

Visualization-aided Multi-criterion Decision-making Using Reference Direction Based Pareto Race

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Abstract—The goal of a multi-criteria decision-making (MCDM) approach is to select one or a few preferred solutions in an iterative manner from a set of Pareto-optimal solutions obtained by a generative or simultaneous evolutionary multi- and many-objective optimization (EMO and EMaO) algorithm. In each iteration, the decision-maker (DM) formulates a suitable scalarized optimization problem using preference information that guides the DM to arrive at the most desirable solution set. Visualization of trade-offs among multiple objectives and their interactions with constraints can provide crucial decision-making information. In this paper, we propose a visualization-assisted MCDM approach that utilizes interpretable Self-Organizing Maps (iSOM) on a well-known MCDM technique known as Pareto Race. The proposed method, applied to one test and two real-world problems involving three to five objectives, demonstrates the usefulness of the iSOM-visualization method in implementing Pareto Race decision-making approach. The study opens up further avenues for integrating iSOM visualization approach with other MCDM techniques.

Index Terms—Multi-criteria decision making, Pareto Race, Pareto-optimal front, Self-Organizing Maps.

I. INTRODUCTION

Real-world optimization problems usually deal with multiple (generally) conflicting objectives subject to several constraints. Evolutionary algorithms (EAs) are the well-known approaches for solving such multi-objective optimization (MOO) problems for evaluating a set of non-dominated solutions that are all equally good without further preference information [1], [2]. The decision maker (DM) may not like to look at a set of non-dominated solutions, instead, the DM prefers one or more specific solutions amongst the ones in the Pareto-optimal front, favoring their preferences or criterion. Multi-criteria decision-making (MCDM) techniques have been widely used for providing such preference-based solutions based on the criteria provided by the DMs [3], [4].

MCDM algorithms can be broadly classified into three categories based on the criteria provided by the DM and evaluation of non-dominated near Pareto-optimal solutions. The first category of techniques evaluates a set of near Pareto-optimal solutions followed by finding the preferred solution(s) based on the criterion provided by the DM [5]. However, computing the complete Pareto-optimal front is computationally

expensive, and most of the solutions are discarded once the aspiration level is attained [4]. However, the advantage is that the DM can compare the preferred solutions with the neighborhood solutions.

The second category of MCDM techniques does not require information on the complete Pareto front. Based on the DM's criteria, a specific part of the Pareto-optimal front is generated using the reference point or reference direction-based evolutionary multi-objective (R-EMO) algorithms that require a less computational cost. The DMs can compare and conveniently select the most preferred solutions amongst the few solutions obtained through the R-EMO algorithms.

The third category of MCDM techniques utilizes the idea of scalarization function based on the reference point, reference direction, and aspiration level to re-formulate the MOO problem into a scalarized sub-problem and provide a single Pareto-optimal solution [6], [7]. This category of MCDM techniques has the least computational cost but it does not permit a comparison of the preferred solution with the neighborhood solutions. From the DM's perspective, a mixed form of these three classes of MCDM technique allows a convenient interaction and intervention of the DM in each iteration of the algorithm as it progresses towards the aspiration level.

During the decision-making task, DM might be interested to infer the specific properties of the preferred solution(s) such as closeness to the- (i) Pareto disjoint boundaries, (ii) edge and corner of the Pareto-optimal front, (iii) constraint boundaries, (iv) trade-off among the objectives, etc. In the process of finding out the most preferred Pareto-optimal solution using MCDM methods, efficient and interactive visualization technique(s) can aid to obtain the Pareto-optimal solution that is very close to the aspiration level [5], [8]. Particularly, when reformulating the MOO problem into a scalarized sub-problem, DM usually provides the criteria in the form of aspiration level, reference point and/or reference direction. These criteria can be interactively provided by the DM with the aid of visualization techniques. Thus, expediting the process of attaining the aspiration level, resulting in the reduction of computational time. Therefore, algorithmic versions of MCDM techniques might not be sufficient and it is desirable to have a user

interactive, preferably visual approach to supply preferences. Current work discusses one such MCDM technique called Pareto Race and uses iSOM, a neural network approach to allow an interactive way to provide the DM's preferences.

The rest of the paper is organized as follows- Section II discusses the literature on visualization-aided interactive MCDM approaches followed by Pareto Race in Section III. A brief introduction to iSOM – a data visualization method– used in this paper is discussed in Section IV. Integration of iSOM into Pareto Race framework is discussed in Section V, followed by the Results in Section VI, Computational Complexity in Section VII, and Conclusion in Section VIII.

II. VISUALIZATION-AIDED INTERACTIVE MULTI-CRITERIA DECISION MAKING (MCDM) APPROACHES

Interactive MCDM approaches such as- NIMBUS [9], Pareto Race [10] and its non-linear version i.e. quadratic Pareto Race [11], Pareto Navigator, surrogate worth trade-off (SWT) [12], satisficing trade-off method (STOM) [13], GUESS [14], and other hybrid methods [15] conveniently and iteratively utilize the combined form of EMO and EMaO algorithms. Generally, the interactive MCDM approaches start with finding out an initial Pareto-optimal solution by projecting the start point (generally, a feasible but not necessarily Pareto-optimal solution) provided by the DM on the non-dominated set of near Pareto-optimal front. Next, the preferred solution(s) is obtained by re-formulating the MOO problem into a sub-problem(s) by applying the concept of scalarization methods such as achievement scalarization function (ASF) or its augmented version (AASF) [7] by utilizing the DM's criteria in the form of classification of objectives or reference direction or aspiration level. These scalarization methods combined with the reference point and reference direction-based evolutionary algorithms (R-EMO) such as RD-NSGA-II [16] and R-NSGA-III [17] provide a set of non-dominated Pareto-optimal preferred solutions in the vicinity of the current solution that can be visually compared using the effective visualization technique(s) to arrive at the most preferred solution or near aspiration level.

