000 \text{ kg}, & m = 30,000 \text{ kg}, & \mu = 0.1 \\ l_o = 25 \text{ m}, & c = 50 \text{ N-s/m}. & \end{array}$$

Here, we use the elitist non-dominated sorting genetic algorithm (GA) or NSGA-II [6] for minimizing the above two objectives. NSGA-II is a multi-objective optimization algorithm based on the evolutionary optimization method. Since its development in 2000 [5], NSGA-II has been popularly used for multi-objective optimization. The main features of NSGA-II are as follows:

1. NSGA-II emphasizes the *non-dominated* solutions [14], thereby allowing the GA population to move towards the Pareto-optimal front.

2. NSGA-II emphasizes the *less-crowded* solutions (solutions which do not have too many solutions in the objective neighborhood), thereby allowing a well-distributed set of solutions to be found in a finite-sized GA population.

3. NSGA-II emphasizes the *elite* solutions (solutions which are best in a population based on a partial-ordering of multiple objectives), thereby allowing it to converge near the Pareto-optimal front in a computationally fast manner.

More detail about the working principle of NSGA-II and a source code in C programming language can be obtained from the first author's web site. Constraints are handled by NSGA-II using a constraint-domination approach suggested elsewhere [3]. We use the following NSGA-II parameters: (i) population size = 150, (ii) maximum number of generations = 1,000, (ii) probability of crossover = 0.9, and (iv) probability of mutation = 0.01.

The decision variables in the crane operating problem are the magnitude and sequence of application of forces on the trolley till it reaches the destination. To keep matters simple, we have used a force F_0 and a sequence of Boolean variables to denote the application of the force. A typical NSGA-II solution is as follows:

$$(F_0 \quad (1110010100))$$

Each time step is assumed to be of $\Delta t = 4$ sec duration. Thus, in the above example, the trolley reaches the destination in (10×4) or in 40 sec. The force F_0 is always applied at the beginning of the first time step. Thus, in the binary string representing the sequence of operation, the first bit is always a 1. It is clear that with the above representation scheme, every solution may have a different size of the binary string. To avoid this problem of coding, we maintain a fixed length (of large size, $\ell_{\max} = 150$) string and use the front part of the string. Thus, a NSGA-II solution will have a maximum trolley time of 150×4 or 600 sec. The motion of the trolley-load is simulated with the pattern of application of force as dictated by a string and the corresponding F_0 and as soon as the trolley reaches the destination, the string is not used further.

The force parameter is also treated as a binary string of length $\ell_F = 10$ initialized in the range $[100, 1670]$ N. Thus, the total string length of a NSGA-II solution is $\ell_F + \ell_{\max} = 10 + 150$ or 160. The overall binary string is operated by a single-point crossover and a bit-wise mutation [13].

Three different implementations are adopted for the energy-time minimizations:

Approach 1: The magnitude of force is varied and the pattern of the application of force is kept periodic, such as on, off, on, off, etc (or (1010...)).

Approach 2: The pattern of application of force is varied as a series of Boolean variables (on or off) and the force is kept to a constant value (F_0).

Approach 3: Both the pattern of application of force and the magnitude of force are varied.

In each case, the maximum string length for representing the pattern of force is kept to be 150. Later, we shall discuss two more case studies: (i) the length of hanging load varies as the crane moves and (ii) a braking operation is introduced in the maneuvering strategy. Now, we discuss each of the above three implementations and results obtained by NSGA-II in the following subsections.

3.1 Approach 1: Variation of Magnitude of Force

Here, a fixed pattern of application of force is applied and the magnitude of the force is dictated by a GA solution. Thus, for each solution we keep track of the total work being done to move the trolley to the final destination and call it the energy to be supplied. The second objective is the total time required to reach the destination and to have the load sway to reduce to a permissible value. Here we call the system is stable if the angular sway is reduced to $\alpha_c = 0.002$ rad.

Figure 2 shows the optimized non-dominated front, trading-off the two conflicting objectives. We observe that a small energy solution takes longer time and a fast solution requires large energy. Although such a trade-off which was anticipated at the start of the study, the figure quantifies the terms and shows a number of such trade-off solutions. It then depends on the operator to choose a particular solution depending on the available time and energy at his/her disposal. The figure also shows the initial random population. This population is shown to get a clear idea of the extent of progress of NSGA-II in the objective space.

