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Introducing Anti-Dominance Structures in Multi-objective Optimization

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***Abstract*—Various dominance structures are proposed in the multi-objective optimization literature. However, a systematic procedure to understand their effect in determining the resulting optimal set for generic domination principles, besides the standard Pareto-dominance principle, is lacking. In this paper, we analyze and lay out properties of generalized dominance structures which help provide insights for resulting optimal solutions. We introduce a concept of anti-dominance structure, derived from the chosen dominance structure, to explain how the resulting non-dominated or optimal set can be identified easily compared to using the dominance structure directly. The concept allows a unified explanation of optimal solutions for both single and multi-objective optimization problems. The anti-dominance structure is applied to analyze respective optimal solutions for most popularly-used static and spatially-changing dominance structures. The theoretical and deductive results of this study can be utilized to create more meaningful dominance structures for practical problems, understand and identify resulting optimal solutions, and help develop better test problems and algorithms for multi-objective optimization.**

I. Summary

In a practical multi-objective optimization study, users should have the flexibility in choosing a dominance structure which would involve objective preferences and priorities of users. In this paper, we introduce the concept of an *anti-dominance* structure derived from a given dominance structure defined in the objective space. For the generalized dominance structure (Ω_0) shown in Figure 1 in which the origin dominates the entire marked blue region in the objective space. The resulting anti-dominance structure (Ω_0') is shown in Figure 2 at any objective vector $\mathbf{f}(\mathbf{x})$. The following key outcomes are reported as an outcome of this study.

- The anti-dominance structure is related to dominance structure: $\Omega_0' = \Omega_0$.
- If $\mathbf{f}(\mathbf{x}) + \Omega_0' = \emptyset$, then $\mathbf{x}$ is a Pareto-optimal solution.
- The knowledge of Ω_0' helps to identify the non-dominated set in a population computationally quicker than Ω_0.
- Population-based evolutionary multi-objective optimization (EMO) algorithms can work with semi-transitive dominance structures better than point-based algorithms.

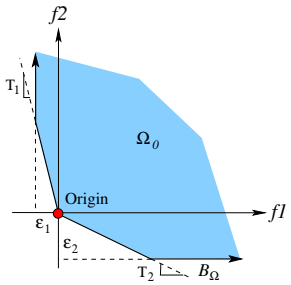


Fig. 1: A generalized dominance structure Ω_0 and its boundary B_Ω.

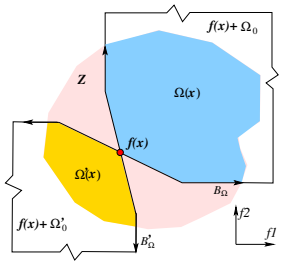
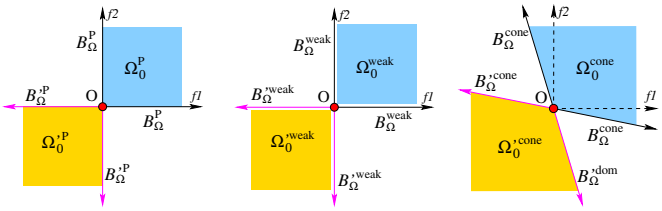


Fig. 2: Relationship between non-overlapping $\Omega(\mathbf{x})$ and Ω_0.

- Ω_0' structures have been presented for a number of standard Ω_0 structures used in the EMO and multi-objective optimization literature, as shown in Figure 3.
- For Ω_0 structures with overlapping region with Ω_0', there is no Pareto-optimal solution. However, the study suggests simple update procedures to Ω_0 to make it usable with EMO algorithms.



(a) Pareto-dominance. (b) Weak dominance. (c) Cone dominance.

Fig. 3: Commonly-used dominance structures. In each case, $\Omega_0 \cap \Omega'_0 = \emptyset$.

For more results, the 2023 original study [1] published in Mathematical and Computational Applications journal can be referred.

References

[1] K. Deb and M. Ehrgott, "On generalized dominance structures for multi-objective optimization," *Mathematical and Computational Applications*, vol. 28, no. 5, p. 100, 2023.