The visualization techniques used for interactive MCDM include parallel co-ordinate plot (PCP), bar chart, petal diagram, scatter plot, RadViz plot, heat maps, among others. The usual limitations of the mentioned approaches are:

- 1) they typically choke with increase in the number of dimensions (objectives and variables) of the MOO problem,
- 2) difficulty in comparing large set of preferred solutions,
- 3) high computational cost,
- 4) need for data pre-processing,
- 5) inability to capture the functionality of the original Pareto-optimal front,
- 6) lack of interactiveness and loss of interpretability.

III. PARETO RACE

Pareto Race, an interactive MCDM approach proposed by Korhonen and Wallenius represents a dynamic, visual, and

interactive procedure for multiple objective linear programming [10]. DM provides an initial reference point (a feasible solution) that is projected to the Pareto-optimal front to obtain the start point, from which the DM provides a reference direction and step size to arrive at the most preferred solution or aspiration level iteratively. In Pareto Race, a DM can freely search the efficient frontier of a multiple objective linear programming problem by controlling the direction (reference direction) and speed (step size) of motion. The DM can visualize the objective function values in the numeric form on the bar graphs whose lengths are dynamically changing as DM moves on the efficient frontier. Based on the step size and reference direction, the user controls include gears, an accelerator, brakes, and a steering mechanism, similar to as a driving vehicle that can attain different stages i.e. 'Accelerator', 'Gear: forward', 'Gear: backward', 'Turn', 'Brake' and 'Exit'. The description of different user-controlled commands used in Pareto Race algorithms are listed as follows:

1. **Accelerator:** Proceed in the current reference direction at a constant speed.
2. **Gears (Forward):** Increase speed in forward direction.
3. **Gears (Backward):** Increase speed in backward direction.
4. **Brakes:** Reduce speed.
5. **Turn:** Change the reference direction.
6. **Exit:** Terminate the Pareto Race.

The non-linear version of Pareto Race i.e. quadratic Pareto Race reduces the original problem into a linear complementarity problem, whereas Pareto Navigator switches the search space from the original Pareto-optimal front to approximated polyhedral space and finds the non-dominated solutions in polyhedral space that is projected on the original Pareto-optimal front. In the current work, Pareto Race based on the reference direction proposed by Deb and Kumar [16] is discussed. The approach can effectively be applied to both the linear and non-linear MOO problems.

The speed of the search in the user-specified reference direction can be changed by 'step size' parameter [18]. The bar graph combined with Pareto Race compares the preferred solution in objective function space obtained in different iteration. However, it is also important for a DM to infer the specific properties of the preferred solution(s) such as closeness to the- (i) Pareto disjoint boundaries, (ii) edge and corner of the Pareto-optimal front, (iii) constraint boundaries, (iv) trade-off among the objectives, etc. In the current paper, while iteratively proceeding from the current solution to the aspiration level; in addition to the direction and step size, specific properties of the preferred solution(s) are also the key factors that are being visualized by an efficient visualization technique known as interpretable Self-Organizing Maps (iSOM).

IV. INTERPRETABLE SELF-ORGANIZING MAPS (iSOM) VISUALIZATION METHOD

Self-organizing Maps (SOM) is a type of Artificial Neural Network (ANN) that maps high-dimensional data to 2-dimensional (usually) representation [19] termed as component plane. Each neuron called as SOM node, is associated

or assigned with a weight vector named as SOM code-book vector whose dimension is equal to the high dimensional data. These nodes are associated with their neighboring nodes in two-dimensional space, to form a rectangle or hexagon. The two-step SOM algorithm includes initialization as the first step followed by training as the second step to distort the SOM grid such that it captures the original high-dimensional dataset in a 2-dimensional representation. The output of the SOM is 2-dimensional component planes for each dimension and a U-matrix that identifies the disjoint regions providing the cluster information of the dataset.

During the initialization, SOM assigns some initial weights to the SOM nodes chosen randomly or linearly from the dataset. The training step includes finding the best matching units (BMUs) based on the euclidean distance calculation between SOM nodes and dataset followed by the weight update rule iteratively, such that the SOM grid maps the high-dimensional data into 2-dimensional representation. However, this initialization and training suffer with self-intersection and folding of the SOM grid resulting in loss of topography [19].

Modification in the initialization and training has been proposed to overcome the issues of self-intersection and folding of SOM [20]. The modification enables inherent interpretability of component planes, hence termed as interpretable Self-Organizing Maps (iSOM). Further details on iSOM initialization and training is documented in [20]. A recent study [21] compared conventional Self-Organizing Maps (cSOM) and iSOM in the context of visualizing multi-objective Pareto-optimal solutions and demonstrated the advantage of iSOM. The current work utilizes the iSOM in the Pareto Race algorithm to visualize the steps for easier, better, and more informed MCDM. iSOM nodes with different face color and edge color allow visualization of the different steps of the Pareto Race algorithm permitting informed decision making.

