Smallest Deviation in Upper-level Due to Variation in Lower-level for Bi-level Problems

Kalyanmoy Deb, Zhichao Lu, Ian Kropp, J. Sebastian Hernandez-Suarez, Rayan Hussein, Steven Miller, and A. Pouyan Nejadhashemi

COIN Report Number 2024012

***Abstract*—Many societal and industrial problem solving tasks involving search, optimization, design and management, are conveniently decomposed into hierarchical sub-problems. While the process allows a systematic procedure to have a multi-stakeholder solution, an independent decision-making process for the lower-level problem causes a deviation in expected outcome at the upper-level problem. In this paper, we provide a new and computationally efficient evolutionary approach for the upper-level decision-makers to analyze vagaries of lower-level decision-making for choosing a preferred solution with the minimum deviation from their expectations. The concept is novel and pragmatic. We demonstrate the concept through a search for optimistic-pessimistic trade-off solutions found by an evolutionary multi-objective optimization approach first on two difficult test problems, followed by a watershed management problem, and a telecommunication management problem. The approach is generic and can be applied to other similar hierarchical management problems for achieving minimum deviation with a more predictive and reliable outcome.**

I. Summary

Many societal and industrial management problems involve multiple levels of hierarchy, mainly due to ease of managing and controlling various aspects related to the problems [1], [2]. Each level may involve an optimization sub-problem with certain decision variables, constraints and objectives, which are dictated and controlled by a set of users or decision-makers (DMs). The variables of both levels are usually non-overlapping and controlled by the DMs of the respective level. However, the sub-problems are linked and each level requires a combination of (upper level) UL and (lower level) LL variables to compute its respective objective and constraint values. Due to the existence of two different independent groups of DMs and the complexity involved in the interconnection of the two sub-problems, an implementation of a theoretical Pareto-optimal solution does not often produce the desired objective outcome.

In this study, we have proposed a systematic multi-criterion decision-making (MCDM) approach which considers the entire variation of UL objectives which can occur due to independent choice of LL solutions from LL Pareto set by its own DMs. UL-DMs do not have a direct control of the actions of LL-DMs, however, UL-DMs have the liberty to choose any of its preferred UL solutions for implementation. Under this scenario, the best approach UL-DMs can follow is to choose a UL solution which produces minimal deviation in its objective values caused by LL-DMs. This way, despite independence of LL-DMs in choosing its own Pareto solution, UL-DMs ensure that the realized objective values would be minimally different from their expected values.

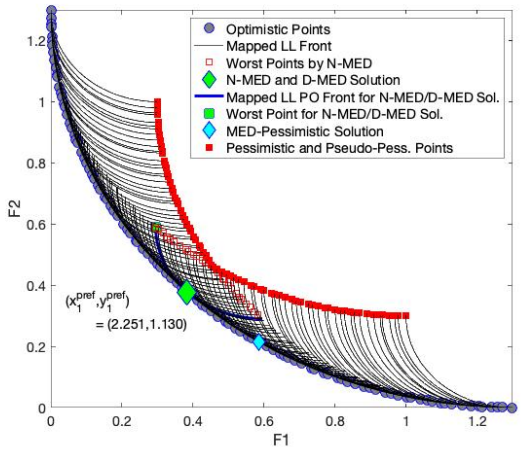

Fig. 1. Optimistic and pseudo-pessimistic solutions for DS1 problem.

Figure 1 plots the variation of UL objective values (black dots) of 100 UL solutions (blue circles) obtained using an EMO algorithm for a bi-level test problem having two conflicting objectives in each level. It is clear that each UL solution has a different variation pattern. By finding the least deviation from the UL efficient points, our approach finds the UL solution marked with a green diamond as the preferred solution.

More details about this work can be found from 2022 IEEE Trans. on Evolutionary Computation paper [3].

References

[1] J. F. Bard, Practical bilevel optimization: algorithms and applications. Springer Science & Business Media, 2013, vol. 30.

[2] A. Sinha, P. Malo, and K. Deb, "A review on bilevel optimization: from classical to evolutionary approaches and applications," IEEE Transactions on Evolutionary Computation, vol. 22, no. 2, pp. 276–295, 2017.

[3] K. Deb, Z. Lu, I. Kropp, J. S. Hernandez-Suarez, R. Hussein, S. Miller, and A. P. Nejadhashemi, "Minimizing expected deviation in upper-level outcomes due to lower-level decision-making in hierarchical multi-objective problems," IEEE Transactions on Evolutionary Computation, vol. 27, no. 3, pp. 505–519, 2022.