

Optimized Electric Machine Design Solutions with Efficient Handling of Constraints

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Abstract—Design optimization of electric machines often requires time-expensive Finite Element Analysis (FEA) to evaluate objective functions accurately. In contrast to time-consuming simulations, the constraints of a feasible design can be based on relatively simple geometric expressions, resulting in a mixed computationally expensive, constrained multi-objective optimization problem. The proposed optimization method incorporates a customized repair operator into the well-known evolutionary multi-objective optimization algorithm NSGA-II. The repair operator exploits the low computational cost of constraints to ensure the feasibility of all solutions. The effectiveness of the proposed method is demonstrated by comparing results from two different optimization approaches. Apart from the optimization method itself, a physical explanation of Pareto-optimal solutions and insights about electric machine design are also discussed. Finally, a posteriori approach for selecting preferred solutions from the non-dominated set is presented while highlighting the trade-off offered by selected solutions.

Index Terms—Engineering Design, Electric Machines, Multi-objective Optimization, NSGA-II, MCDM.

I. INTRODUCTION

Electric machine design is an iterative process where each iteration focuses on improving the design's quality. Machines are complex systems where many variables, geometric and physical, interact non-linearly and affect the machine's performance. These performance measures can include evaluating electromagnetic, thermal, and structural performances, making electric machines a multi-physics, multi-objective optimization problem (MOOP). The need for design optimization is mainly application-driven, and since the field of applications for electric machines is large, there is much scope for improvement.

Research efforts in machine design optimization have been focused on improving solution accuracy and quality while reducing the optimization run-time. For this purpose, optimization algorithms and machine evaluation tools have been explored extensively. Some of the early electric machine design optimization studies include using pattern search, and sequential unconstrained minimization techniques to optimize induction motor [1], [2]. However, it was demonstrated that evolutionary algorithms (EAs) are superior to point-by-point methods in finding global optimums for complex systems

like electric machines [3]. Consequently, the use of EAs has increased with time in the optimization of machine design [4]–[9]. Since the electric machine is a nonlinear system, finite element analysis (FEA) is the most preferred method for their evaluation. Examples of optimization studies where EAs were combined with FEA can be found [10], [11]. Since EAs require many function evaluations to converge to global optimum, using FEA to evaluate each case during optimization requires substantial computational resources. In this regard, researchers have explored techniques, including surrogate-assisted optimization methods, to substitute FEA for reducing computation time, however, at the expense of solution accuracy [12]–[16]. For example, the authors used a combination of second-order response surface model (RSM) and GA to optimize the magnet shape and placement in the rotor of an interior permanent magnet (IPM) machine to achieve the largest constant power speed region (CPSR) [12]. The second-order RSM was used to predict d-axis and q-axis inductances and magnet flux linkage of an IPM machine. Similarly, a surrogate-based optimization approach using multi-objective differential evolution (MODE) algorithm was employed to minimize active material mass, and total losses of an axial flux PM (AFPM) machine at rated operation [15]. In another study, a local refinement strategy to improve a Pareto-optimal design further after the optimization terminated was presented [17]. Results showed that a posteriori local search, even with fewer function evaluations, produced similar results to those by an approach solely relying on global optimization.

Although researchers have explored different optimization algorithms and methods to evaluate objective functions, constraint handling in literature for electrical machine optimization is inefficient. In optimizing electric machines, constraints define feasible regions in the design and objective space [18]. Typically, every machine optimization problem involves some geometric constraints that must be respected to generate feasible designs. Unlike objective functions, these geometric constraints can be quickly evaluated through analytical expressions. A procedure to identify the geometric feasibility of a candidate solution can be included in the optimization

algorithm itself. A common approach to handle geometrically infeasible solutions is to discard them and rely on random initialization of variables and repeat this process until a feasible solution has been found [19]. However, random sampling becomes relatively inefficient as the number of geometric variables and constraints increases, which is eventually reflected in the quality of Pareto-optimal solutions when the computational resources are limited. To tackle this problem, we propose an embedded optimization problem where the information from constraint violation is used to repair geometrically infeasible solutions and improve the Pareto-optimal front.