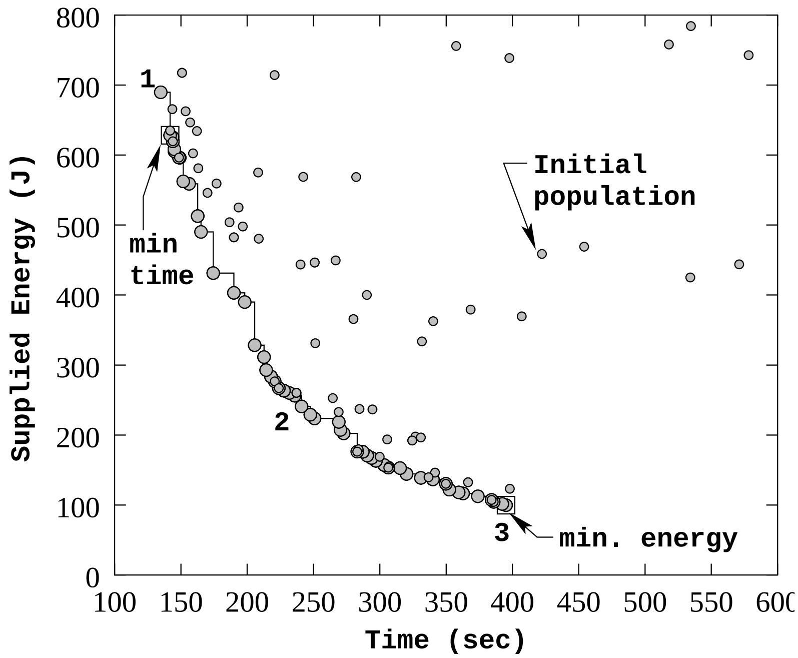

Figure 2: Optimized trade-off solutions for Approach 1 are compared with the initial random population. Single-objective optimized solutions are also marked.

Figure 2 also shows two individual minima for the objectives. Since the individual minima comes on the the optimized front, it can concluded that the obtained NSGA-II solutions are the true non-dominated solutions or are very close to them.

To show the trade-off further, we choose three solutions from the optimized front (marked as 1, 2 and 3 in Figure 2) and show the time-variation of the trolley's velocity with time in Figure 3. Since a periodic pattern is used for applying the force, a periodic variation in the velocity is observed. Solution 1 is the minimum-time solution and hence require a large energy. On the other hand, solution 3 is the minimum energy solution. This solution suggests the smallest velocity of the trolley as it moves towards the destination, but requires the longest time to reach there.

3.2 Approach 2: Variation of Pattern of Application of Force

Here, we keep a constant force $F_0 = 500$ N and vary the pattern of application of force. Such an implementation is quite practical as with a fixed energy source it can be assumed that the force applied to the trolley would be identical at different time steps and the user only needs to know with what sequence the force is to be applied.

Figure 4 shows the obtained optimized front for the same two objectives. The initial population is also marked. The inset figure shows the trade-off between supplied energy and the time more clearly. Individual minima are also shown in the plot. NSGA-II solutions are found to be non-dominated with these solutions.

Some interesting observations can be made when we investigate the force patterns, as shown in Table 1. With the increase in supplied energy the strings get smaller, meaning that the system reaches the destination quickly. In almost all solutions the force is continuously applied in the initial few time steps. Once the trolley has acquired the required energy to overcome the friction and other dynamics, only occasionally the force is required to be applied. A general pattern for an optimal maneuvering seems to be to apply the force early on and let the frictional force reduce

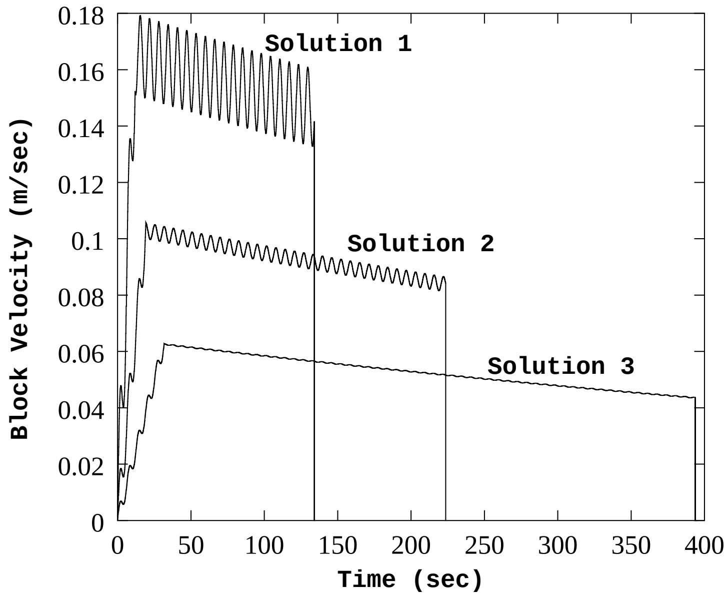

Figure 3: The velocity variation of the trolley as it moves towards the destination for three different non-dominated solutions. The trade-off in their variations is clear.

Table 1: Trade-off solutions and corresponding patterns of application of force.

