

# A Mixed-Fidelity Evaluation Algorithm for Efficient Constrained Multi- and Many-Objective Optimization: First Results

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**Abstract.** A low-fidelity model using surrogate functions helps reduce the overall computational time for optimizing problems involving time-consuming evaluation of objectives and constraints. Most surrogate-assisted evolutionary multi- and many-objective optimization (SA-EM(a)O) algorithms evaluate a newly-created population member either using high-fidelity or low-fidelity models for all objectives and constraints. However, recent studies have shown that a mixed-fidelity evaluation of a population member, in which some objectives and constraints are evaluated using a high-fidelity model and others evaluated using a low-fidelity model, may be more efficient. This is because the time saved by not evaluating constraint functions for largely infeasible or largely feasible solutions, and objective functions for largely infeasible solutions can be utilized to evaluate potential solutions near the constraint boundaries and feasible regions, respectively. In this paper, we propose a metric that determines the potential benefit for evaluating every objective and constraint function for every population member independently. The metric uses the potential for the solution's probability to dominate other neighboring members, modeling error, and extent of constraint violation. We demonstrate the efficacy of the proposed approach by presenting results on two to 10-variable test and engineering design problems having up to eight objectives and up to 10 constraints. We compare the proposed approach with a SA-EMO algorithm which evaluates every constraint and objective function for every new offspring. This preliminary study shows promise as a new direction of research in the area of surrogate-assisted evolutionary multi- and many-objective optimization.

**Keywords:** Constrained surrogate-assisted optimization, Selective evaluation, Multi-objective optimization, Independently evaluable objectives and constraints

## 1 Introduction

Optimization methods require a considerable number of solution evaluations before near-optimal solutions are found from an initial point or set of random solutions. This is particularly challenging in problems where each evaluation is computationally expensive, such as when using one or more third-party modeling and evaluation software. One practically viable way to execute an optimization algorithm in such scenarios is to construct a low-complexity surrogate model for each objective and constraint function and use the surrogate models to run the majority of optimization iterations. Since surrogate models are usually approximate mathematical representations of the original functions, they are computationally cheap to compute. It is not surprising that a number of surrogate-assisted optimization (SAO) algorithms have been proposed for this purpose [4, 5, 8, 9, 17, 20, 25, 28, 33].

Despite the idea being appealing, optimization researchers need to figure out several matters related to SAO algorithms: (i) which new solutions must be computed for high-fidelity evaluation in addition to the already-evaluated set of high-fidelity solutions? (ii) which surrogate modeling approach would be most appropriate for the current set of high-fidelity evaluated solutions? and (iii) how should the optimization with surrogates be integrated with the overall optimization algorithm (referring to the surrogate management problem)? The first question requires a methodology to analyze the current set of high-fidelity evaluation solutions to predict new solutions at the potential search regions providing better solutions and/or improving the surrogates to produce lesser prediction error for better solutions. Thus, this first task is an important step towards any SAO algorithm. The second question usually demands the SAO algorithm to try a number of surrogate modeling approaches in competition and discover the most appropriate approach for the current phase of the SAO algorithm. Notice that such an ensemble-based surrogate modeling method has the self-adaptive ability to choose a different surrogate modeling approach at different stages of the SAO algorithm. The third question arises from the fact that after the surrogate models are developed, they must be exploited to optimize and find new (infill) solutions for the next set of high-fidelity evaluations. Since surrogate models are cheaper to evaluate, direct optimization of surrogate models, instead of high-fidelity models, will help reduce computational time to arrive at better regions of the search space. The flip side of surrogate optimization is that since surrogates are approximate functions of the original objective and constrained functions, the true optimal solutions of the surrogate models may not corroborate to the better regions of the original objectives. Thus, this task requires estimating how much the developed surrogate models can be trusted to avoid selecting infill solutions that are far from the true optimal regions of the original problem.

Clearly, the above three tasks must be coordinated in an adaptive manner so that progressive surrogate modeling with new infill solutions and by using appropriate surrogate modeling approaches monotonically progresses towards the near-optimal search region with a minimal number of high-fidelity evaluations.

When multiple objectives and constraints are present, the SAO becomes more challenging, as there exist multiple objectives and constraints to be modeled using surrogates. A recent study [13] has suggested a taxonomy of 10 different surrogate methods for modeling each individual objective independently to modeling a single combined scalarized and composite function combining objectives and constraints for surrogate-assisted evolutionary multi-objective optimization (SA-EM(a)O) algorithms. A later study [16] has shown that an adaptive choice of the most appropriate one out of 10 methods at every iteration of the overall optimization algorithm produces the best performing outcome. Since an optimization process goes through different types of search landscapes (constrained, isolated, multi-modal, strongly interconnected objectives, etc.) at different phases of the run, a different surrogate modeling method was found to be most appropriate. The adaptive surrogate modeling (ASM) [16] approach discovered the right method adaptively to produce the best performing result.

However, in most existing SA-EM(a)O studies, every infill solution, found worthy to be evaluated with high-fidelity models, is eventually evaluated for all objectives ( $M$ ) and constraints ( $J$ ) either individually or in an aggregate fashion, irrespective of whether surrogate prediction error of each objective function, whether it has the potential to be feasible, and whether it has the potential to have the best objective value compared to its neighboring and competing other solutions. Since the entire solution is marked to be evaluated for all objectives and constraints, in many-objective optimization, this may cause unnecessary high-fidelity evaluation of some objectives and constraints, which have an apparent correlation with other computationally fast objectives/constraints. This aspect of not evaluating all objectives and constraints for a solution leading to a mixed-fidelity evaluation (MFE) of objectives and constraints of population members is not addressed that well in the existing SA-EM(a)O literature [1–5, 19, 25, 27, 29].