The main contributions of this work are presented as follows:

- The proposal of a repair operator ensuring the feasibility of designs. This operator exploits inexpensive constraints, e.g., geometric constraints calculated using analytical expressions, while respecting manufacturing accuracy limitations.
- A demonstration of improvement in quality of Pareto-optimal solution set when the proposed repair operator is integrated for multi-objective optimization of an electrical machine design.
- A detailed physical explanation of Pareto-optimal solutions and recommendations for selecting preferred solutions based on performance measures related to the machine manufacturing cost, maximum achievable speed, and noise, vibration, and harshness (NVH).

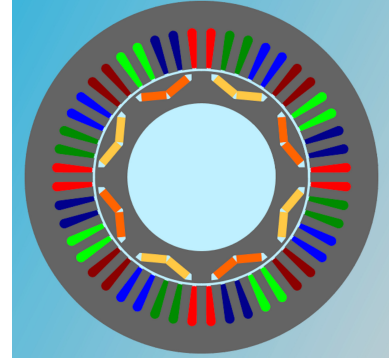
The remainder of this paper is structured as follows. Section II discusses the formulation of the optimization problem for the electric machine design. Section III proposes an optimization method addressing directly the computationally inexpensive constraints of the application problem. The impact of the proposed repair operator and Pareto-optimal electric machine designs are discussed in Section IV. Finally, the conclusions are presented in Section V.

II. ELECTRIC MACHINE DESIGN AND OPTIMIZATION PROBLEM FORMULATION

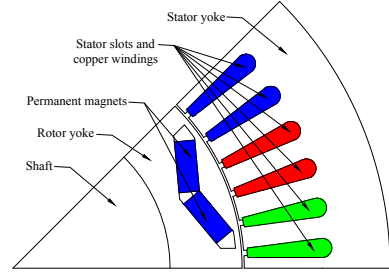
Optimization of electrical machines typically starts with the selection of a machine template. Based on machine performance requirements, objective functions are decided, followed by the selection of the design variables, the variable ranges, and constraints. Average torque, torque pulsations, losses, and efficiency are some of the most common objective functions used in optimizing machine design. Designers usually select variables based on the domain knowledge and then proceed to sensitivity analysis to keep only the most significant variables during optimization [20]. Variable ranges can be selected arbitrarily or based on the machine designer's experience. Since the search space is generally high dimensional, a proper definition of geometric constraints can ensure the reliability of obtained solutions.

TABLE I
PARAMETERS OF IPM MACHINE USED FOR OPTIMIZATION.

Parameters	Values	Parameters	Values
Mechanical power	69 kW	Turns per coil	11
Rated speed	3000 rpm	Slot/ pole/ phase	2
Peak current	177 A	Slot fill factor	0.46
Stator outer diameter	264 mm	Air-gap	0.75 mm
Rotor outer diameter	160.4 mm	Stack length	50.8 mm
Average torque	214.8 Nm	Magnet type	NdFeB
Torque pulsations	36.2 Nm	DC-link voltage	650 V



(a) 2d model.



(b) Reduced model.

Fig. 1. IPM machine used for optimization.

A. Selection of Machine Topology, Objective Functions, and Evaluation Method

Permanent magnet synchronous machines (PMSMs) are known for their high torque density and efficiency. They are extensively used in hybrid and electric vehicle applications. In this work, an electrical machine used in the 2010 Toyota Prius is chosen for optimization. The machine is a 3-phase, 48-slot, 8-pole IPM machine with a single layer of V-shaped magnet, Fig. 1(a) [21]. Details of machine parameters are given in Table I. Since IPM machines are highly nonlinear, FEA is chosen as the objective function evaluation tool. As FEA is time-consuming, periodicity is exploited, and only $1/8^{th}$ of the model is used for evaluation as shown in Fig. 1(b). The goal of the optimization is to maximize average torque while minimizing torque pulsations, where pulsations are defined from peak-to-peak as shown in Fig. 2.