V. ISOM-PARETO RACE TECHNIQUE FOR INTERACTIVE MCDM

The original Pareto Race method is only applicable for multi-objective linear programming problems. Quadratic Pareto Race is applicable for problems with one quadratic and multiple linear objectives subject to linear constraints. Pareto Navigator is the non-linear extension of the Pareto Race that requires only a few representatives of the Pareto-optimal set to approximate the polyhedral for searching for the most preferred solution. Then, the preferred solutions in the approximated polyhedral space are projected into the original Pareto-optimal front to arrive at the final solution.

The proposed iSOM visualization-aided Pareto Race method uses the idea of the original Pareto Race (navigating on the Pareto-optimal front) by utilizing the reference direction-based evolutionary algorithm (RD-NSGA-II) [16] that can be effectively applied on both linear and non-linear MOO problems.

Consider the MOO problem

$$\begin{aligned} \min_{(\mathbf{x})} \quad & \{f_1(\mathbf{x}), \dots, f_M(\mathbf{x})\}, \\ \text{subject to:} \quad & \mathbf{g}(\mathbf{x}) = (g_1(\mathbf{x}), \dots, g_J(\mathbf{x}))^T \leq 0, \end{aligned} \quad (1)$$

where, $\mathbf{x} = (x_1, \dots, x_n)^T$ is a n dimensional design variable vector and $\mathbf{f}(\mathbf{x}) = \{f_1(\mathbf{x}), \dots, f_M(\mathbf{x})\}^T$ is a vector of M objective functions subjected to J constraints $\mathbf{g}(\mathbf{x})$.

Pareto Race algorithm starts with finding a non-dominated Pareto-optimal start point \mathbf{z}_0 in objective function space obtained from projecting a user-supplied feasible point \mathbf{x}_s or a reference point \mathbf{z}_s (not necessarily a Pareto-optimal solution) on the Pareto-optimal set, based on the DM's aspiration level. Next, DM provides a reference point \mathbf{z}_1^r to the objectives based on the aspiration level to define the reference direction $\bar{\mathbf{z}}_1^r$ as follows

$$\bar{\mathbf{z}}_1^r = \mathbf{z}_1^r - \mathbf{z}_0. \quad (2)$$

If values of reference point \mathbf{z}_s are not known explicitly, DM can also classify (' $I^<$ ', ' $I^='$ ' and ' $I^>$ ') the objectives based on the aspiration level. Classification ' $I^<$ ' implies that the assigned objective function should be improved, ' $I^='$ ' implies that the current value of the objective function is satisfactory and should be retained for the next iteration, whereas, ' $I^>$ ' implies that assigned objective function can change freely. Next, nadir point (\mathbf{z}^{nad}) and ideal point (\mathbf{z}^*) are used to estimate the reference point \mathbf{z}_0 . Consider the classification defined to a 5 objective MOO problem, at a point \mathbf{z} as following

$$I^< = \{1, 5\}, I^= = \{3\}, \text{ and } I^> = \{2, 4\};$$

then the reference point for this classification is

$$\mathbf{z}_1^r = [z_1^*, z_2^{\text{nad}}, z_3, z_4^{\text{nad}}, z_5^*].$$

Finally, this reference point \mathbf{z}_1^r is used for calculating the reference direction using (2). This reference direction ($\bar{\mathbf{z}}_1^r$) along with the appropriate step size parameter (t) is used in the RD-NSGA-II [16] method for evaluating the preferred solution(s) for the next iteration using the achievement scalarized function (ASF) or its augmented version (AASF)

$$\begin{aligned} \min_{\mathbf{z}} \quad & s(\mathbf{z}, \mathbf{r}, \mathbf{w}) = \max_{i \in I} \left[\frac{z_i - r_i}{w_i} \right]; \\ \text{subject to:} \quad & \mathbf{r}(t) = \mathbf{z}_{t-1} + t \times \bar{\mathbf{z}}_1^r, \quad t \in (0, \infty]. \end{aligned} \quad (3)$$

Selection of reference direction ($\bar{\mathbf{z}}_1^r$) and step size (t) allows the DM to navigate or freely search on the Pareto-optimal front by assigning the positive weights w_i to the objective functions.

The steps used in iSOM-Pareto Race method are described in the following

Step 1: Compute a well-distributed set of non-dominated Pareto-optimal set using the evolutionary algorithm. It is to be noted that ideal point (\mathbf{z}^*) is evaluated by minimizing each objectives function, whereas nadir point (\mathbf{z}^{nad}) is estimated as discussed in [22]. For each objective function, plot iSOM component plane along with U-matrix. iSOM component planes permit identifying the interaction among the objective

TABLE I

COLOR CODE FOR ISOM VISUALIZATION: EDGE (EDGE OF ISOM NODE), FACE (FACE OF ISOM NODE), COLOR (EDGE COLOR/ FACE COLOR OF CORRESPONDING ISOM NODE), NODE/ ARROW (FOR DEMO PURPOSE)

S.No.	Points	Edge	Face	Color	Node
1	Boundary points	Yes	No	Black	
2	Near-constraint points	No	Yes	Grey	
3	Start point/current solution	No	Yes	Black	
4	Turn	No	Yes	Magenta	
5	Accelerator	No	Yes	Green	
6	Brakes, Gears	Yes	No	Red	
7	Final solution	No	Yes	Red	
8	Progress of Pareto Race	—	—	—	

functions, whereas, Pareto disjoint regions can be identified from the U-matrix and highlighted using the color code defined in Table I (S.No. 1).