In this paper, we propose an MFE-EM(a)O framework that determines a high-fidelity selection (HFS) criterion for each objective and constraint for each population member, thereby allowing a population to have mixed-evaluated members, in which some objectives and constraints carry surrogate-evaluated values and some carry high-fidelity evaluated values. An EM(a)O algorithm then uses the current surrogate models to find a new infill population without much computational effort. The old and new infill populations are combined, and then the proposed HFS criteria for each objective and constraint are invoked. This process considers the probability of a solution being feasible, the extent of constraint violation, and metrics used in [30]. As a result, we can determine the objectives and constraints that need to be high-fidelity evaluated and a selection of the new infill population members for the next generation. Those selected members will undergo high-fidelity evaluation if not previously evaluated.

In the remainder of the paper, we discuss the proposed mixed-fidelity evaluation based evolutionary multi- and many-objective optimization (EM(a)O) algorithm in Section 2 with detailed sketches. Section 3 presents results on multi- and

many-objective test and engineering problems including unconstrained problems. Conclusions and future extensions of this study are discussed in Section 4.

## 2 Proposed Mixed-Fidelity Evaluation (MFE) Based EM(a)O Algorithm

We modify the elitist evolutionary many-objective optimization algorithm – NSGA-III [14] – to implement the MFE algorithm, which can also be used in other popular reference vector based EM(a)O algorithms [22, 32]. A set of  $N$  reference vectors are first created using the Das and Dennis [10] approach or using the Riesz energy method [7]. The goal of the proposed MFE-NSGA-III is to find at most  $N$  non-dominated (ND) solutions close to the Pareto front (PF) of the problem using a mixed-fidelity evaluation technique.

Each member of the initial population  $P_{doe}$  of size  $N_{doe}$ , generated using the Latin hypercube sampling (LHS) method [26], is evaluated with high fidelity for all objectives ( $M$ ) and constraints ( $J$ ). Although we do not consider the exact computational time for evaluating objectives and constraints in this study, we simply assume that each objective and constraint takes an equal amount of time to evaluate. Assuming that the total time required to evaluate all objectives and constraints is one unit, each objective or constraint takes  $1/(J + M)$  units of time. Thus, the initial design of experiments (doe) population  $P_{doe}$  of size  $N_{doe}$  consumes  $T = N_{doe}$  units of time before starting the proposed EM(a)O procedure. A maximum of  $T_{\max}$  time is allowed for finding the non-dominated (ND) set, of which the final  $N$  units time is reserved for evaluating at most  $N$  final population members with high-fidelity evaluations. Thus, a total of  $T_{\max} - N$  time is kept for MFE-NSGA-III generations.

A surrogate for each objective or constraint function is constructed with  $N_{doe}$  high-fidelity evaluations. We select an initial population  $P_0$  of size  $N$  (with  $N \leq N_{doe}$ ) by using a chosen EM(a)O survival operator from  $N_{doe}$  solutions. Then, an EM(a)O algorithm is run with initial population  $P_0$  for  $t_S$  generations using the surrogates, thereby not causing any new high-fidelity evaluation. The final population  $Q_0$  at the end of the surrogate-assisted NSGA-III run is then merged with  $P_0$  and a combined population  $R_0 = P_0 \cup Q_0$  is formed. Note that no member of  $Q_0$  is high-fidelity evaluated yet. We set the iteration counter as  $t = 0$ .

Each population member  $s$ , in the combined population of  $R_t$  is then associated with one of the supplied reference vectors. To make a fair computation, all solutions for which a high-fidelity evaluation of one or more objectives and constraints were already made are re-evaluated with the respective surrogate model for executing surrogate-assisted NSGA-III run. Then, a prospect metric  $\rho_s^m$  is calculated for every objective ( $m$ ) and solution ( $s$ ) using an overall probability of constraint satisfaction ( $PG_s$ ), as follows:

$$\rho_s^m = (1 + PG_s) P_s^m \left( 1 + \left( \frac{\sigma_s^m}{|\Delta f_m|} \right)^{\frac{1}{\eta}} \right). \quad (1)$$

Here,  $P_s^m$  refers to the probability of solution  $s$  being non-dominated among its neighboring members, (in the sense of reference vectors [14, 32]), discussed in a previous study [30].  $\sigma_s^m$  and  $\Delta f_m$  are surrogate uncertainty and range of  $m$ -th objective in the population, respectively, discussed in [30] as well. Parameter  $\eta = 20$  has worked well in this study. The structure of  $\rho_s^m$  is obtained by numerous trial-and-error experiments and with an intuitive understanding of each constituent term.

The overall probability of constraint satisfaction ( $PG_s$ ) is calculated as follows:

$$PG_s = \prod_{j=1}^J P(g_j(s) \leq 0), \quad (2)$$

where  $P(g_j(s) \leq 0) = \Phi(Z_s^j) = \frac{1}{2} \left(1 + \operatorname{erf}\left(\frac{Z_s^j}{\sqrt{2}}\right)\right)$  and

$$Z_s^j = \frac{-\mu_s^j}{\sigma_s^j}. \quad (3)$$

The parameters  $\mu_s^j$  and  $\sigma_s^j$  are prediction and associated uncertainty in prediction in  $j$ -th constraint for solution  $s$  using Kriging surrogate model. We used the same Kriging models [6, 21, 23] as in our previous study [30]. The motivation for the structure of Equation 1 comes from our successful previous study on heterogeneous evaluation of objectives with unequal computational times. We add the first term involving constraint satisfaction in the above equation to handle constraints.

The procedure for deciding which objectives and constraints to be high-fidelity evaluated is described below, but it requires a threshold parameter  $e$ , which is calculated from the median value of the non-dimensional entity  $Z_s^j$  for all constraints of the entire  $R_t$  population:

$$e = \min(\operatorname{Median}_{\forall(s,g)} Z_s^g, -1). \quad (4)$$

Figure 1 illustrates the philosophy of the high-fidelity selection (HFS) procedure.