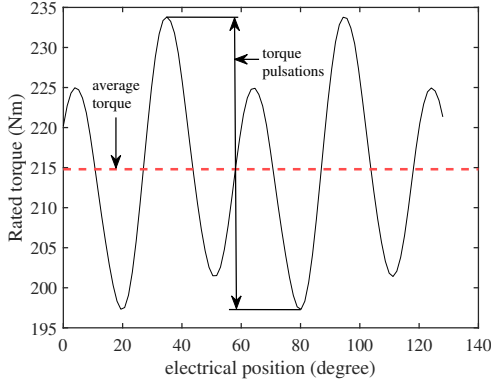


Fig. 2. Torque profile of reference design at rated operating conditions.

TABLE II
VALUES OF GEOMETRIC VARIABLES USED FOR OPTIMIZATION.

x_i	Variable	Unit	$x^{(ref)}$	$x^{(L)}$	$x^{(U)}$
x_1	Height of rotor pole cap	mm	9.56	7.65	11.47
x_2	Magnet thickness	mm	7.16	5.73	8.59
x_3	Magnet width	mm	17.88	14.30	21.46
x_4	Angle between magnets	degree	145.35	116.28	174.42
x_5	Bridge height	mm	1.99	1.59	2.39
x_6	Q-axis width	mm	13.9	11.12	16.68
x_7	Slot height	mm	30.9	24.72	37.08
x_8	Slot width	mm	6.69	5.35	8.03
x_9	Height of slot opening	mm	1.22	0.98	1.46
x_{10}	Width of slot opening	mm	1.88	1.50	2.26

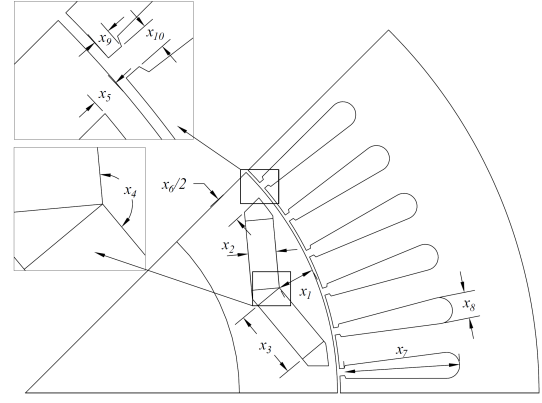


Fig. 3. Geometric variables used for optimization

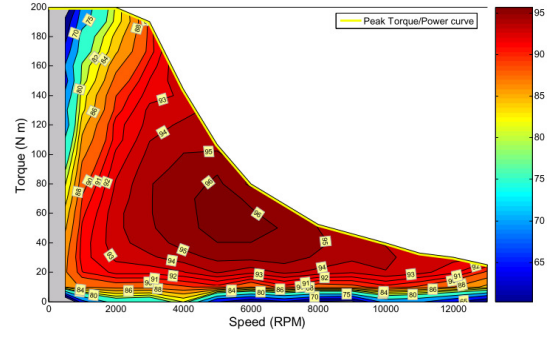


Fig. 4. 2010 Prius motor efficiency contours for 650 Vdc [22].

B. Definition of Feasible Search Space

The following geometric variables were kept constant as in the original 2010 Prius motor.

- Inner diameter (ID) and outer diameter (OD) of stator and rotor
- Air-gap length between rotor and stator
- Stack length of the machine
- Number of turns per coil
- Slot fill factor and maximum current density in stator slot

Next, a sensitivity analysis was performed to identify the ten most significant variables, which are shown in Fig. 3. Out of the ten variables used, six define magnet shape and placement in rotor yoke while four control slot size and shape. Variable ranges were defined to be within 20% variation from the reference design. All variable values were limited to have only two decimal places because of manufacturing accuracy limitations. Variable values for the reference design ($x^{(ref)}$) along with lower ($x^{(L)}$) and upper bounds ($x^{(U)}$) are given in Table II. A total of ten geometric constraints are considered for the optimization problem. The geometric constraints allow the quick check of a design for geometric feasibility without running the FEA simulation. These ten constraints were defined considering domain knowledge vital for a good torque profile and a typical manufacturing tolerance of ± 0.05 mm.