Step 2: Plot a component plane indicating the average normalized constraint violation ‘ G ’ to highlight the feasible near-constraint violation solutions [17]. The values for plotting ‘ G ’ is computed using the following equation

$$G(\mathbf{x}) = \frac{1}{J} \sum_{j=1}^J \hat{g}_j(\mathbf{x}) [\mathbb{1}(\hat{g}_j(\mathbf{x}) \leq 0)] \quad (4)$$

where, \hat{g} is the normalized constraint, $\mathbb{1}$ is the indicator function that takes value 1 if the solution is feasible and 0 otherwise. G is sorted in descending order and the top 10% are assigned as near constraint violation solutions and highlighted using the color code defined in Table I (S.No. 2).

Step 3: Plot the other component plane indicating the trade-off solutions ‘ T ’. This allows the DM to navigate toward the high trade-off region(s). The values for iSOM component plane ‘ T ’ is computed based on the minimum loss versus maximum gain for a non-dominating point while moving to any neighboring point defined within a Pareto-optimal set.

Step 4: Plot \mathbf{z}_0 of the Pareto Race on iSOM component planes using the color code defined in Table I (S.No. 3).

Step 5: Find the reference direction ($\bar{\mathbf{z}}_1^r$) by classifying the objectives or providing the aspiration level (\mathbf{z}_1^r).

Step 6: Provide the new reference direction for changing the current direction and highlight it using the color code defined in Table I (S.No. 4).

Step 7: Navigate in the current direction by changing the size parameter ‘ t ’. Keep ‘ t ’ constant for ‘**Acceleration**’ or decrease ‘ t ’ for applying ‘**Brakes**’ and highlight the obtained solutions on iSOM component plane using the color code defined in Table I (S.No. 5). Else go to Step 6.

Step 8: Increase ‘ t ’ in the direction of $\bar{\mathbf{z}}_1^r$ or $-\bar{\mathbf{z}}_1^r$ for ‘**Gears (Forward)**’ or ‘**Gears (Backward)**’ respectively and highlight the obtained solutions on iSOM component plane using the color code defined in Table I (S.No. 6). Else go to Step 6.

Step 9: If aspiration level is reached, select the final solution and highlight as per Table I (S.No. 7) and terminate. Else go to Step 6.

Step 10: The black color arrow (S.No. 8 in Table I) indicates the progress of preferred solutions highlighted by letters- a, b, \dots . The speed of each iteration is defined by the numbers ranging from 1 to 5. 1 signifying the lowest speed, and 5, the highest speed.

VI. RESULTS

One benchmark problem and two engineering design problems are considered to demonstrate the proposed iSOM-Pareto Race approach.

A. Constrained problem: C2-DTLZ2

Decision-making on a constrained MOO problems benefits from a visualization framework revealing the near constraint solutions. A three-objective C2-DTLZ2 problem is considered to demonstrate the effectiveness of the iSOM visualization method in identifying the near constraint points.

For the given problem, ideal point ($\mathbf{z}^* = [0, 0, 0]$) and nadir point ($\mathbf{z}^{\text{nad}} = [1, 1, 1]$) are given. A well-distributed set of non-dominated Pareto-optimal front is used for plotting the iSOM component planes and U-matrix of objectives functions (f_i) as shown in the Fig. 1. The color of iSOM nodes in the component plane indicates the values of the respective objective function. Hence the correlation amongst component planes of the objective functions (f_i) is inherently interpretable. U-matrix in Fig. 1 represents 4 disjoint regions that have been highlighted with black edge colored iSOM nodes in component planes of objectives (f_i). This completes Step 1 of iSOM-Pareto Race method.

An iSOM component plane for the average of normalized constraint violation values (G) has been plotted using (4) and top 10% near constraint violation solutions have been highlighted by iSOM nodes with gray face color in iSOM component planes of f_i and G as shown in Fig. 1. This completes Step 2.

Component plane T (shown in Fig. 1) indicating the trade-off values among the objective functions aids the DM to choose the reference direction or define the aspiration level with high trade-off values. This completes Step 3.

The progress of Pareto Race is in iSOM component plane ‘ T ’ (Fig. 1), in which, each solution is associated with a letter and a number. The letter indicates the current solution whereas the number indicates the speed of search. ‘0’ indicates the termination of the algorithm and the associated letter indicates the final solution.

A reference point $\mathbf{z}_s = [0.2, 0.8, 0.5]$ based criteria is given as an initial point. The resulting Pareto-optimal start point is $\mathbf{z}_0 = [0.047, 0.956, 0.286]$. The start point \mathbf{z}_0 highlighted with