Time (sec)	Work (J)	Pattern
104.5	1440.45	111111000000000000000000
98.5	1938.64	1111110000001000000000
97.2	2487.75	10111100010001000000100
95.5	2520.04	111111000001000100000
85.8	2531.83	11111100011000000000
83.7	2539.40	11111101010000000000
79.0	3933.45	1111110000111001000
78.5	3937.73	1111110001111000000
78.4	4399.62	1111110001110001000
71.9	4759.73	11111110111000010
71.1	4779.47	11111111110001000
68.0	5670.62	1111111111001100

the motion of the trolley later on and till the trolley reaches its destination. Such a pattern is not hard-coded in NSGA-II. Such a property of the optimized solutions emerge as a desired mode of operation in an optimal manner. Such informations are useful to the operators and can be quite useful in real-world applications [4].

3.3 Approach 3: Variation of Force and Application Pattern

Next, we keep both the force and the application of force pattern as variables. Figure 5 shows the obtained non-dominated front. Once again, the individual minima (obtained using a single-objective GA with identical representation scheme and operators as in NSGA-II) are also shown on the plot. It can be observed that the NSGA-II frontier is non-dominated with these individual

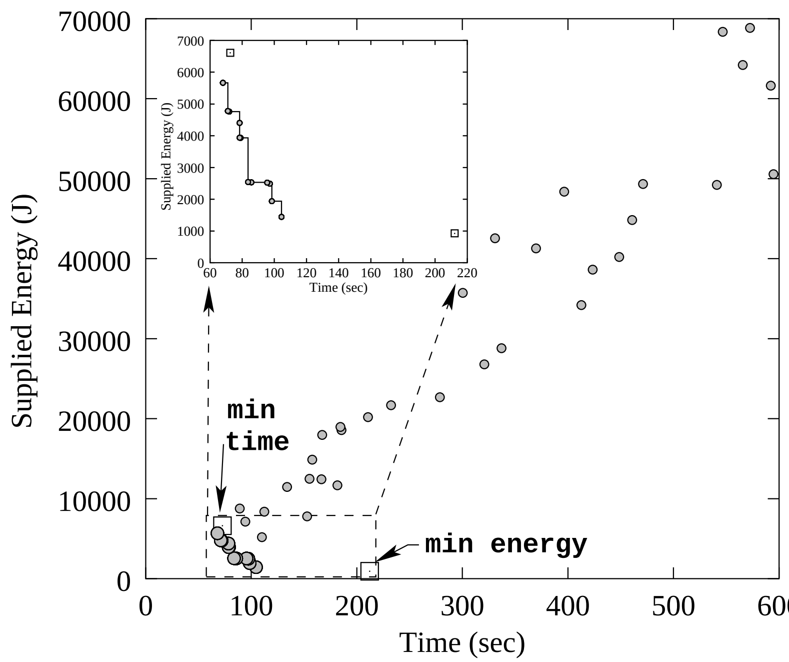

Figure 4: Optimized trade-off solutions for Approach 2 are compared with the initial random population. Single-objective optimized solutions are also marked.

minima. Also, since the representation involves both F_0 and force pattern, a wider non-dominated

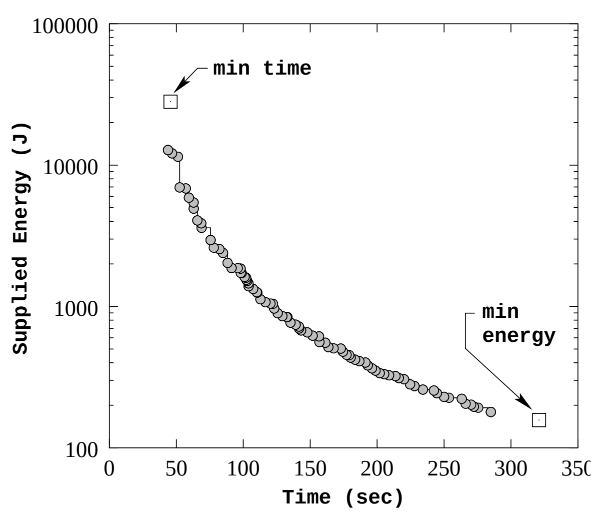

Figure 5: Optimized trade-off solutions for Approach 3. Single-objective optimized solutions are also marked.

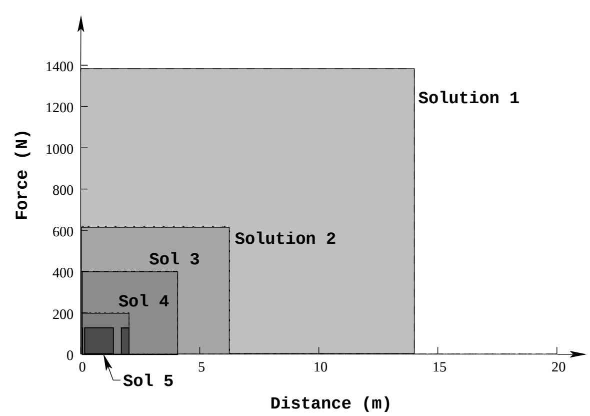

Figure 6: Force versus distance moved by the trolley for five widely-distributed solutions on the obtained front.

front is discovered compared to Figures 2 and 4.