C. Selection of Operating Point for Optimization

The performance of an IPM machine directly depends on the speed and torque requirements. Fig. 4 shows the efficiency contour map of the 2010 Prius motor with a dc-link voltage of 650 V [22]. The peak torque rating of the machine stays constant till a particular speed, called the base speed of the machine, which is also very close to the rated speed of the machine. PMSMs are famous for their torque density, and therefore, their operation at rated speed is of high importance. To increase the efficiency of PMSMs in the speed region up to the base speed, they are operated in a way to minimize the excitation current fed to stator copper windings and, therefore Joule losses, while meeting the torque requirements. This specific mode of operation is called maximum torque per ampere (MTPA) operation. For our optimization problem formulation, the rotational speed of the rotor and the excitation angle is kept constant to those of reference design at rated MTPA operation. It should be noted that the excitation current is not kept constant during optimization. The slot fill factor denotes the proportion of slot area filled by copper windings. As the slot fill factor is kept constant in contrast to the slot cross-section, it results in different current ratings for different designs.

Mathematically, the MOOP is now defined as:

$$\begin{aligned}
& \text{Maximize Average Torque}(\mathbf{x}) \\
& \text{Minimize Torque Pulsation}(\mathbf{x}) \\
& \text{subject to } g_j(\mathbf{x}) \leq 0 \quad \forall j \in 1, \dots, 10 \quad (1) \\
& \quad x_i^{(L)} \leq x_i \leq x_i^{(U)}, \quad \forall i \in 1, \dots, 10 \\
& \text{where } \mathbf{x} \in \mathbb{R}^N.
\end{aligned}$$

where $g_j(\mathbf{x})$ represent the geometrical constraints, $\mathbf{x}$ the variables to optimize, and $x_i^{(L)}$ and $x_i^{(U)}$ the lower and upper bound of the i -th variable respectively. All variables are restricted to have a precision of two decimal places due to manufacturing accuracy limits. Both objective functions, Average Torque($\mathbf{x}$) and Torque Pulsation($\mathbf{x}$), are based on the result of a 2D transient electromagnetic analysis.

III. METHODOLOGY

When solving real-world optimization problems, the definition of the problem itself can be challenging, and numerous design decisions have to be made. A careful observation of the optimization problem in the previous section reveals that two objectives as well multiple constraint functions need to be dealt with during optimization. In particular, constraints are of geometric nature, and therefore, they are mathematically expressed and are relatively inexpensive to evaluate. However, the objective functions require the use of a finite element method; hence they are relatively expensive to compute. We exploit the computational inexpensiveness of constraint evaluations by using a repair operator to satisfy them.

In this study, we use the well-known evolutionary multi-objective algorithm NSGA-II [23] and customize it with a repair operator that converts every infeasible solution into a feasible solution (described in the next paragraph). NSGA-II is a modular, parameter-less optimization algorithm well suited for bi-objective optimization problems including optimization of electric machines. NSGA-II starts with a population of random solutions, called the parent population. After evaluating the population members, pair-wise comparisons are made to select non-dominated and less-crowded solutions in order [24] to meet the main goals of multi-objective optimization. The selected population members are then recombined and mutated to create an offspring population of the same size as the parent population. After evaluating offspring population is merged with the parent population to execute a final survival selection to pick the top half of the population. The selected population becomes the parent population of the next generation. This process is continued until a termination criterion is satisfied.

The goal of the repair operator is to convert an infeasible solution to a feasible one so that all constraints get satisfied. A recommended property of the repair procedure is to find a feasible solution close to the original infeasible solution but at the intersection of the infeasible constraints. We propose a repair operator implementing in *two* phases: first, the geometric constraints are satisfied by using an embedded but relatively simple optimization procedure; second, a precision of two decimals is satisfied by rounding each variable up or

down, preserving the satisfaction of the geometric constraints for an easy implementation purpose. Thus, the repair operator finally returns a feasible solution considering all the electric engine design problem specifications. More details of each of the two phases shall be given below.