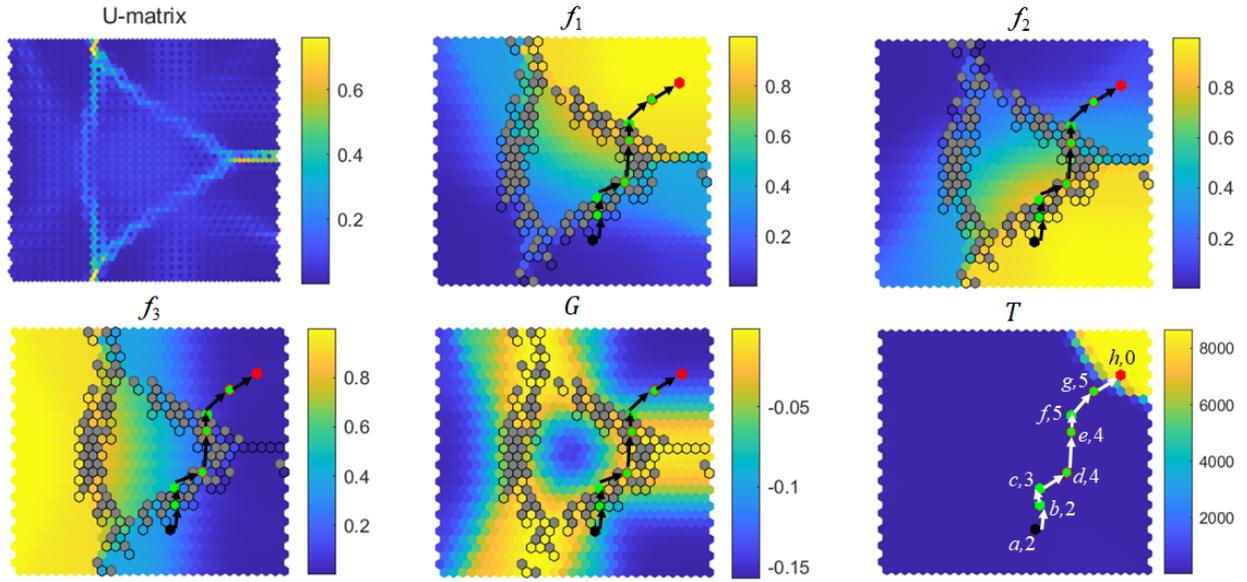


Fig. 1. C2-DTLZ2 problem: U-matrix identifies the Pareto disjoint regions, f_1 , f_2 and f_3 are iSOM component plane of objectives, G and T are iSOM component plane of average of normalized constraint violation and trade-off values, respectively. Color code for the Pareto Race points is defined in Table I. Arrow indicates the progress of the Pareto Race-based search, the letter indicates the solution and the number indicates the speed.

iSOM nodes with black face color in iSOM component planes of f_i and G are shown in Fig. 1. This completes Step 4. The start point \mathbf{z}_0 is very close to the constraint boundary (near gray face color iSOM nodes) and Pareto disjoint boundary (near to iSOM nodes with black edge color). In addition, it corresponds to the lower trade-off (T) values. Hence, DM would decide to proceed in a direction to obtain better trade-off values utilizing the visual aid of iSOM.

In this regard, from the start point \mathbf{z}_0 , the following preferences to the objectives will be provided by the DM-

$$I^< = \{2, 3\}, I^= = \phi, \text{ and } I^> = \{1\},$$

that result in the following reference direction

$$\bar{\mathbf{z}}_1^r = \mathbf{z}_0 - [z_1^{\text{nad}}, z_2^*, z_3^*],$$

as the current search direction that completes Step 5 of the procedure. From the start point 'a', in the current search direction $\bar{\mathbf{z}}_1^r$ with the speed of '2', a preferred solution 'b' ($\mathbf{z}_1 = [0.158, 0.925, 0.343]$) is computed which again lies near to the constraint boundary and boundary of the Pareto disjoint region. In the current direction the following solutions-

- 'c': $\mathbf{z}_2 = [0.387, 0.836, 0.388]$, Speed set at 2,
- 'd': $\mathbf{z}_3 = [0.486, 0.793, 0.366]$, Speed set at 3,
- 'e': $\mathbf{z}_4 = [0.807, 0.508, 0.299]$, Speed set at 4,
- 'f': $\mathbf{z}_5 = [0.929, 0.319, 0.185]$, Speed set at 4;

are computed in successive iterations with increased speed heading towards the larger trade-off values. However, it can be visualized from the iSOM plot that these solutions correspond to the large values of f_2 and are near the constraint boundary and Pareto disjoint boundary. The solutions at constant speed are highlighted by the iSOM nodes with green face color,

whereas, the solutions with change in the speed are highlighted by the iSOM nodes with red edge color and green face color according to the color code defined in Table I. This completes Step 7 of the procedure.

Following the same reference direction $\bar{\mathbf{z}}_1^r$, DM would wish to increase the speed to attain the solution with a higher trade-off value region faster, as shown in iSOM component plane ' T '. Therefore, from the current solution i.e. 'f' ($\mathbf{z}_5 = [0.929, 0.319, 0.185]$) and in the current direction i.e. $\bar{\mathbf{z}}_1^r$, DM increases the speed and sets it at '5', to arrive at a new solution 'g' ($\mathbf{z}_6 = [0.968, 0.217, 0.121]$) that is relatively closer to the higher trade-off region. Again, DM would move in the current direction with the constant speed and arrive at the solution 'h' ($\mathbf{z}_7 = [0.985, 0.155, 0.067]$) that corresponds to a high trade-off region, far from the constraint boundary and Pareto disjoint region, and relatively smaller values of f_2 and f_3 . Finally, 'h' ($\mathbf{z}_7 = [0.985, 0.155, 0.067]$) is declared as final solution (iSOM node with red face color) and Pareto Race is terminated assigning the current speed as '0'. This completes Step 8 of the iSOM-Pareto Race procedure.

This benchmark example demonstrates the visualization-aided decision-making on a constrained MOO problem, by allowing the DM to visually compare and control the search of the preferred solution by avoiding the solutions closer to the constraint boundary and Pareto disjoint region; and arrive at a solution with high trade-off values. It is to be noted that the Pareto Race algorithm operates in the objective function space. However, in the proposed approach we are able to identify the potential interest points by being able to visualize the search point in ' G ' and ' T ', permitting intuitive decision making.