In Table 2, we show the force and its application pattern for a few obtained trade-off solutions. It is clear that quicker solutions require more energy and a larger magnitude of force. Importantly, the force is required to be applied early on and then the trolley moves with its acquired energy till it reaches the destination. Smaller force solutions consume smaller energy and moves slowly

Table 2: Objective values and corresponding solutions for a few non-dominated solutions.

Time (sec)	Force (N)	Pattern
285.0	176.9	(1110111000000000000000000000000000000000000000000000000000000000000000)
266.1	189.2	(11101110000000000000000000000000000000000000000000000000000000000)
213.7	237.0	(111011100000000000000000000000000000000000000000000)
191.3	264.7	(1110111000000000000000000000000000000000000000)
167.5	297.0	(11101110000000000000000000000000000000000)
116.6	432.5	(1110111000000000000000000000)
98.0	484.8	(1111011100000000000000)
75.6	717.3	(111011100000000000)
68.9	792.8	(1110111000000000)
63.0	926.7	(11101110000000)
51.2	1060.7	(11111111000)
43.8	1493.3	(1110111000)

to the destination. A similar pattern was also found in Approach 2.

To show the above pattern and force graphically, we have chosen five solutions from the entire range of the obtained solutions and the force variation is plotted with distance moved by the trolley in Figure 6. Since the area covered by such variations is related to the supplied energy, it is clear that the minimum time solution (solution 1 marked in Figure 6) requires the largest energy to complete the task, while solution 5 requires a small fraction of the energy needed in solution 1 to complete the task, but the time taken to reach the destination is 6.5 times more.

4 Trolley Time and Sway Time Minimizations

Now, we consider two components of the overall time of completion of the task: (i) minimization of the trolley time — time needed by the trolley to reach the destination and (ii) sway time — time needed for the hanging load to stabilize (maximum angular displacement comes within a small limit $\alpha_c = 0.0002$ rad). In this case, we consider $\Delta t = 6.67$ sec and a maximum string length of $\ell_{\max} = 750$, so that a maximum trolley time of 750×6.67 or $5,000$ sec can be achieved. For this case, the a NSGA-II solution is 760 bit long.

In this case, we only show the approach in which both the force and the pattern of application of the force are varied. Figure 7 shows the initial population, obtained trade-off front by NSGA-II, and two individual minima for the objectives obtained using a single-objective GA. A comparison with the individual minima indicates that the obtained NSGA-II solutions are non-dominated with the individual minima.

To get a better idea of the trade-off between these two objectives, we have shown the time-variation of the trolley and hanging load for three solutions – two extreme solutions and one intermediate solution on the obtained front in Figures 8 to 10. The bottom figure shows the variation of the trolley mass. It can be seen that when the trolley reaches 20 m destination mark, it is forced to stop. At that instant, the energy gets transferred to the hanging mass and it started to sway. Its motion decays due to the damping into the system. The critical angular displacements in positive and negative directions are marked with dotted lines in the figure.

The trade-off between the trolley time and the sway time is clear from these figures. Solution 1 requires smallest trolley time (34.6 sec), but gets into a larger amplitude oscillation which requires 4,248.1 sec to get damped to the critical α_c. On other hand, solution 2 (intermediate one) takes slightly more trolley time (37.9 sec), but gets damped out to the limit within 870.6 sec. Solution 3 reaches the destination slowly requiring as large as 88.9 sec. But the slow arrival at the destination

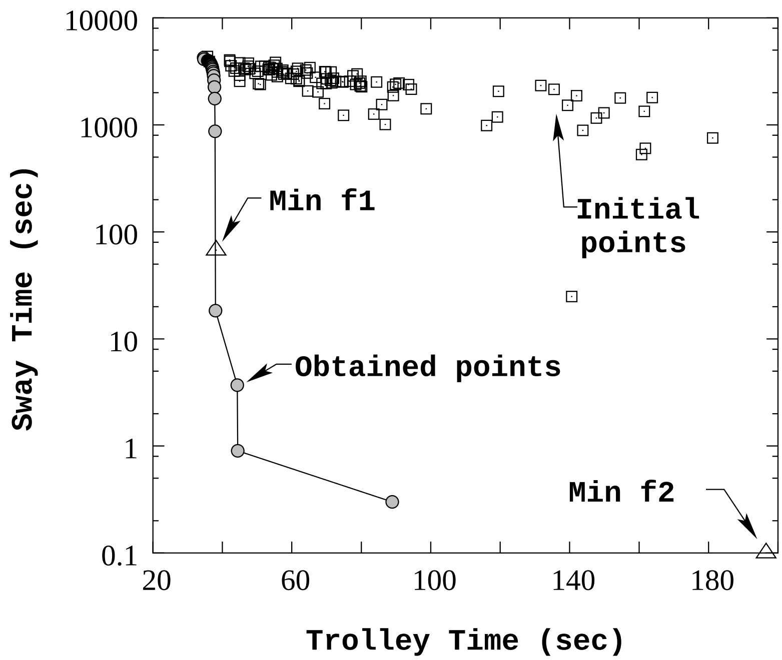

Figure 7: Optimized trade-off solutions for the trolley time ($f1$) and sway time ($f2$) minimizations. Single-objective optimized solutions are also marked.

causes the hanging mass to get damped out to the limit almost immediately (in only 0.3 sec).

