A Comparative Study of Pareto-front of Optimal Solutions Set for NAO Robot's Gait Optimization Using the Dominance Move Indicator based on Mixed Integer Programming

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Abstract. Evolutionary multi-objective optimization (EMO) algorithms generate solution sets representing tradeoffs between conflicting objectives. The comparison between two EMO algorithm performances is challenging for real-world problems lacking reference Pareto fronts (PFs). Most performance indicators require a reference PF or point to compare the Pareto optimal solutions. This study uses Dominance Move (DoM), formulated as a Mixed Integer Programming (MIP), as it compares sets without any reference PF or points and avoids information loss. The GD+, IGD+, HV, ϵ, and MIP-DoM indicators have been used to compare solution sets from two popular EMO algorithms– NSGA-II and MOPSO. EMO algorithms are used to minimize power consumption and maximize stability for a 25-DOF humanoid robot gait optimization problem. The results show that NSGA-II outperforms MOPSO on this highly-constrained problem. The MIP-DoM exhibits the strongest correlation with the IGD+ indicator, whereas weaker correlations are seen for the Hypervolume and Epsilon indicators. The EMO performance has also been tracked over generations using IGD+, which provides additional insight into algorithm dynamics. The proposed techniques could be extended to other real-world optimization problems.

Keywords: Evolutionary Algorithms · NAO Humanoid Robot · Multi-objective Optimization · Performance Indicators · Dominance Move

1 Introduction

Designing and controlling humanoid robots that mimic human-like movements and perform various tasks in real-world environments is a challenging problem. Achieving efficient and stable walking with bipedal locomotion is a complex problem, as it requires different computations for energy and stability during the single support phase (SSP) and the double support phase (DSP) [3]. Power consumption and stability are two essential objectives for optimal bipedal locomotion, but they have a trade-off relationship [9]. Researchers have mostly focused on finding a balance between them to achieve optimal bipedal locomotion. Evolutionary multi-objective optimization (EMO) can generate a set of optimal solutions representing different trade-offs between multiple objectives for complex bipedal walking patterns, unlike traditional methods, which get stuck in local minima.

Researchers have been widely using different EMO algorithms to improve gait planning for humanoid robots [13,9,8]. However, not all EMO algorithms are equally effective for each problem. According to the No Free Lunch theorem [12], no single algorithm can outperform all other algorithms on all problems. Therefore, algorithm selection is essential in applying EMO to robotics problems. Several researchers had compared different EMO algorithms on humanoid robot problems to find the best algorithms for their problem [13,10]. Various performance indicators [6], such as Generational Distance Plus (GD+), Inverted Generational Distance Plus (IGD+), Hypervolume (HV), and Epsilon (ϵ)- indicator, can be used to compare the quality of the solutions generated by different EMO algorithms. However, these indicators have drawbacks, such as requiring a reference set (in GD+ and IGD+), a reference point and normalization (in HV), or considering only one solution during comparison (in ϵ). Moreover, GD+ and IGD+ are hard to extend for many objectives, whereas HV and ϵ are computationally expensive with increasing objectives. Therefore, this study adopts Dominance Move (DoM) based on Mixed Integer Programming (MIP) [7] as a performance indicator, which does not have these limitations. The details of DoM are given in section 3.

The present study extends our prior work on single-objective optimization [3] to a multi-objective optimization (MOO) problem on efficient and stable gait generation. This study focuses on the 25-DOF NAO humanoid robot, a small robot developed by SoftBank Robotics. A constrained MOO problem has been formulated for minimizing power consumption while maximizing the dynamic stability margin within the torque fluctuation limits. The two most popular EMO algorithms, namely elitist non-dominated sorting genetic algorithm (NSGA-II) [2] and multi-objective particle swarm optimization (MOPSO)[1], have been used to solve the MOO problem for both SSP and DSP separately. MIP-DoM is used to evaluate the Pareto-front optimal solutions obtained by each EMO algorithm. IGD+, which has the highest correlation coefficient with MIP-DoM, is also used as a running metric to analyze the best run of each algorithm. This metric measures the IGD+ value at each generation and captures the progress of the algorithms over time. This work will provide new insights into the selection of EMO algorithms. The proposed techniques could generalize to other real-world optimization problems with complex constraints and tradeoffs.

The rest of the paper is organized as follows: Section 2 discusses the NAO robotics problem and introduces the multi-objective optimization problem formulation. Section 3 presents DoM based on MIP. Section 4 discusses the results, and the conclusions are presented in Section 5.

2 Problem Definition

This section introduces the multi-objective optimization (MOO) problems of gait generation for the NAO robot. The kinematic analysis and design of gait parameters for NAO's walking cycle are carried out based on our prior work on single objective optimization [3].