In short, if a solution  $s$  is highly infeasible with respect to any of the constraints, then no constraint or objective function needs to be high-fidelity evaluated for the solution, as the solution cannot be a potentially good solution. On the other hand, if a constraint value is within one-sigma (surrogate's error) from zero, the constraint is worth evaluating with a high-fidelity model. If a solution is feasible for all constraints or marginally infeasible (within one-sigma from zero, see Figure 1) for some constraints, then, at least one objective must be evaluated with its high-fidelity model. The step-by-step procedure is given below:

**Step 1:** For every reference vector ( $k$ ) starting with  $k = 1$ , rank all associated population members based on the  $\rho_s^m$  (Equation 1). Then, pick the solution  $\bar{s}$  and respective  $\bar{m}$  corresponding to the largest  $\rho_s^m$ . If  $\bar{s}$  has already been selected for high-fidelity evaluation (in  $P_0$  or  $Q_0$ ), include this solution for survival in  $P_{t+1}$  and move to Step 3. Else, consider this solution for its prospect for high-fidelity evaluation of constraints and move to Step 2.

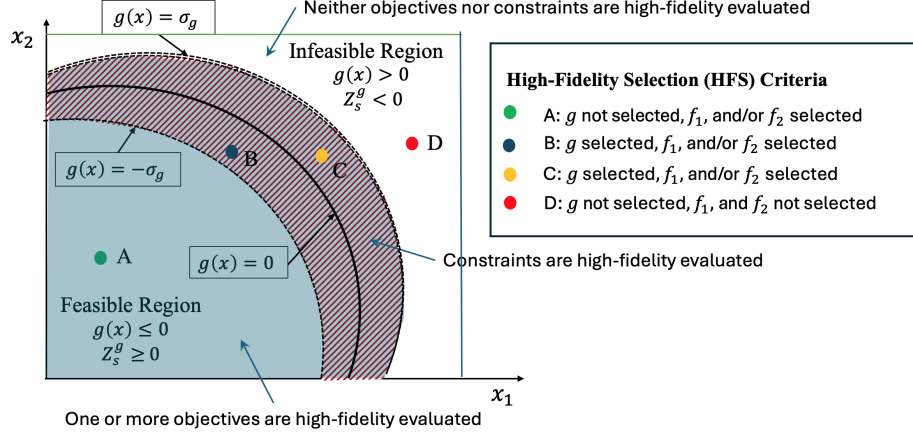


Fig. 1: Selection of solutions to be high-fidelity evaluated. Solutions A, B, and C are either feasible or close to the constraint boundary, hence some of their objective functions are selected for high-fidelity evaluation based on  $\rho_s^m$  and  $e$  values. Solution D is infeasible and far away from the constraint boundary, hence it is not selected for either constraints or objectives for high-fidelity evaluations. Solutions B and C are close to the constraint boundary and are evaluated using high-fidelity constraint models.

**Step 2:** Use the following sub-steps to decide which constraints need to be high-fidelity evaluated for  $\bar{s}$ :

**2.1:** If  $\min_j (|Z_{\bar{s}}^j|) > |e|$ , meaning that  $\bar{s}$  is not close to the boundary of any constraint, this solution does not need to be evaluated for any constraint function.

**2.2:** Else, identify all constraints ( $j$ ) for which  $Z_{\bar{s}}^j$  is within  $[e, -e]$ , meaning that  $\bar{s}$  is close to the boundary of these constraints, select  $\bar{s}$  for a high-fidelity evaluation of all of these constraints. If  $C$  constraints are high-fidelity evaluated, a total time of  $C/(J+M)$  is added to the time counter ( $T$ ). The solution is also picked for survival. Additionally, Step 3 is executed to determine which objective function needs to be high-fidelity evaluated.

**Step 3:** If  $\min_j (Z_{\bar{s}}^j) > e$ , meaning that  $\bar{s}$  is either feasible or is infeasible but lies close to all constraint boundaries, then perform a high-fidelity evaluation of the objective  $\bar{m}$  (if not selected before) having the largest  $\rho_{\bar{s}}^m$ . Time  $1/(J+M)$  is added to  $T$ . Select  $\bar{s}$  for survival as well.

**Step 4:** Repeat above steps for the reference vector by incrementing  $k$  by one. This process is continued until  $N$  members are selected for survival and saved in  $P_{t+1}$ .

Figure 2 illustrates the above procedure of combining  $P_t$  and  $Q_t$  to form  $R_t$  and arriving at the next generation population  $P_{t+1}$ . It is interesting to note how our proposed technique handles a mixed-fidelity of objectives and constraints