Table 3 shows the objective values and corresponding solutions for a few of the obtained trade-off solutions. It can be observed that the applied force is inversely proportional to the elapsed

Table 3: Objective values and corresponding solutions for a few of the non-dominated solutions.

Trolley Time (sec)	Sway Time (sec)	Force (N)	Pattern
34.6	4248.1	1673.46	(11111)
36.5	3800.2	1618.03	(11110)
37.8	1758.1	1413.27	(11111)
37.9	870.6	1404.03	(11111)
38.0	18.4	1402.50	(11111)
44.3	3.7	1031.45	(111111)
88.9	0.3	260.12	(11100101011010011)

trolley time, that is, for a quick (small time) arrival at the destination, more force (hence more energy) must be applied. Although most solutions require an early application of the force, as dictated by the pattern in the table, the smallest sway time requires a careful on-off application of the force till it reaches the destination.

If the time of completing the task is important, the summation of trolley and sway times can be optimized. For the solutions mentioned in the above table, the second-last solution seems to be the optimal solution. Although the above consideration of two-objective minimization would usually include this optimized solution, it also provides other useful information about the problem which would be useful to the operators or users.

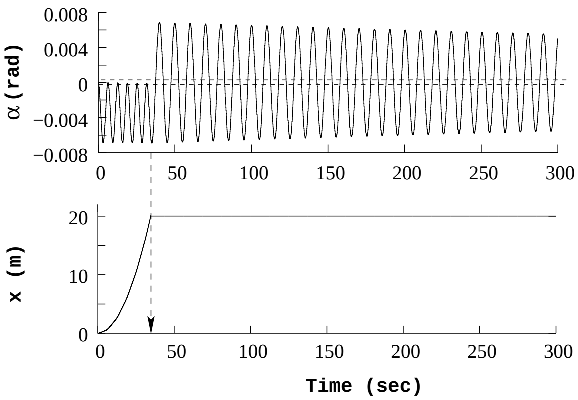

Figure 8: Minimum trolley-time solution (sol. 1). Sway stabilizes after 4,248.1 sec.

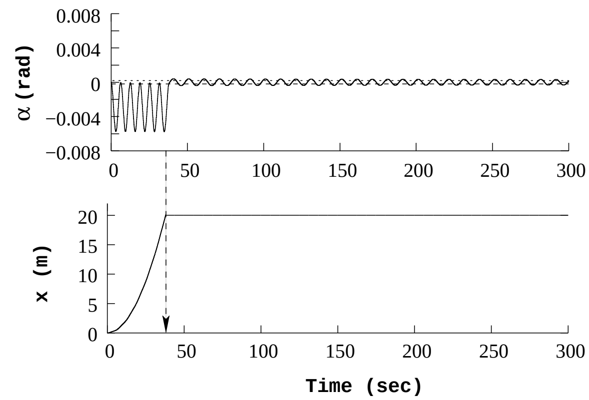

Figure 9: Intermediate solution (solution 2). Sway stabilizes after 870.6 sec.

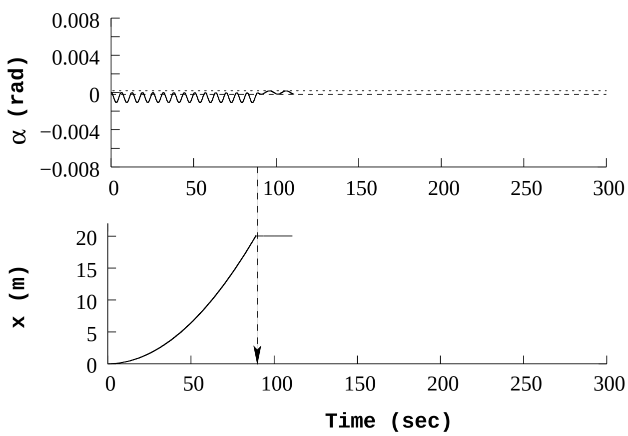

Figure 10: Minimum sway time solution (solution 3). Sway stabilizes after 0.3 sec.