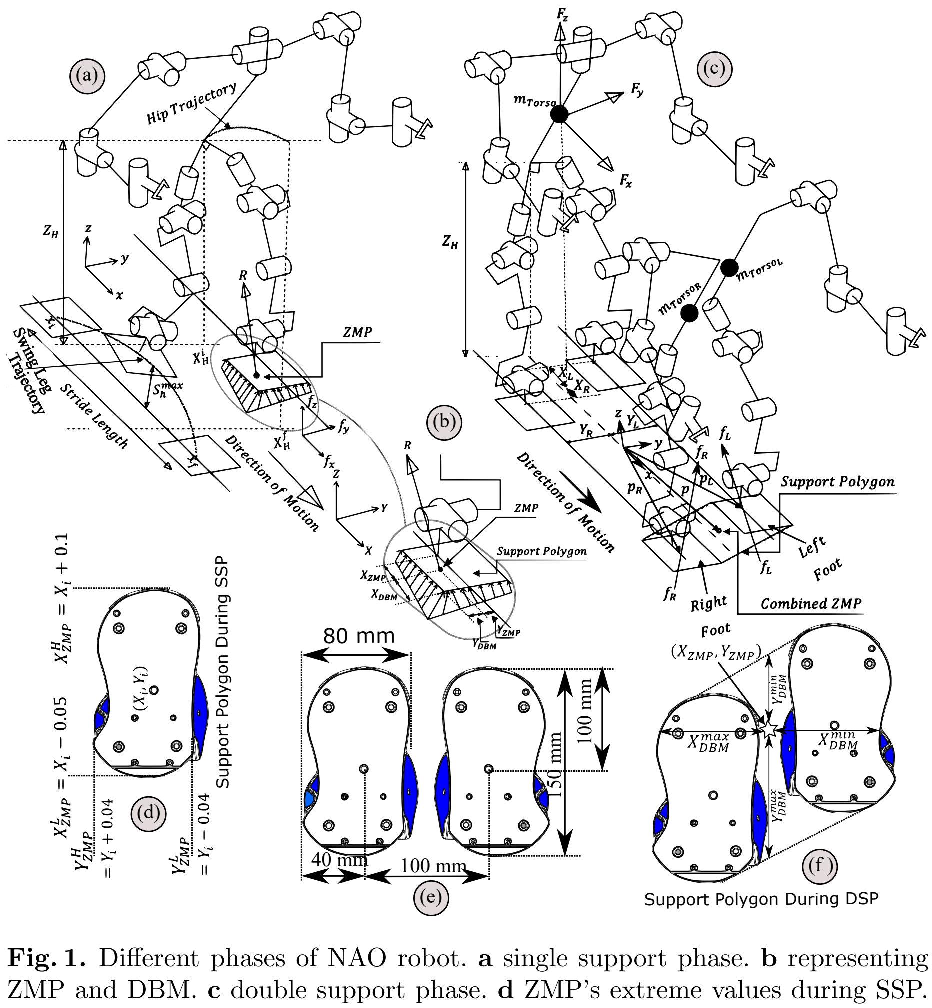

Fig. 1. Different phases of NAO robot. **a** single support phase. **b** representing ZMP and DBM. **c** double support phase. **d** ZMP's extreme values during SSP. **e** foot dimensions. **f** DBM values during DSP.

The SSP and DSP of the walking cycle for the NAO robot were defined in our previous work. Here, the same problem has been extended to handle multiple objectives: power consumption and stability. The kinematic model is developed based on the DH parameters, and the mass and dimensions of the NAO robot are obtained from the official websites. The hip moves from $[0.07, Y_H, Z_h]^T$ to $[0.11, -Y_H, Z_h]^T$ during DSP, and the movement of the hip during SSP is from $[x_i + x_f/4, Y_H, Z_h]^T$ to $[x_i + 3x_f/4, Y_H, Z_h]^T$. Y_H is kept equal to 0.025. The support leg during SSP is kept fixed at $[x_i + x_f/2, 2Y_H, 0]^T$. The swing leg moves from $[x_i, -2Y_H, 0]^T$ to $[x_f, -2Y_H, 0]^T$. The step length (S_L) is taken as 0.06 m. x_i and x_f are the initial and final positions in the x-direction, as shown in Fig. 1 (a), along with the hip height Z_h and maximum swing height S_h^{max}.