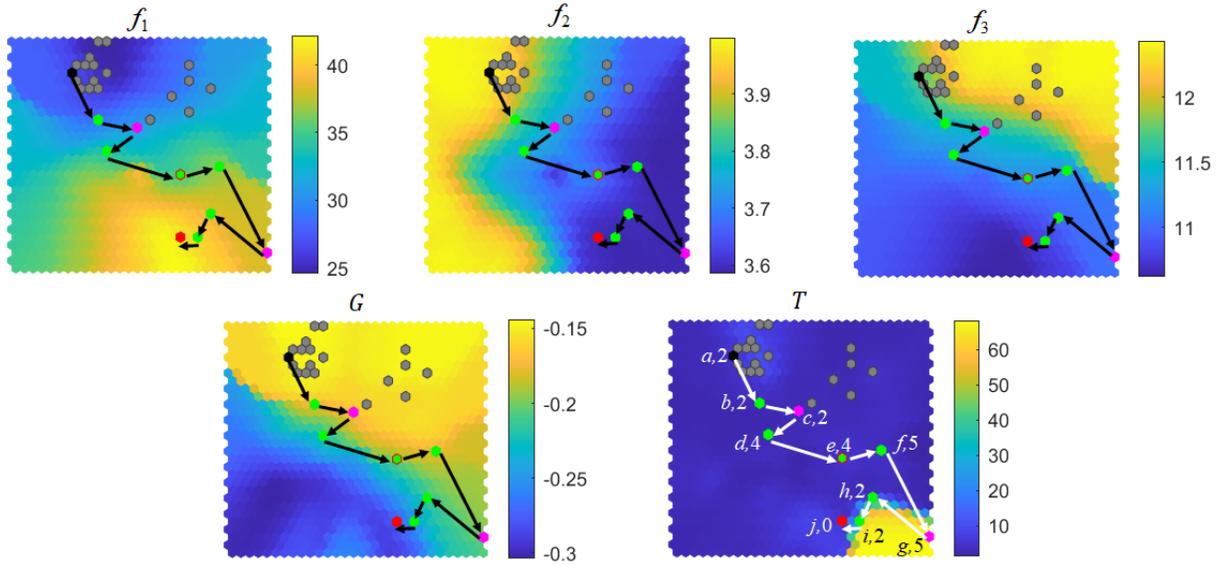


Fig. 2. Car side-impact problem: iSOM component plane of objectives, G and T . Color code for the Pareto Race points is defined in Table I. Arrow indicates the progress of the Pareto Race-based search, the letter indicates the solution and the number indicates the speed.

B. Car side-impact problem

In this section, we demonstrate the proposed approach to the real-world multi-objective optimization car side-impact problem. This problem has 7 design variables and it is subjected to 10 constraints to minimize the 3 objectives namely- the weight of the car, the pubic force experienced by a passenger, and the average velocity of the B-pillar responsible for withstanding the impact load [16].

For the given problem ideal point (\mathbf{z}^*) and nadir point (\mathbf{z}^{nad}) are evaluated. A well-distributed set of Pareto-optimal solutions are used to generate the iSOM component plane of objectives f_i , average constraint violation values G , and trade-off values T . Top 10% of near CV solution has been highlighted with iSOM nodes with gray face color shown in Fig. 2. This completes Steps 1, 2, and 3 of iSOM aided Pareto Race method. A start point ‘a’ ($\mathbf{z}_0 = [26.899, 3.999, 11.504]$) is computed using the weighted sum of the objectives, that is close to the near constraint violation solutions. It has higher f_2 whereas moderate f_3 values and relatively small trade-off values. DM would wish to improve the objectives f_2 and f_3 at the cost of f_1 , and arrive at a solution far from the constraint boundary with a better trade-off value by defining the following criteria to objective functions

$$I^< = \{2, 3\}, I^= = \phi, \text{ and } I^> = \{1\},$$

that result in the following reference direction-

$$\bar{\mathbf{z}}_1^r = \mathbf{z}_0 - [z_1^{\text{nad}}, z_2^*, z_3^*],$$

as the current direction of search that completes Step 4 and 5 of the procedure. Setting the speed of ‘2’, the following two solutions (‘b’ and ‘c’) are computed in the reference direction

$\bar{\mathbf{z}}_1^r$ in two successive iterations-

$$\text{‘b’}: \mathbf{z}_1 = [28.704, 3.915, 11.406], \text{ Speed set at } 2,$$

$$\text{‘c’}: \mathbf{z}_2 = [31.091, 3.788, 11.352], \text{ Speed set at } 2;$$

and highlighted in the Fig. 2 as per color code defined in Table I. From the iSOM component plane of T , it is evident that the current direction is not heading towards the region of large trade-off values. Hence, DM would provide the following criteria

$$I^< = \{2\}, I^= = \{3\}, \text{ and } I^> = \{1\},$$

that results in the following reference direction-

$$\bar{\mathbf{z}}_2^r = \mathbf{z}_2 - [z_1^{\text{nad}}, z_2^*, z_3^2],$$

leading towards the higher T value. This completes Step 6 of the procedure. In the new reference direction, $\bar{\mathbf{z}}_2^r$ the following four solutions (‘d’, ‘e’, ‘f’ and ‘g’) have been computed with increased speed to quickly obtain large trade-off region