5 Variable-Length Maneuvering

Often, a load is required to be lifted from a higher elevation and placed on to a truck or a railway wagon by lowering the load. In such a scenario, it is desirable to establish the optimal maneuvering strategy for achieving the task for a lesser supplied energy and for a smaller completion time. Here, we extend our dynamic model so as to allow the length of the load to vary in the following manner as a function of position (x) from the starting point of the trolley:

$$l(x) = l_0 + \left(\frac{x}{x_f}\right)^{2^\gamma} (l_f - l_0). \tag{3}$$

The length $l(x)$ is computed downwards and l_0 is the length of the cord connecting the hanging load at $x = 0$ and l_f is the length at $x = x_f$ (the destination of trolley). A condition of $l_f > l_0$ is assumed here. The parameter γ is kept as a variable in the NSGA-II and is allowed to vary within $[-3, 3]$ so that different strategies of lowering the load from initial to the destination point are possible, as shown in Figure 11.

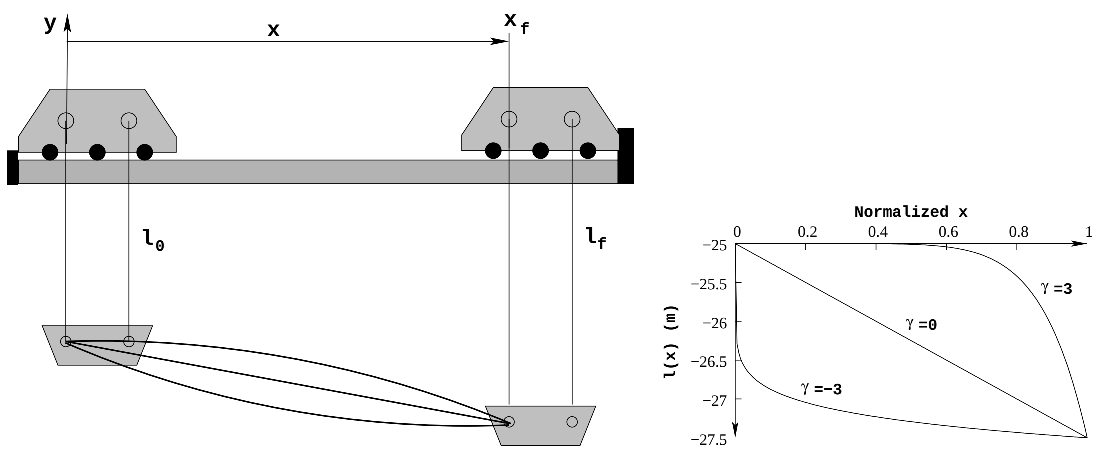

Figure 11: A schematic of the varying-length case study. The right plot shows a true variation with $\gamma = -3$, 0 and 3 with the normalized x (or x/x_f).

For this problem, NSGA-II now has three sets of variables: (i) magnitude of applied force F_0 (varying in $[100, 1675]$ N, coded in a 10-bit string), (ii) pattern of applying force F_0 (coded in a 500-bit string and applying the force at an interval of $\Delta t = 4$ sec, thereby keeping a maximum of 500×4 or 2,000 sec time for the trolley to reach the destination point), and (iii) γ (treated as a real-valued variable to lie within $[-3, 3]$). NSGA-II allows mixed variables (discrete and real) to be handled together. The variable γ is operated by using the simulated binary crossover (SBX) and the polynomial mutation operators [3]. Each of these two operators requires a parameter to be set specifying the extent of search and here we use standard values of $\eta_c = 5$ and $\eta_m = 20$ for crossover and mutation, respectively. Like before, the time of completing the task is computed by summing the time required for the trolley to reach the destination and the time required for the hanging load to reach a small angle of sway of $\alpha_c = 0.0002$ rad. NSGA-II is run with a population of size 100 and the non-dominated solutions found after 1,000 generations are shown in Figure 12.

The trade-off between the two objectives of minimizing supplied energy and minimizing overall time of completing the task is clear from the figure. On the figure, we have also shown the individual minimum solutions of each objective obtained using a single-objective GA. The minimum-energy NSGA-II solution is slightly better than the minimum-energy single-objective

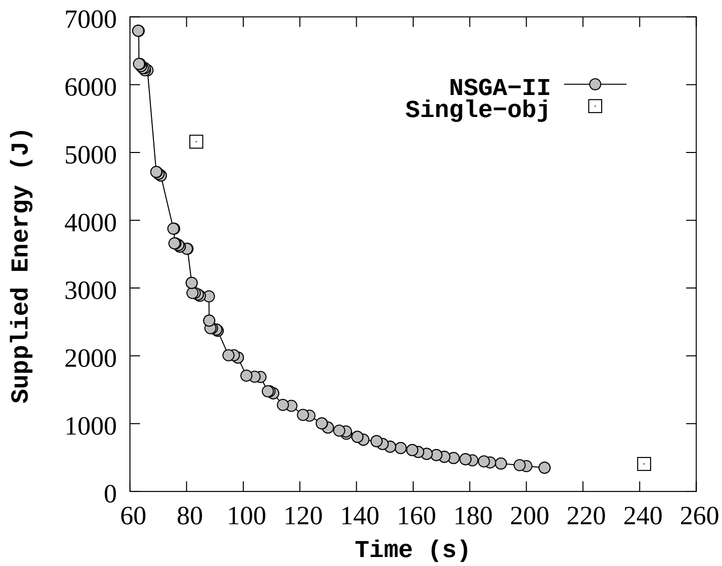

Figure 12: Non-dominated solutions for the varying length study are compared with the fixed-length study.