The corresponding joint angles for the legs are then found by applying inverse kinematics [3]. These joint angles are then used to calculate the required joint torques using the Lagrange-Euler formulation and later to evaluate the power consumption. The concept of zero moment point (ZMP) [11] is used to check, whether the NAO robot is in the dynamic equilibrium or not, and to prevent it from falling on the ground. The external force experienced by the robot is calculated from the equation of translational motion of the robot. The resultant force R, shown in Figs. 1(a) and (b), should be within the foot support polygon to maintain stability during the motion. Figs. 1(d) and (e) show the extreme ZMP values based on the foot location and its dimensions. Fig. 1(c) shows the NAO robot model during DSP. The DSP, being a closed-loop structure, is simplified into two SSPs for the analysis. The torso mass: m_{Torso}, acting on both legs during DSP, is distributed in two parts according to their sagittal and lateral distances from both feet. These distances are shown by $[X_L,\ X_R]^T$ and $[Y_L,\ Y_R]^T$ in the figure. Instead of one ZMP as in the case of SSP, during DSP two ZMP positions: p_L and p_R are determined from each SSP, as shown in Fig. 1(c), and, later, these are combined to find the ZMP of the whole robot by putting the moments' x and y components about point p to zero [5]. The points: p_L and p_R are the ZMPs for the left and right legs, respectively. DBM during DSP is determined by considering the minimum DBM available in any direction, as shown in Fig. 1(f). This approach was applied to the NAO robot by Gupta et al. [3], and their work provides more details on this analysis.

The conflicting objectives of power consumption and stability margin, with eight functional constraints, as a multi-objective optimization problem during SSP and DSP separately have been formulated. The average power P to be minimized is determined with the heat loss over the time period T for the j^{th} joint given in (1). The second objective function considers maximizing the stability margin given in (2). We retain all the gait parameters from our previous study[3], and four more parameters are added to control the arm movement. Therefore, we have 13 gait parameters for SSP and 12 for DSP (due to the absence of a swing height parameter). During DSP, the rest of the decision parameters $(x_1 - x_7)$ are similar to SSP, and the next five decision variables' lower and upper limits are taken as $[0.2, 0, 1, -0.50, -1.50]^T$ to $[2, \pi/2, 2, -0.04, -0.2]^T$, respectively. The first two constraints (g_1(x) and g_2(x)) given as $\theta^j \geq \theta^j_{min}$ and $\theta^j \leq \theta^j_{max}$ allow the joint movement between θ^j_{min} to θ^j_{max} for the j^{th} joint.

$$\underset{\mathrm{x}\in X\subseteq\mathbb{R}^k}{\text{minimize}} \quad P(\mathrm{x}) = \sum_{j=1}^{n} \frac{1}{T} \int_0^T \left(\left| \tau_j \dot{\theta}_j \right| + 0.025 \times \tau_j^2 \right) dt, \tag{1}$$

$$\underset{\mathrm{x}\in X\subseteq\mathbb{R}^k}{\text{minimize}} \quad 1/X_{DBM}, \tag{2}$$

$$\text{subject to Constraints } \ g_1(\mathrm{x}) \text{ through } \ g_8(\mathrm{x})$$
$$x_i^L \leq x_i \leq x_i^U, \quad \text{with}$$

x_i^L = [0.25, -0.05, -0.05, 0.001, 0.001, -0.2, -0.2, 0.015, 0.4, 0, 1, 0.04, 0.2 $]^T$, x_i^U = [0.31, 0.05, 0.05, 0.2, 0.2, 0.2, 0.2, 0.030, 4, $\pi/2$, 2, 0.5, 1.5 $]^T$, $\theta^j_{min} = -$[1.14, 0.38, 1.53, 0.09, 1.19, 0.39, 0.79, 1.53, 0.10, 1.18, 0.76 $]^T$, and, θ^j_{max} = [0.74, 0.79, 0.48, 2.11, 0.92, 0.76, 0.38, 0.48, 2.12, 0.93, 0.39 $]^T$.
The symbol: Avg ($\Delta\tau_{ij}^{max}$) represents the average of maximum torque value in the j^{th} joint for the i^{th} time interval given in Eq. Avg($\Delta\tau_{ij}^{max}$) $\leq$ 0.4 Nm is the third constraint g_3(x). The fourth g_4(x) and fifth g_5(x) constraints in Eqs. ${}^{\min}X^i_{DBM} \geq 0.001$ m and ${}^{\min}Y^i_{DBM} \geq 0.001$ m are the minimum required DBM in x and y directions during the motion. The next two constraints (g_6(x) and g_7(x)) in Eq. ${}^L\theta^f_{SP} \geq {}^L\theta^i_{SP}$ and ${}^L\theta^f_{ER} \geq {}^L\theta^i_{ER}$ control the correct arm movement. The last constraint g_8(x) considers the minimum average DBM in y direction (${}^{min}Y^{Avg}_{DBM}$) in Eq. $Y^{Avg}_{DBM} \geq {}^{\min}Y^{Avg}_{DBM}$ during SSP and DSP are considered as 0.02 m and 0.05 m, respectively.