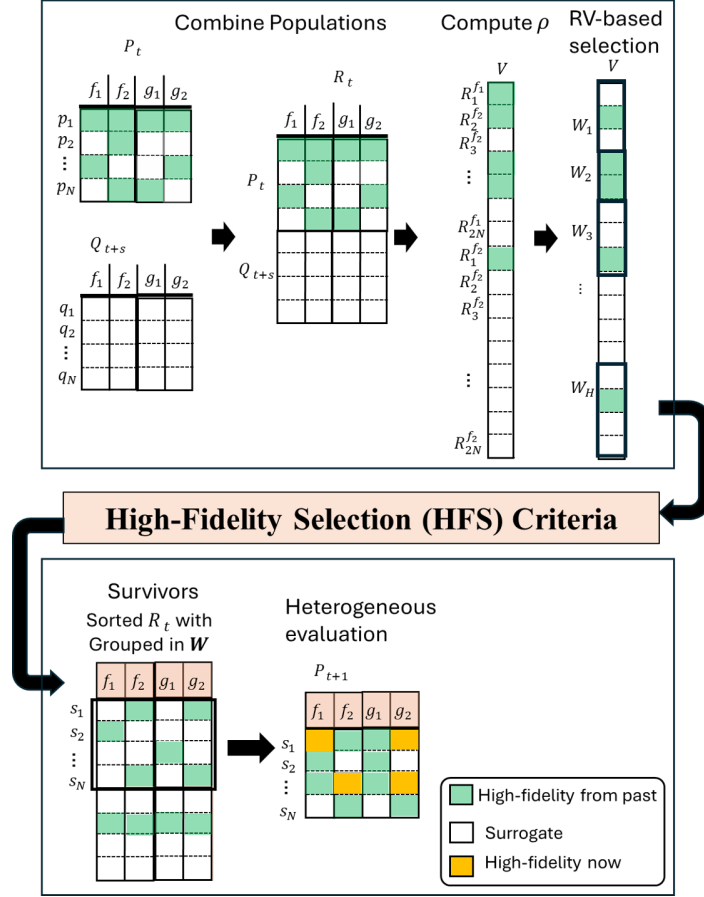


Fig. 2: Survival selection of  $P_{t+1}$  from  $R_t$  based on  $\rho_s^m$  metric and high-fidelity selection (HFS) criterion.

(green entities were previously high-fidelity modeled, white entities are surrogate modeled, and orange entities are freshly high-fidelity evaluated).

Notice also that for unconstrained problems, every  $s$  is feasible. In this case,  $Z_s^j = \infty$  and Step 2 does not get executed and the above technique simply chooses a single objective  $\bar{m}$  for high-fidelity evaluation for each solution  $\bar{s}$  at a time. This process is similar to the heterogeneous unconstrained surrogate-assisted algorithm proposed and was shown to perform extremely well on unconstrained problems by the authors recently [30].

### 3 Results and Discussions

We test the proposed approach MFE-NSGA-III with two other algorithms run for an identical  $T_{\max}$ : (i) the original NSGA-III [14] in which every population

member is high-fidelity evaluated for all objectives and constraints without any use of surrogates, and (ii) SA-NSGA-III, in which surrogates are used for optimization for  $t_S$  generations, but all offspring population members of  $Q_{t+t_S}$  are high-fidelity evaluated for all objectives and constraints and surrogates are then updated for next generation.

In this study, we test these three methods on several multi and many-objective problems: ZDT1 [34], DTLZ2 [15], BNH, OSY, SRN, TNK, CTP1 [12], C2DTLZ2 [18], MW3, MW7, MW14 [24], and three engineering design problems [11, 31]. In these problems, we consider 2-10 variables, up to 10 constraints, and 2-8 objectives.

For all problems, the population size ( $N$ ) is set to 50, number of reference directions chosen to be same as the population size ( $N$ ). The overall termination budget  $T_{\max}$  is set as 300 and the initial design of experiments (doe)  $N_{doe}$  is set as 100 for two and three objective problems. For five and eight objective problems, the termination budget  $T_{\max}$  is set as 400, and the initial doe  $N_{doe}$  is set as 150. The surrogate generations ( $t_S$ ) to create offspring population  $Q_{t+t_S}$  is set as 10. The parameter  $\eta$  in Equation 1 to emphasize uncertainty in surrogate modeling of objectives is 20.

For each problem, we run all algorithms 15 times with different initial populations. Hyper-volume (HV) is chosen as the performance metric. For HV computation, we normalize the population members using the known Pareto front's nadir and ideal points. If the Pareto front is unknown in the engineering design problems, we obtain the approximated Pareto fronts from the work of Tanabe et al. [31]. We used a reference point vector  $[1.05]_{1 \times M}$  to compute HV. Wilcoxon signed-rank test with  $p = 0.05$  is used to determine statistically best, similar, and worst-performing algorithms.

### 3.1 Unconstrained Problems

First, we consider two bi-objective unconstrained test problems (ZDT1 [34] and DTLZ2 [15]) to illustrate the working of our proposed mixed-fidelity evaluation algorithm on unconstrained problems.

For HV growth plots, the time  $T$  is used in the  $x$ -axis for a fair comparison of the three algorithms. The HV growth plots are presented in Figures 3c and 3d. We observe that the proposed MFE-NSGA-III has a much better performance compared to the other two methods. The obtained final generation solutions of all three algorithms are shown in Figures 3a and 3b. We observe that MFE-NSGA-III obtains a well-distributed near-Pareto-optimal solutions. The obtained median HV values of the final generation members are shown in Table 1. The statically best and similar performing algorithms for each problem are marked in bold and bold italics text, respectively. The MFE-NSGA-III is statically better compared to the other two algorithms on these unconstrained problems.

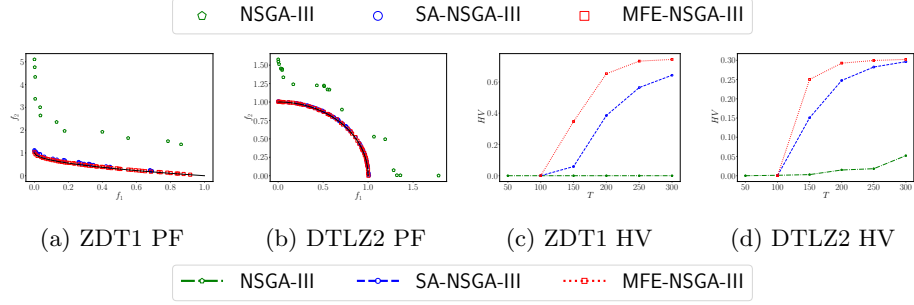


Fig. 3: Comparison of three algorithms for two-objective unconstrained optimization problems. SA-NSGA-III produces near-PF solutions, but the distribution of solutions is not as good as MFE-NSGA-III.