1) *Geometric Constraint Repair*: Given an infeasible solution $\mathbf{x}$, the goal of this phase is to convert it to a feasible solution $\mathbf{x}'$. For this purpose we solve the optimization problem defined in (2). Both the geometric constraints and box constraints ensure that the resulting solution ($\mathbf{x}'$) of (2) is feasible, whereas the objective $\|\mathbf{x} - \mathbf{x}'\|^2$ favors the feasible solution having the closest Euclidean Distance (in ℓ_2 -norm) to the original solution $\mathbf{x}$.

$$\begin{aligned}
& \text{Minimize } \|\mathbf{x} - \mathbf{x}'\|^2 \\
& \text{subject to } g_j(\mathbf{x}') \leq 0 \quad \forall j \in 1, \dots, 10 \quad (2) \\
& \quad x_i^{(L)} \leq x'_i \leq x_i^{(U)}, \quad \forall i \in 1, \dots, 10 \\
& \text{where } \mathbf{x}' \in \mathbb{R}^N.
\end{aligned}$$

In our implementation, this optimization problem is solved by the gradient-free simplex optimization algorithm proposed by Nelder and Mead [25]. The algorithm requires an initial solution set to $\mathbf{x}$, thereby searching is focused and quick. The output of the first phase is $\mathbf{x}'$, which is a feasible real-valued vector.

2) *Precision Repair*: After the first phase of the repair has been completed, every variable x'_i needs to be modified to make it a floating-point number with a precision of two. This can be achieved by rounding each variable x'_i ($1 \leq i \leq N$) to its floor or ceil value to the desired precision. Because such rounding will result in 2^N different possibilities, we propose an efficient rounding scheme.

The method starts by creating a random permutation P of the first N consecutive natural numbers ($|P| = N$). The permutation P provides the order of the variables to be rounded sequentially. After that, a local search method inspired by Hooke-Jeeves pattern moves [26] is implemented to find a feasible solution having two-decimal precision in all variables. For the current variable under consideration, two changes (its floor and ceil) are evaluated for the entire solution's feasibility. By keeping the rounding that caused feasibility, the subsequent variable in P is checked. A new random permutation is then tried if the process does not find a feasible solution after all variables are explored. If a solution $\mathbf{x}$ cannot be repaired in a maximum of ρ ($= 100$ used here) attempts, then we discard the solution $\mathbf{x}$. This is an efficient way of finding a feasible solution out of all 2^N neighbouring possibilities.

An illustration of the repair operator and its two phases is shown in Figure 5. The two-dimensional search space is split up into two regions, the feasible one where for $\forall j \in J : g_j(\mathbf{x}) \leq 0$ and infeasible one where $\exists j \in J : g_j(\mathbf{x}) > 0$. A solution $\mathbf{x}$ which is located in the infeasible area, the repair operator attempts to make it feasible. The first phase executes the embedded optimization method obtaining $\mathbf{x}'$, which is feasible and minimizes the Euclidean distance to $\mathbf{x}$. In the

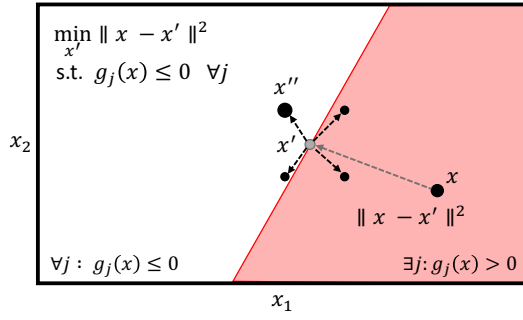


Fig. 5. Illustration of the repair operator in a two-dim. search space.

second phase, $\mathbf{x}'$ is modified to be a float of precision two by rounding. In this example, in a two-dimensional search space ($N = 2$), the rounding can result in $2^N = 2^2 = 4$ different solutions where 50% turn out to be still feasible. Nevertheless, as soon as the first attempt of randomly ordered rounding is successful, the repair is completed, and $\mathbf{x}''$ is returned. In all cases of our simulation, the above two phases can produce feasible solutions with two-decimal places of rounding given that the constraints are inexpensive to evaluate.