$$\text{‘d’}: \mathbf{z}_3 = [33.853, 3.696, 11.290], \text{ Speed set at } 2,$$

$$\text{‘e’}: \mathbf{z}_4 = [35.336, 3.642, 11.247], \text{ Speed set at } 4,$$

$$\text{‘f’}: \mathbf{z}_5 = [36.538, 3.585, 11.256], \text{ Speed set at } 4,$$

$$\text{‘g’}: \mathbf{z}_6 = [38.121, 3.585, 11.044], \text{ Speed set at } 5.$$

The solutions with constant and varying speeds have been highlighted as per the color code defined in Table I. Though the current solution ‘g’ = \mathbf{z}_6 corresponds to the higher trade-off values, DM further would wish to improve on the objective f_3 by providing the following criteria

$$I^< = \{3\}, I^= = \{2\}, \text{ and } I^> = \{1\},$$

that provides a new reference direction-

$$\bar{\mathbf{z}}_3^r = \mathbf{z}_6 - [z_1^{\text{nad}}, z_2^6, z_3^*],$$

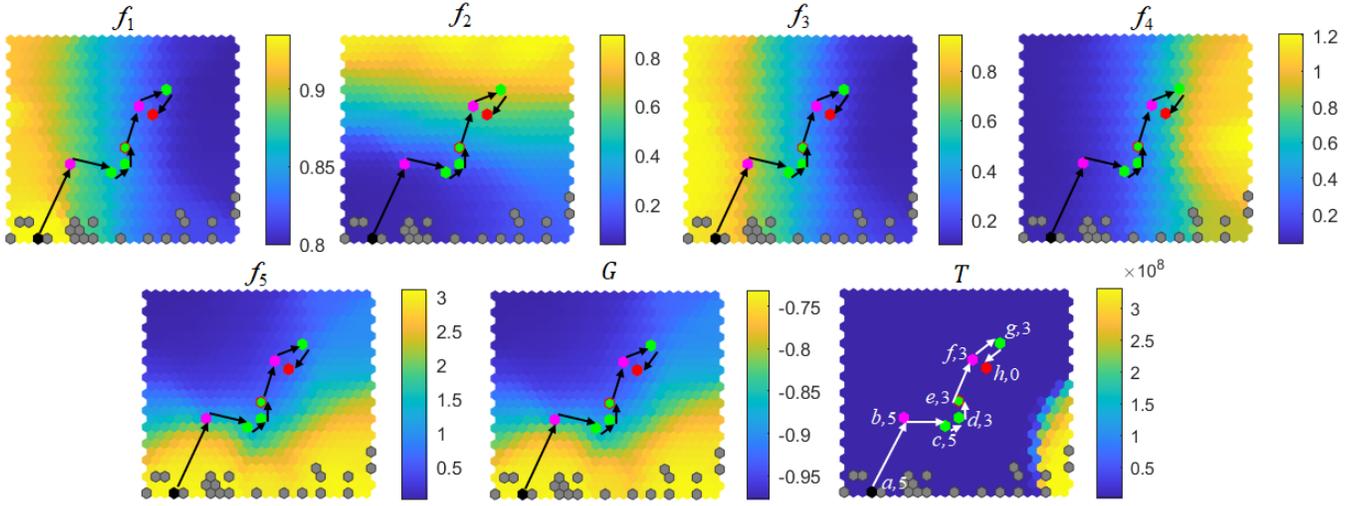


Fig. 3. WATER problem: iSOM component plane of objectives, G and T . Color code for the Pareto Race points is defined in Table I. Arrow indicates the progress of the Pareto Race-based search, the letter indicates the solution and the number indicates the speed.

along which the following solutions (' h ', ' i ' and ' j ') have been evaluated with reduced speed and highlighted as per color code provided in Table I

- ' h ': $\mathbf{z}_7 = [38.676, 3.585, 10.997]$, Speed set at 5,
- ' i ': $\mathbf{z}_8 = [40.237, 3.585, 10.854]$, Speed set at 2,
- ' j ': $\mathbf{z}_9 = [41.197, 3.585, 10.756]$, Speed set at 2;

DM would select the solution ' j ' = \mathbf{z}_9 that is far from the constraint boundary, corresponding to the moderate trade-off value, and relatively smaller f_2 and f_3 values at the cost of higher f_1 value as the final solution highlighted by iSOM nodes with red face color and terminated the Pareto Race by setting the speed to '0'. This example demonstrates the usefulness of iSOM-visualization aided Pareto Race by assisting the DM to change the reference direction and set the speed of the search for preferred solutions based on the user-defined criteria. There is a saving in computational time as iSOM visualization-aided Pareto Race reduces the number of iterations taken to arrive at the aspiration level.

C. Water problem

In this section, we demonstrate the proposed iSOM-Pareto Race method on a 5-objective WATER problem [23]. iSOM component plane of f_i , ' G ' and ' T ' are plotted in Fig. 3. It is evident that the region corresponding to the high trade-off values is near the constraint boundary. A start point ' a ' ($\mathbf{z}_0 = [0.934, 0.0449, 0.879, 0.050, 2.300]$) using the weighted sum of the objectives is evaluated and plotted on iSOM component planes shown in the Fig. 3. The start point corresponds to the higher values of f_1 , f_3 , and f_5 that are closer to the constraint boundary. With the visual aid of iSOM, the following criteria are provided by the decision maker

$$I^< = \{1,3,5\}, I^= = \{4\}, \text{ and } I^> = \{2\},$$

resulted in the following reference direction

$$\bar{\mathbf{z}}_1^r = \mathbf{z}_0 - [z_1^*, z_2^{\text{nad}}, z_3^*, z_4^0, z_5^*],$$

that leads toward the solutions away from the constraint boundary and improves the current values of objective functions f_1 , f_3 , and f_5 . In the current reference direction ($\bar{\mathbf{z}}_1^r$), a new solution ' b ' ($\mathbf{z}_0 = [0.888, 0.110, 0.735, 0.076, 0.925]$) is computed by setting the speed at '5'.