### 3.2 Constrained Multi-Objective Problems

Next, we consider two and three-objective constrained test and engineering problems. The final generation solutions and HV growth plots for OSY, SRN, BNH, MW3, CTP1, and C2DTLZ2 problems are shown in Figure 4. It is evident from

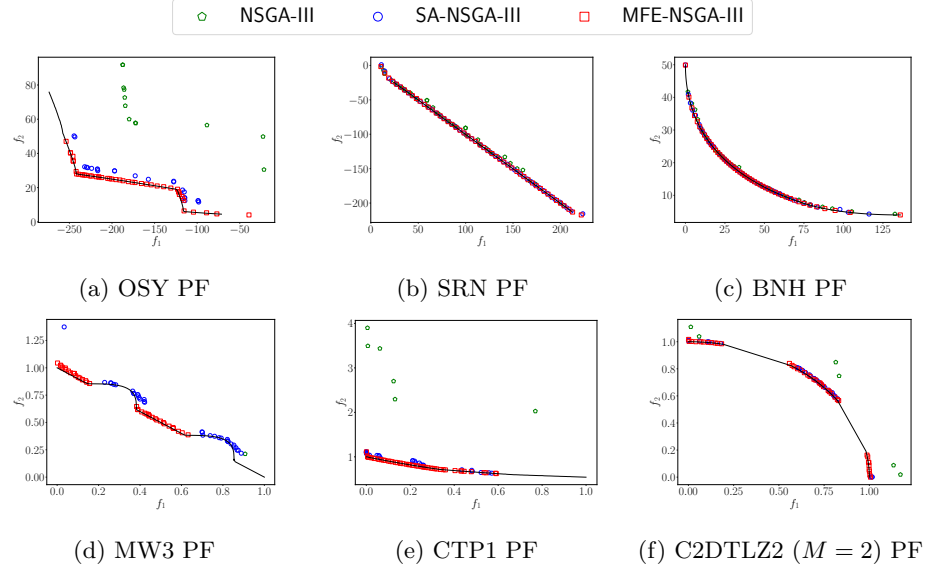


Fig. 4: Comparison of PFs for two-objective constrained optimization problems. With only  $T_{\max} = 300$  (meaning equivalent to a maximum of 300 complete solution evaluations), no algorithm is perfect, but MFE-NSGA-III produces better PFs in all problems.

HV growth plots (Figure 5) that MFE-NSGA-III outperforms NSGA-III and

Table 1: Median hypervolume (HV) comparison of three algorithms for all problems used in this study. HVs for statistically equivalent algorithms (with 95% confidence level) are marked in bold italics and for the best performing algorithm is marked in bold.

| Problem         | #Obj. | #Constr. | $T_{max}$ | NSGA-III          | SA-NSGA-III       | MFE-NSGA-III      |
|-----------------|-------|----------|-----------|-------------------|-------------------|-------------------|
| ZDT1            | 2     | 0        | 300       | 0.0000e+0         | 6.4321e-1         | <b>7.4482e-1</b>  |
| DTLZ2           | 2     | 0        | 300       | 5.2156e-2         | 2.9640e-1         | <b>3.0179e-1</b>  |
| BNH             | 2     | 2        | 300       | 8.9956e-1         | 9.0599e-1         | <b>9.0682e-1</b>  |
| SRN             | 2     | 2        | 300       | 5.6782e-1         | 5.9049e-1         | <b>5.9130e-1</b>  |
| TNK             | 2     | 2        | 300       | 3.0120e-1         | <b>3.8077e-1</b>  | <b>3.8900e-1</b>  |
| OSY             | 2     | 6        | 300       | 1.8338e-1         | 6.9623e-1         | <b>7.9869e-1</b>  |
| MW3             | 2     | 2        | 300       | 1.1953e-1         | 4.1405e-1         | <b>4.7122e-1</b>  |
| MW7             | 2     | 2        | 300       | 4.7374e-2         | 2.3489e-1         | <b>3.0304e-1</b>  |
| CTP1            | 2     | 2        | 300       | 0.0000e+0         | 6.2555e-1         | <b>7.1673e-1</b>  |
| C2DTLZ2         | 2     | 1        | 300       | 7.8029e-2         | 2.4434e-1         | <b>2.6294e-1</b>  |
| WELD            | 2     | 4        | 300       | 8.6389e-1         | 8.5806e-1         | <b>9.6595e-1</b>  |
| MW14            | 3     | 1        | 300       | 6.3110e-3         | <b>1.0374e-1</b>  | <b>1.2581e-1</b>  |
| C2DTLZ2         | 3     | 1        | 300       | 1.0415e-1         | 3.4501e-1         | <b>4.0335e-1</b>  |
| Carside         | 3     | 10       | 300       | 4.0586e-1         | 5.8093e-1         | <b>6.2751e-1</b>  |
| C2DTLZ2         | 5     | 1        | 400       | 2.0309e-1         | 4.8035e-1         | <b>5.6943e-1</b>  |
| WATER           | 5     | 7        | 400       | 4.6298e-1         | <b>5.2203e-1</b>  | <b>5.2791e-1</b>  |
| C2DTLZ2         | 8     | 1        | 400       | 3.4028e-1         | 5.0270e-1         | <b>8.0004e-1</b>  |
| Total + / = / - |       |          |           | 0 / 0 / <b>17</b> | 0 / 3 / <b>14</b> | <b>14</b> / 3 / 0 |

SA-NSGA-III. The final generation median HV values for all two objective constrained problems are presented in Table 1. MFE-NSGA-III outperforms the other algorithms eight times and performs similarly to SA-NSGA-III once.