3 Dominance Move

This section introduces the DoM indicator based on MIP, which can be extended to many-objective optimization problems. DoM does not require any reference set or point, and it considers all the important aspects of solution quality, such as convergence, diversity, uniformity, and cardinality. MIP-DoM has a physical meaning and applies to many-objective optimization problems. It uses all the information from both sets, thus also avoiding information loss, unlike the Epsilon indicator, which only uses one solution per set. However, DoM is difficult to compute, especially for the higher dimensions. Therefore, a MIP approach is used to calculate DoM efficiently and exactly for two or more objectives. The MIP-DoM approach has already been tested on the benchmark problem [7]; however, a comparison with other performance indicators for a real-world robotics problem will provide more information on its use. MIP-DoM measures the total sum of each solution's movement in a set that is required to dominate another set. MIP-DoM is a binary indicator that compares two Pareto optimal sets, A and B. The DoM of A to B can be denoted by DoM(A, B) and if DoM(A, B) is smaller than DoM(B, A), then set A is better than set B. Gurobi (version 9.5.2 build v9.5.2rc0) [4] software is used to solve the MIP-DoM problem.

4 Results and Discussion

The NSGA-II and MOPSO algorithms are used to solve the NAO robot's multi-objective problem in SSP and DSP scenarios. The algorithms are run six times with a population size of 100 for 200 generations, and the non-dominated solution sets from the first to the last generation are stored. The non-dominated solution sets of the final generation obtained from each run are shown in Figs. 2 (a) and (b) for the SSP and DSP problems, respectively. The blue and red dots represent the NSGA-II and MOPSO solutions set, respectively. The SSP problem reveals a single high trade-off region, known as the knee region, while the DSP problem reveals two knee regions. Rectangles enclose these key regions in the figure, which offer better trade-offs than the rest of the Pareto front. Since the GD+, IGD+, and epsilon indicators require a reference Pareto front to compute their values, and this is unavailable for these real-world problems, all the Pareto fronts are combined to extract the non-dominated solution set, shown as the approximated Pareto set with green dots for both problems.

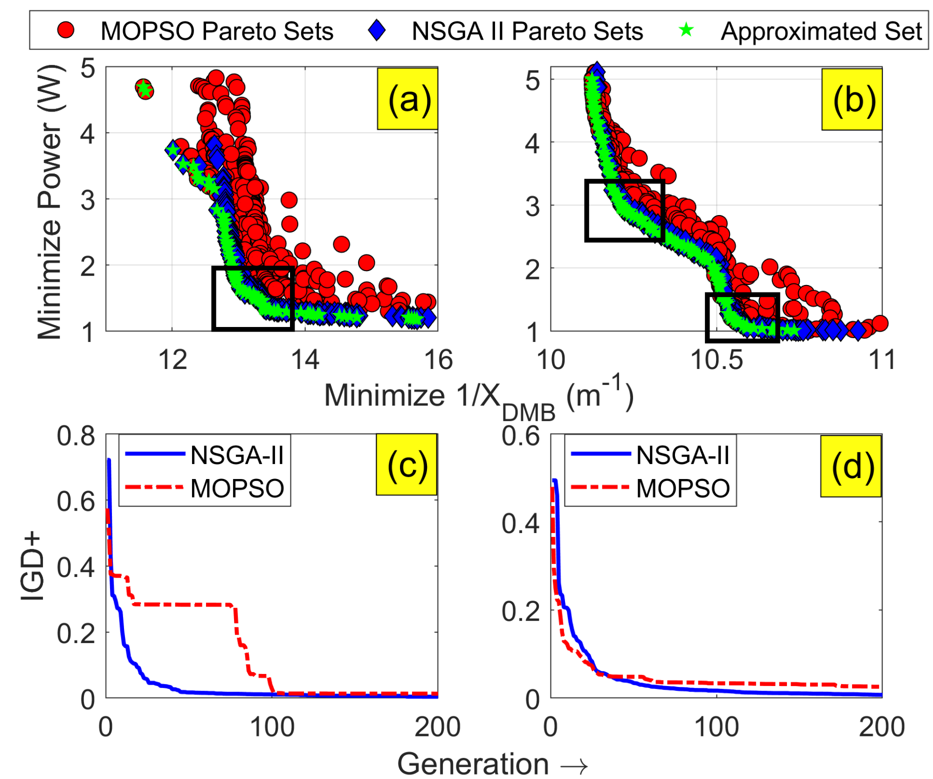

Fig. 2. Approximated Pareto front using NSGA-II and MOPSO algorithm during **a** SSP problem and **b** DSP problem. Running IGD+ metric during **c** SSP problem and **d** DSP problem.