solution. However, the minimum-time solution obtained using the single-objective GA is not at all close to the true optimum solution, as NSGA-II is able to find much better minimum-time solution among its non-dominated solutions. If a particular objective function is difficult to optimize in the presence of constraints, the use of two or more objectives make the search process more flexible. In the past, better solutions are also reported using a multi-objective optimization method in such cases [7, 10].

Table 4 shows the variables (force, pattern of applying force and γ) of a few well-distributed set of non-dominated solutions. Although the variations in force and its application patterns are

Table 4: Variables for optimal solutions in the varying-length study arranged in increasing order of supplied energy.

γ	Force (N)	Pattern
2.98	149.26	111111110010001000000000000000000000000000000000000
3.00	187.76	11111111001000100000000000000000000000000
3.00	238.56	111111110010001000000000000000000
3.00	327.86	11111011001001100000000000
2.98	427.93	111110110010011000000
3.00	544.94	11111111001000100
2.87	629.62	111111110010001
2.88	818.99	110111010010001

similar to that observed in the previous case studies, an extremely interesting observation can be made from the variation in γ values. Although this variable can take any value within $[-3, 3]$, all solutions seem to have been fixed close to its upper bound (≈ 3.00). A plot of this variation ($\gamma = 3$ marked on the right plot in Figure 11) reveals that all non-dominated solutions portrays a strategy in which the load should not be dropped early on during the course of the trolley movement and should be lowered only at the end. Such a strategy emerging out from all optimal solutions is certainly interesting and can be explained easily when the phenomenon is revealed.

Assuming the hanging load to be an ideal pendulum, we can equate the supplied energy (E) to the maximum change in potential energy and obtain the following relationship between the required energy E to the length of pendulum:

$$E = mg(1 - \cos\theta)l. \tag{4}$$

For a fixed termination condition of a critical $\theta = \alpha_c$, a pendulum having a larger length requires more supplied energy. Thus, for minimal energy consideration a smaller length is better. This is the reason why the optimal strategy for energy-time considerations is to keep the length fixed to its lower value (l_0) initially and then lower the load as late as possible.

We would like to highlight here that the NSGA-II procedure used here is capable of finding such interesting and useful operating principles in a problem and are demonstrated to achieve the same in other problems as well [8, 9].

6 Adding a Braking Operation

So far, we have only considered two options in the pattern of applying the force: (i) apply the force (represented by a `1`) and (ii) do not apply the force (represented by a `0`). Here, we add another option of applying a brake (represented by a `2`) in a time step. This way the string representing a force application pattern will now be a ternary string, taking one of the three options. Identical NSGA-II algorithm can be used, except by using a mutation scheme in which the existing bit value is changed to one of the other two options with a equal probability of 0.5. If a brake is to be applied, an application force of $-F_0$ is used.

To demonstrate the flexibility of NSGA-II's working principle, we apply this modified approach only to the varying length problem discussed in the previous section. The problem parameters and NSGA-II parameters are kept the same as in the previous study. The non-dominated front obtained by NSGA-II is similar to that obtained in the previous section, however an interesting observation is made from obtained solutions shown in Table 5. Although three options (force (`1`),

Table 5: Variables for optimal solutions in the braking option. The braking option (`2`) hardly appears in the obtained patterns of force application.

Energy (J)	Time (s)	γ	Force (N)	Pattern
282.24	228.70	3.00	133.87	11111111001000100000000000000000000000000000000000000000
427.01	187.20	2.99	164.66	1111111100100010000000000000000000000000000000
601.65	162.20	2.99	195.45	111111110010001000000000000000000000000
884.78	136.30	2.99	237.02	111111110010001000000000000000000
1116.66	123.40	3.00	266.28	111111110010001000000000000000
1688.62	106.10	3.00	327.86	11110110010011000000000000
2926.70	82.90	3.00	431.01	11111111001000100000
4833.20	62.90	3.00	689.66	111111110000000
9375.87	52.30	2.87	883.65	1211111100100
13364.44	42.70	2.82	1528.74	1101110100

no force (`0`) and braking (`2`)) are considered at each time step, all optimized trade-off solutions did not choose the braking option. Since braking can be considered as a procedure in which energy is wasted by friction, the time which may have been saved with a braking operation may be better achieved with an alternate application of a small-magnitude force. This is not quite an intuitive matter and NSGA-II is able to arrive at such a strategy by clearing the braking option from all initial randomly-created solutions and finding a set of optimized solutions without recommending any braking operation. Once again, a large positive value of γ (close to the upper bound of 3.00) turns out to be an optimal strategy for maneuvering the crane.