Figure 6a shows the number of high-fidelity evaluations for  $f_1, f_2$  and  $g_1$  with respect to  $T$  in MFE-NSGA-III for C2DTLZ2 problem. This figure clearly demonstrates selective high-fidelity evaluations for objective and constraint functions. In this problem, the constraint function is less frequently modeled using high-fidelity models due to relatively less complex structure of it compared to objective functions. Figure 6b shows the total number of high-fidelity evaluations in all three algorithms, showing fewer overall evaluations of the constraint function.

For three objective C2DTLZ2, Carside impact, and MW14 problems, final generation solutions in objective space and HV growth plots are presented in Figure 7. The final generation median HV values of all three objective problems are presented in Table 1. For MW14, MFE-NSGA-III performs similar to SA-NSGA-III. For all other problems, MFE-NSGA-III performs the best.

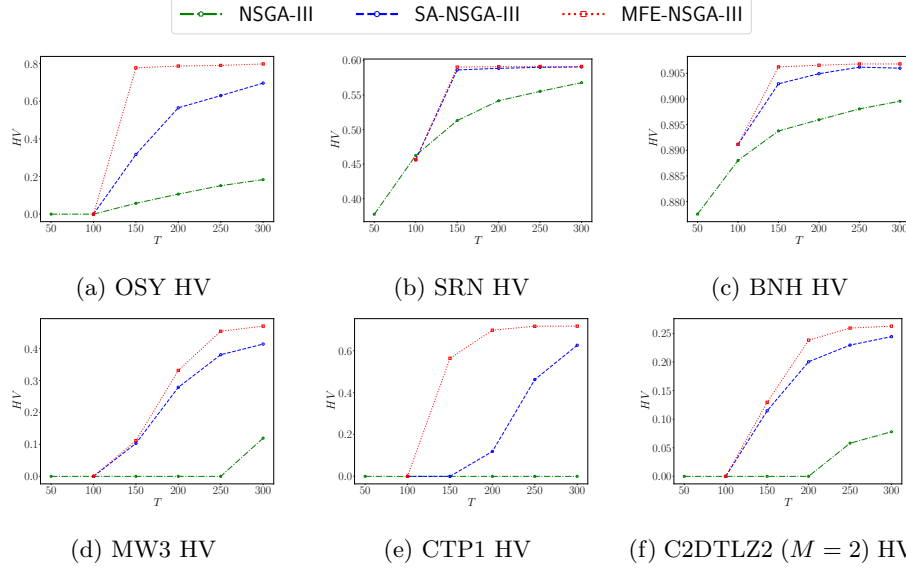


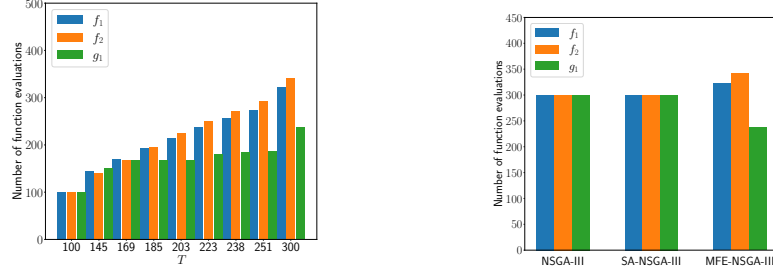
Fig. 5: Comparison of HV growth for three algorithms for two-objective constrained optimization problems. Different algorithms start from a different initial  $T$  due to each algorithms evaluation scheme of initial DOE set for surrogate model creation.

### 3.3 Constrained Many-Objective Problems

Finally, we present results on five and eight-objective C2DTLZ2 problems and a five-objective engineering problem (WATER [18]). The first row in Figure 8 shows the final generation objective function solutions on PCP. The second row in Figure 8 shows HV growth plots of five and eight-objective C2DTLZ2 problems, which clearly demonstrates the superiority of the proposed MFE-NSGA-III. Table 1 shows the median HV values of final generation population members. For five and eight-objective C2DTLZ2 problems, the proposed MFE-NSGA-III performs statistically better than other methods. For the WATER problem, MFE-NSGA-III is statically similar to SA-NSGA-III.

## 4 Conclusions and Future Extensions

In this paper, we have proposed a mixed-fidelity evaluation algorithm mainly for constrained optimization problems, which also degenerates to handle unconstrained problems. The proposed technique allows the objectives and constraints of an evolving population member to be evaluated using a surrogate, or, a high-fidelity model, as needed, determined by a proposed prospect metric. Results on a number of multi- and many-objective test and engineering problems show the efficacy of the proposed algorithm compared to two other algorithms. MFE-NSGA-III makes unequal calls to objectives and constraints, as demanded by



(a) #Hi-fi evals. for  $f_1$ ,  $f_2$  and  $g_1$  with  $T$ . (b) Total #hi-fi evals. for  $f_1$ ,  $f_2$  and  $g_1$ .

Fig. 6: Number of high-fidelity evaluations recorded for  $f_1$ ,  $f_2$  and  $g_1$  on C2DTLZ2 ( $M = 2, J = 1$ ) using MFE-NSGA-III, which uses less evaluation of the constraint compared to objectives to produce a better performance (HV=0.263) than NSGA-III (HV=0.078) and SA-NSGA-III (HV=0.244).