Table 1 shows the MIP-DoM and other indicator values for both phases. The MIP-DOM (A, B) values are calculated by considering set A as the solution generated from the NSGA-II or MOPSO algorithms and set B as the approximated solution set. The GD+, IGD+, and ϵ values are calculated using set B as the reference set. The HV is computed using $[1, 1]^T$ as the reference point. Lower minimum values of all the indicators except HV show better performance of the EMO algorithm since maximum HV indicates better performance. The GD+, IGD+, HV, and epsilon indicator values are found after normalizing set A with respect to set B.

Table 1 presents the average value of the six runs obtained by each indicator. Moreover, a correlation analysis is also performed to assess the relationship between MIP-DoM and other indicators. All six values obtained from each indicator are passed to the MATLAB *corrcoef* function to find the correlation of MIP-DoM with other indicators. HV is taken as a negative to properly align with the other indicators. IGD+ showed a high correlation, while ϵ showed the least correlation due to information loss, as it mainly relates to one solution rather than considering the entire Pareto front set. A hypothesis test with a significance level of 0.05 is also carried out. Values marked with $*$ are found to be statistically significant. MIP-DoM demonstrated promising results as a performance indicator. It performed consistently compared to HV and epsilon indicators, which can be affected by reference point selection and single solution consideration, respectively.

Table 1. MIP-DOM(A, B) values for SSP and DSP problem when compared between NSGA-II and MOPSO algorithms. A is the generated solutions set, but B is the approximated solution set. GD+, IGD+, HV, and ϵ values are also presented. MIP-DoM correlation with other indicators is also given.

Problem	Algorithms	MIP-DoM	GD+	IGD+	HV	ϵ-Indicator
SSP	NSGA-II	1.730	0.011	0.022	0.682	0.238
	MOPSO	2.490	0.123	0.098	0.580	0.259
DSP	NSGA-II	0.439	0.007	0.010	0.823	0.032
	MOPSO	0.617	0.041	0.038	0.760	0.128
	Pearson correlation coefficient (ρ) between MIP-DoM and other indicators					
			GD+	IGD+	−HV	ϵ-Indicator
SSP	NSGA-II		0.393	0.967*	0.780	0.610
	MOPSO		0.956*	0.953*	0.830	−0.169
DSP	NSGA-II		0.112	0.846	0.798	0.575
	MOPSO		0.937	0.982*	0.130	0.097

Although MIP-DoM is a superior indicator, it is computationally costly for use as a running performance metric. Therefore, IGD+ is used to track the performance of the best-run generation-wise for both EMO algorithms during their respective problems. The IGD+ indicator correctly tracked the algorithms' progress and stagnation in solving the given robotics problem using the approximated set as a reference. Figs. 2(c) and (d) show the SSP and DSP results of the best runs obtained from the NSGA-II and MOPSO algorithms. The results reveal that NSGA-II consistently performed better in both cases at achieving superior convergence and diversity, outperforming MOPSO. MOPSO stagnated during SSP from generation 20 to 70 without any solution improvement, as shown by the horizontal line in the IGD+ plot in Fig. 2(c). Interestingly, MOPSO demonstrated rapid convergence in DSP (refer to Fig. 2(d)), but failed to find better solutions after generation 30. It required many more generations to achieve the same convergence level close to NSGA-II, which reached it much faster. Thus, NSGA-II proved more effective when working with higher decision variables and constraints, as it handles constraints better than MOPSO.

5 Conclusions

This study compared the performance of two EMO algorithms, NSGA-II and MOPSO, for the NAO robot gait generation problem in single and double support phases. The comparison used several metrics, such as MIP-DoM, GD+, IGD+, HV, and epsilon indicators. The results revealed that NSGA-II has a clear advantage over MOPSO in terms of convergence, diversity, and trade-off regions of the Pareto front. The MIP-DoM indicator also has a high correlation with IGD+ and a poor correlation with HV and epsilon indicators. IGD+ is used to monitor the progress of the algorithms over generations, which showed that MOPSO has a poor performance and stagnated for many generations, while NSGA-II can explore the search space more effectively and efficiently. Therefore, NSGA-II has been concluded to be a better algorithm for solving the problem with multiple constraints. For future work, testing these indicators for more than two objectives on this complex robotics gait generation problem could further validate the advantages of the MIP-DoM indicator.

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