The proposed repair ensures that a solution is feasible before its evaluation of objectives. In order to ensure feasibility, the constraint functions are called more frequently than the objective functions, exploiting inexpensiveness in constraint evaluations. Incorporating this repair procedure into an evolutionary algorithm, in this study NSGA-II, is relatively straightforward and yet an effective strategy of adapting an existing optimization method to the needs of an application problem.

IV. RESULTS AND DISCUSSION

In the following, the performance contribution of the repair operator shall be examined. This includes answering the following key questions:

- Is the repair operator helpful during optimization, and what is its impact?
- What insights are gained from the Pareto-optimal designs, and what can we learn from them for the electric machine design?

The first question is related to the optimization method itself and the convergence. The latter is vital as it addresses the ultimate goal of optimization, which is gaining more insights and finally choosing an electric machine design.

A. Analysis of Constraints

Before analyzing the impact of the repair operator, the optimization constraints are investigated. As an initial study, we have randomly sampled 100 solutions using Latin hypercube sampling. To ensure statistical significance, we have repeated the experiment 100 times. Results show that 69.7% of these randomly sampled solutions are infeasible, or in other words, only 30.3% are feasible. A solution is considered infeasible if at least one constraint is violated. Thus, an analysis of each constraint separately shall provide more information on what

type of constraints are more difficult to satisfy. In Table III, the percentage of infeasible solutions is shown for each constraint separately. The percentages reveal that some constraints seem to be more difficult to satisfy than others. The constraints g_8 and g_9 , which control the slot shape, and g_1 , which controls the magnet placement close to the shaft, have not been violated in any of the 10,000 solutions. In contrast, some other constraints such as g_2 and g_7 , related to magnet placement close to rotor OD, are responsible for infeasible solutions 31.43% and 36.28% of the time. It is worth mentioning that these 10,000 solutions were generated from the variables ranges defined in Table II. For a different search space with larger or arbitrarily defined variable bounds, the percentage of infeasible solutions could be even larger.

B. Convergence Analysis

To investigate the impact of the proposed approach, we have experimented with two different optimization methods: the conventional NSGA-II, and NSGA-II-WR which refers to NSGA-II with the proposed repair operator. Both approaches compared in this work use binary tournament selection, simulated binary crossover (SBX) operator with a probability of 0.9, and polynomial mutation. Distribution index used for crossover and mutation operators are $\eta_c = 15$ and $\eta_m = 20$ respectively. A population size of 100, with 20 offspring and 1500 total evaluations, was selected for an optimization cycle for both methods. For each method, five such optimization runs were completed making the total number of evaluations 7500, where each optimization run was initialized with a different seed. The same five seeds were used for both methods to make a fair comparison among the non-dominated fronts. The overall setup and the results of all experiments are shown in Table IV and Fig. 6(a) and (b). First of all, one can note that overall the optimization runs seem to have successfully converged to a set of Pareto-optimal solutions trading of average torque and torque pulsation for both methods. Second, NSGA-II-WR outperforms NSGA-II, which demonstrates the positive impact of the repair operator. One might argue how such a repair could have harmed the convergence? One possible danger of adding a deterministic repair of infeasible solutions is a diversity loss because the natural exploration of an evolutionary algorithm has been interfered with. Thus, influencing the exploration by adding a repair can also have a negative effect. However, results indicate that our proposed customization is well-suited for optimizing the design of an IPM machine and increases the diversity of obtained solutions on the non-dominated front.