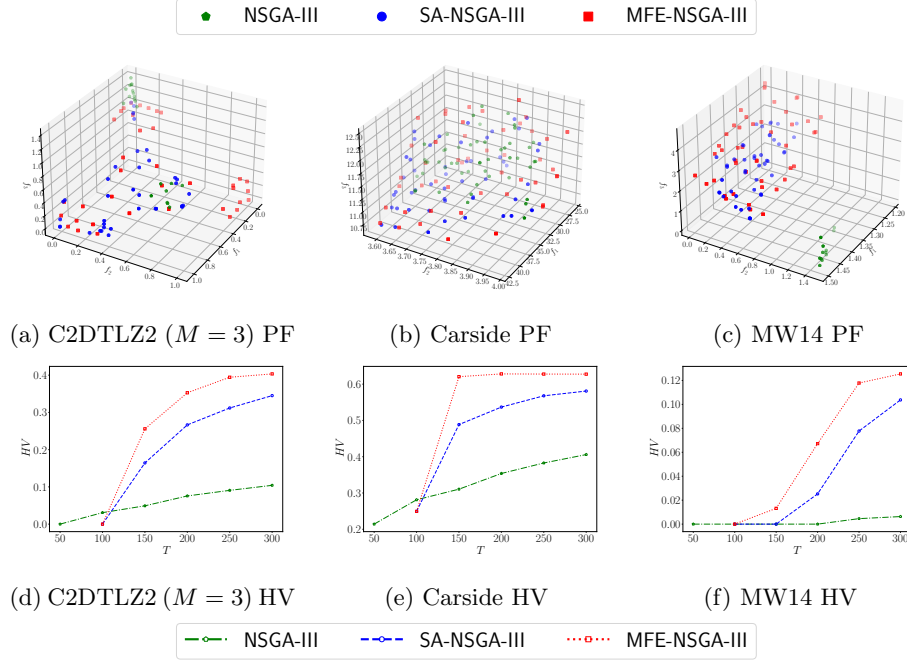


Fig. 7: Comparison of algorithms for three-objective constrained problems.

the relative ease with which the surrogate models can be achieved, to produce an efficient algorithm.

The proposed algorithm is now ready to be applied to more challenging multi- and many-objective problems. The mixed-fidelity evaluation algorithm can now be extended for problems involving evaluation models requiring heterogeneous computational times for objectives and constraints. Other interesting methods for calculating the prospect metric may also be explored. Nevertheless, results of

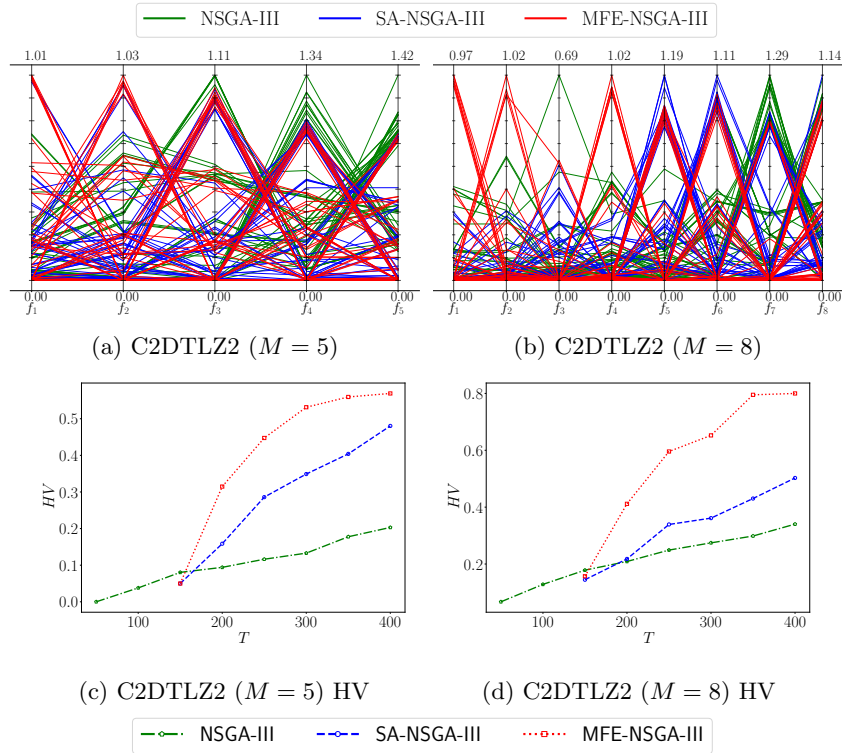


Fig. 8: Comparison of algorithms for five and eight-objective C2DTLZ2.

this first study are extremely encouraging and show a direction for developing a more efficient and flexible search algorithm for computationally expensive multi- and many-objective constrained optimization problems. Due to the flexibility of evolutionary algorithm framework, mixed-fidelity evaluation of objectives and constraints for every solution is possible and EMO researchers must now take advantage in creating more such computationally efficient constrained optimization algorithms.

## References

1. Allmendinger, R., Handl, J., Knowles, J.: Multiobjective optimization: When objectives exhibit non-uniform latencies. *European Journal of Operational Research* **243**(2), 497–513 (2015)
2. Allmendinger, R., Knowles, J.: ‘hang on a minute’: Investigations on the effects of delayed objective functions in multiobjective optimization. In: *International Conference on Evolutionary Multi-Criterion Optimization*. pp. 6–20. Springer (2013)
3. Allmendinger, R., Knowles, J.: Heterogeneous objectives: state-of-the-art and future research. In: *Many-Criteria Optimization and Decision Analysis: State-of-the-Art, Present Challenges, and Future Perspectives*, pp. 317–335. Springer (2023)