7 Conclusions

In this paper, we have attempted to find optimal operating conditions of an overhead crane in carrying a load over a distance. First, the task has been optimized for two conflicting goals of design: the supplied energy and the task completion time. Using a multi-objective GA (NSGA-II), we have obtained a number of trade-off solutions. It has been observed that an operation requiring minimal time of completion demands for a large supplied energy; on the other hand, an operation requiring minimal energy demands for a longer time of completion. In each case, an optimization of individual objectives has been performed to build confidence on the obtained non-dominated solutions. Two other case studies have been considered to demonstrate the flexibilities of using NSGA-II. In one case, the length of the hanging load is varied as the overhead crane moves towards its destination and in the other case a braking option is also introduced in the maneuvering.

An investigation of the obtained trade-off solutions reveal the following important operating principles:

1. The 'bang-off' force model used in the study requires the forces to be applied early on so that the system acquire enough energy to complete the task. Although not obvious, such a strategy would enable the trolley to reach its destination with a minimal energy so that when stopped suddenly at the destination the hanging load does not sway much.

2. The applied force is inversely proportional to the time to reach the destination.

3. When a load needs to be lowered from its initial position to a final destination, the optimal maneuvering strategy is to delay the lowering of the load as much as possible from energy and time minimization point of view.

4. A braking option in the maneuvering strategy is not an optimal choice from energy and time minimization point of view.

In another case study, the trolley time and the sway time are minimized using NSGA-II and a trade-off relationship between them is also observed.

Although the application study considered here is a specific one related to the overhead crane operating conditions, the methodology used here can be used in other engineering design and applications. A consideration of more than one objective (with a conflict in them) in the optimization process is expected to produce a set of trade-off solutions. An investigation of such trade-off solutions should reveal important information about the problem, which may be difficult to obtain by any other means.

References

[1] V. Chankong and Y. Y. Haimes. *Multiobjective Decision Making Theory and Methodology.* New York: North-Holland, 1983.

[2] D. Corne, J. Knowles, and M. Oates. The Pareto envelope-based selection algorithm for multiobjective optimization. In *Proceedings of the Sixth International Conference on Parallel Problem Solving from Nature VI (PPSN-VI)*, pages 839–848, 2000.

[3] K. Deb. *Multi-objective optimization using evolutionary algorithms.* Chichester, UK: Wiley, 2001.

[4] K. Deb. Unveiling innovative design principles by means of multiple conflicting objectives. *Engineering Optimization*, 35(5):445–470, 2003.

[5] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. In *Proceedings of the Parallel Problem Solving from Nature VI (PPSN-VI)*, pages 849–858, 2000.

[6] K. Deb, S. Agrawal, A. Pratap, and T. Meyarivan. A fast and elitist multi-objective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, 6(2):182–197, 2002.

[7] K. Deb, P. Jain, N. Gupta, and H. Maji. Multi-objective placement of VLSI components using evolutionary algorithms. *To appear in IEEE Transactions on Components and Packaging Technologies*, Also KanGAL Report No. 2002006, 2002.

[8] K. Deb and S. Jain. Multi-speed gearbox design using multi-objective evolutionary algorithms. *ASME Transactions on Mechanical Design*, 125(3):609–619, 2003.

[9] K. Deb, K. Mitra, R. Dewri, and S. Majumdar. Towards a better understanding of the epoxy polymerization process using multi-objective evolutionary computation. *To appear in Chemical Engineering Science (Also as KanGAL Report No. 2004001)*, 2004.

[10] K. Deb and S. Tiwari. Multi-objective optimization of a leg mechanism using genetic algorithms. Technical Report KanGAL Report Number 2004005, Kanpur Genetic Algorithms Laboratory, IIT Kanpur, India, 2004.

[11] G. Dissanayake and G. Fang. Minimum-time trajectory for reducing load sway in quay-cranes. *Engineering Optimization*, 33:643–662, 2001.

[12] M. Ehrgott. *Multicriteria Optimization.* Berlin: Springer, 2000.

[13] D. E. Goldberg. *Genetic Algorithms for Search, Optimization, and Machine Learning.* Reading, MA: Addison-Wesley, 1989.

[14] K. Miettinen. *Nonlinear Multiobjective Optimization.* Kluwer, Boston, 1999.

[15] E. Zitzler, M. Laumanns, and L. Thiele. SPEA2: Improving the strength pareto evolutionary algorithm for multiobjective optimization. In K. C. Giannakoglou, D. T. Tsahalis, J. Périaux, K. D. Papailiou, and T. Fogarty, editors, *Evolutionary Methods for Design Optimization and Control with Applications to Industrial Problems*, pages 95–100, Athens, Greece, 2001. International Center for Numerical Methods in Engineering (Cmine).