From the iSOM component plane, it is evident that improvement in the objectives f_1 , f_3 , and f_5 comes at the cost of f_4 , hence the following criteria is provided by the DM

$$I^< = \{1,3,5\}, I^= = \phi, \text{ and } I^> = \{2,4\},$$

resulting in the following reference direction

$$\bar{\mathbf{z}}_2^r = \mathbf{z}_1 - [z_1^*, z_2^{\text{nad}}, z_3^*, z_4^{\text{nad}}, z_5^*],$$

along which the following solutions (' c ', ' d ', ' e ', ' f ' and ' g ') have been evaluated with the visual aid of iSOM

- ' c ': $\mathbf{z}_2 = [0.859, 0.120, 0.532, 0.176, 1.197]$, Speed set at 5,
- ' d ': $\mathbf{z}_3 = [0.847, 0.201, 0.428, 0.278, 0.896]$, Speed set at 5,
- ' e ': $\mathbf{z}_4 = [0.842, 0.311, 0.405, 0.299, 0.561]$, Speed set at 3,
- ' f ': $\mathbf{z}_5 = [0.831, 0.615, 0.329, 0.412, 0.316]$, Speed set at 3,
- ' g ': $\mathbf{z}_6 = [0.812, 0.713, 0.193, 0.722, 0.549]$, Speed set at 3.

in order to improve the values of objective functions f_1 , f_3 , and f_5 . The computed solutions are highlighted on the iSOM component plane shown in Fig. 3 as per the color code defined in Table I.

It is evident from the iSOM component plane that the current solution (' g ' = \mathbf{z}_6) improved with respect to f_1 , f_3 and f_5 but it has higher f_2 and f_4 values that can be improved by providing the following criteria

$$I^< = \{2,4,5\}, I^= = \phi, \text{ and } I^> = \{1,3\},$$

allowing the search in the following reference direction

$$\bar{\mathbf{z}}_3^r = \mathbf{z}_6 - [z_1^{\text{nad}}, z_2^*, z_3^{\text{nad}}, z_4^*, z_5^*].$$

In the current reference direction $\bar{\mathbf{z}}_3^r$ a new solution ‘ h ’ ($\mathbf{z}_7 = [0.817, 0.551, 0.227, 0.657, 0.6226]$) has been evaluated that improved with respect to f_1, f_3 and f_5 and has moderate value of f_2 and f_4 . Finally, \mathbf{z}_7 is assigned as the final solution (iSOM node with red face color) and Pareto Race has been terminated by setting the speed to ‘0’.

iSOM-visualization aids the DM in inferring the fact that the high trade-off region and near to constraint boundaries appear together in this example. Hence, DM changes the criteria from obtaining a better trade-off solution to moving away from constraint boundary so that all the objectives attain either moderate or smaller values.

VII. COMPUTATIONAL COMPLEXITY

The number of samples (N) has a linear $O(N)$ effect, whereas the map unit (m) has a quadratic $O(m^2)$ effect on the computational complexity of iSOM. However, the computational complexity of iSOM has a linear effect of $O(n+M+2)$ with respect to the number of design variables (n), the number of objective functions (M), and iSOM component planes of average constraint (G) and trade-off (T) metrics. Hence, the iSOM algorithm has an overall computational complexity of order $O(Nm^2(n+M+2))$.

VIII. CONCLUSIONS

This paper has highlighted the fact that MCDM methods can be implemented more effectively by using a suitable visualization technique. An MCDM method involves the DM providing preferential information about potential trade-off solutions at every iteration. However, treating a solution as a vector of real objective values or comparing two or more trade-off solutions based on their objective values as a computational or mathematical task may not result in finding the most desired solution. The DM needs to anticipate a solution’s proximity to constraint boundaries, and its local trade-off among objectives, and clearly visualize the possible extent of gains and sacrifices before choosing a preferred solution. A suitable visualization of objectives, their interactions, and feasible region must accompany any effective decision-making task.

To this end, the current work has demonstrated how the iSOM visualization can aid in executing the Pareto Race MCDM concept. The proposed combined approach has been demonstrated on one analytical benchmark and two real-world examples. The detailed description with iSOM plots has revealed the usefulness of the interactive visualization-aided MCDM approach. The iSOM visualization has been able to provide a clear idea of Pareto disjoint boundaries, near constraint boundary regions, high trade-off solutions, and interactions among the objectives in deciding on choosing appropriate step sizes and directions of the Pareto Race method. The study paves the way to actively involve the DM in the decision process toward a more effective decision-making task.

As a future extension of this work, first, we propose to extend the iSOM visualization to other MCDM methods. Multiple MCDM methods can be applied at each iteration, but the final selection of the next iterative solution can be resolved with the user’s intervention with the aid of iSOM visualization method. On the flip side, the iSOM plots can provide vital information on the classification of objectives for possible improvements, sacrifice or status quo, and the extent of such changes allowed by constraint boundaries and Pareto set boundaries in objective directions. The iSOM plots can provide a birds-eye view of all objective, constraint satisfaction, and trade-off landscapes which no other computational, mathematical, or visual aid can provide. The demonstration of the iSOM’s visualization aid to Pareto Race concept demonstrated here stays as a testimony to this effect.

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