4. Blank, J., Deb, K.: Constrained bi-objective surrogate-assisted optimization of problems with heterogeneous evaluation times: Expensive objectives and inexpensive constraints. In: EMO. pp. 257–269 (2021)
5. Blank, J., Deb, K.: Handling constrained multi-objective optimization problems with heterogeneous evaluation times: proof-of-principle results. *Memetic Computing* **14**(2), 135–150 (2022)
6. Blank, J., Deb, K.: pysamoo: Surrogate-assisted multi-objective optimization in python (2022)
7. Blank, J., Deb, K., Dhebar, Y., Bandaru, S., Seada, H.: Generating well-spaced points on a unit simplex for evolutionary many-objective optimization. *IEEE Trans. on Evol. Computation* (2020)
8. Chugh, T., Jin, Y., Miettinen, K., Hakanen, J., Sindhya, K.: A surrogate-assisted reference vector guided evolutionary algorithm for computationally expensive many-objective optimization. *IEEE Transactions on Evolutionary Computation* **22**(1), 129–142 (2018)
9. Chugh, T., Sindhya, K., Hakanen, J., Miettinen, K.: A survey on handling computationally expensive multiobjective optimization problems with evolutionary algorithms. *Soft Computing* **23**, 3137–3166 (2019)
10. Das, I., Dennis, J.: Normal-boundary intersection: A new method for generating the Pareto surface in nonlinear multicriteria optimization problems. *SIAM Journal of Optimization* **8**(3), 631–657 (1998)
11. Deb, K.: Optimal design of a welded beam via genetic algorithms. *AIAA journal* **29**(11), 2013–2015 (1991)
12. Deb, K.: Multi-objective optimization using evolutionary algorithms. Wiley, Chichester, UK (2001)
13. Deb, K., Hussein, R., Roy, P., Toscano, G.: A taxonomy for metamodeling frameworks for evolutionary multi-objective optimization. *IEEE Transactions on Evolutionary Computation* **23**(1), 104–116 (2019)
14. Deb, K., Jain, H.: An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, Part I: Solving problems with box constraints. *IEEE Transactions on Evolutionary Computation* **18**(4), 577–601 (2014)
15. Deb, K., Thiele, L., Laumanns, M., Zitzler, E.: Scalable test problems for evolutionary multi-objective optimization. In: Abraham, A., Jain, L., Goldberg, R. (eds.) *Evolutionary Multiobjective Optimization*, pp. 105–145. London: Springer-Verlag (2005)
16. Deb, K., Roy, P.C., Hussein, R.: Surrogate modeling approaches for multiobjective optimization: Methods, taxonomy, and results. *Mathematical and Computational Applications* **26**(1), 5 (2020)
17. Hussein, R., Deb, K.: A generative kriging surrogate model for constrained and unconstrained multi-objective optimization. In: *Proceedings of the Genetic and Evolutionary Computation Conference 2016*. pp. 573–580 (2016)
18. Jain, H., Deb, K.: An evolutionary many-objective optimization algorithm using reference-point based non-dominated sorting approach, Part II: Handling constraints and extending to an adaptive approach. *IEEE Transactions on Evolutionary Computation* **18**(4), 602–622 (2014)
19. Jin, Y.: Surrogate-assisted evolutionary computation: Recent advances and future challenges. *Swarm and Evolutionary Computation* **1**(2), 61–70 (2011)
20. Knowles, J.: ParEGO: A hybrid algorithm with on-line landscape approximation for expensive multiobjective optimization problems. *IEEE Transactions on Evolutionary Computation* **10**(1), 50–66 (2006)

21. Krige, D.G.: A statistical approach to some basic mine valuation problems on the witwatersrand. *Journal of the Southern African Institute of Mining and Metallurgy* **52**(6), 119–139 (1951)
22. Li, H., Zhang, Q.: Multiobjective optimization problems with complicated Pareto sets, MOEA/D and NSGA-II. *IEEE Trans on Evolutionary Computation* **12**(2), 284–302 (2009)
23. Lophaven, S.N., Nielsen, H.B., Sondergaard, J., Dace, A.: A matlab kriging toolbox. Technical University of Denmark, Kongens Lyngby, Technical Report No. IMM-TR-2002-12 (2002)
24. Ma, Z., Wang, Y.: Evolutionary constrained multiobjective optimization: Test suite construction and performance comparisons. *IEEE Transactions on Evolutionary Computation* **23**(6), 972–986 (2019)
25. Mamun, M.M., Singh, H.K., Ray, T.: An approach for computationally expensive multi-objective optimization problems with independently evaluable objectives. *Swarm & Evol. Compt.* **75**, 101146 (2022)
26. Montgomery, D.C.: *Design and Analysis of Experiments*. Wiley, 8th edition edn. (2012)
27. Rahi, K., Singh, H., Ray, T.: Towards solving expensive optimization problems with heterogeneous constraint costs. In: *Proceedings of the Genetic and Evolutionary Computation Conference Companion*. pp. 2032–2040 (2024)
28. Rahi, K.H., Singh, H.K., Ray, T.: Investigating the use of sequencing and infeasibility driven strategies for constrained optimization. In: *2019 IEEE congress on evolutionary computation (CEC)*. pp. 1642–1649. IEEE (2019)
29. Rahi, K.H., Singh, H.K., Ray, T.: Partial evaluation strategies for expensive evolutionary constrained optimization. *IEEE Transactions on Evolutionary Computation* **25**(6), 1103–1117 (2021)
30. Santoshkumar, B., Deb, K.: Surrogate-assisted multi-objective optimization for handling objectives with heterogeneous evaluation times: Unconstrained problems. In: *2024 IEEE Congress on Evolutionary Computation (CEC)*. pp. 1–8. IEEE (2024)
31. Tanabe, R., Ishibuchi, H.: An easy-to-use real-world multi-objective optimization problem suite. *Applied Soft Computing* **89**, 106078 (2020)
32. Zhang, Q., Li, H.: MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *Evolutionary Computation, IEEE Transactions on* **11**(6), 712–731 (2007)
33. Zhang, Q., Liu, W., Tsang, E., Virginas, B.: Expensive multi-objective optimization by MOEA/D with gaussian process model. *IEEE Transactions on Evolutionary Computation* **4**(3), 456–474 (2010)
34. Zitzler, E., Deb, K., Thiele, L.: Comparison of multiobjective evolutionary algorithms: Empirical results. *Evolutionary Computation Journal* **8**(2), 125–148 (2000)