C. Analysis of Pareto-Optimal Solutions

After discussing the optimization methods themselves, more details and insights about the obtained Pareto-optimal designs are provided. Fig. 6(c) visualizes the distribution of geometric variables of the Pareto-optimal solutions obtained throughout all ten runs. In the parallel-coordinate plot, each vertical axis represents the normalized optimization variable x_i with its lower and upper bounds as 0 and 1, respectively, and each

TABLE III
THE CONSTRAINT VIOLATION OF EACH CONSTRAINT VALUE FROM g_1 TO g_{10} .

Value	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}
Infeasible	0.0%	31.43%	19.16%	19.16%	21.94%	19.59%	36.28%	0.0%	0.0%	20.25%
Rank	7	2	6	6	3	5	1	7	7	4

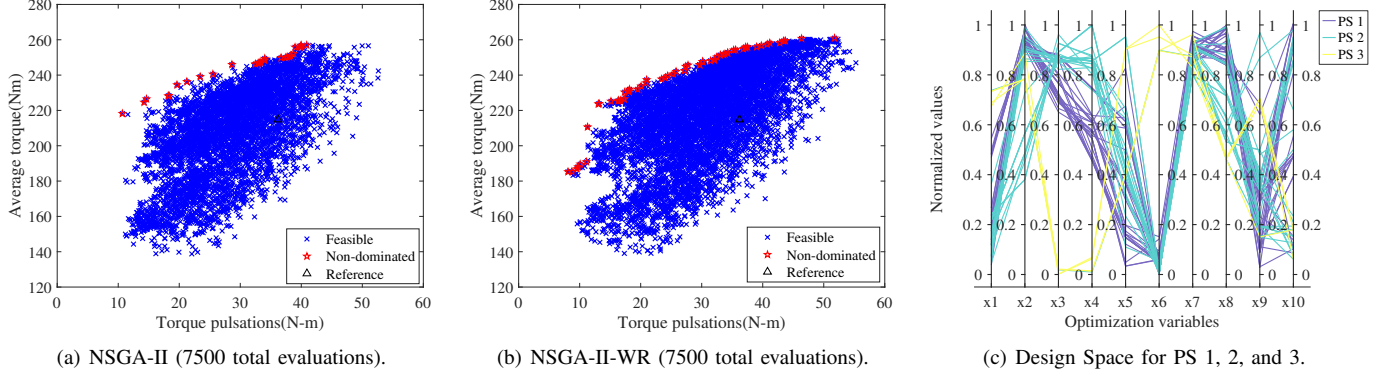


Fig. 6. Objective space illustrating dominated and non-dominated solutions for each method in (a) and (b). Design space of the Pareto-optimal set obtained from all 10 runs combined is shown in (c).

TABLE IV
OPTIMIZATION SETUP AND RESULTS.

Parameters	NSGA-II	NSGA-II-WR
Description	Conventional	With repair
Evaluations	7500	7500
Feasible	5446	7500
Non-dominated	27	59
Hypervolume	0.7206	0.7382

horizontal line represents a solution. Non-dominated solutions originated from the merged results of all runs are divided into 3 Pareto-sets based on the torque pulsation value.

- PS 1 — pulsation $> 30\text{Nm}$
- PS 2 — $30\text{ Nm} \geq \text{pulsation} \geq 10\text{Nm}$
- PS 3 — pulsation $< 10\text{Nm}$

Since the size of the machine is kept constant, magnet width (x_3), slot height (x_7), and slot width (x_8) converge to the higher end of variable ranges for most solutions. This can be explained as more magnet width increases the magnet flux linkage and, therefore, average torque. Larger magnet width also reduces the q-axis width (x_6), which has converged to the lower end of the variable range. For further analysis, q-axis width can be removed from optimization variables and set to a minimum value. Similarly, an increase in slot cross-section area allows more excitation current to be used by means of more copper and increases the average torque.

On the other hand, torque pulsations are affected by the material saturation and the magnet pole arc. While material saturation is associated with the non-linearity of electrical steel, magnet pole arc is a measure of the span of magnets defined by magnet width (x_3) and angle between the magnets (x_4). The effect of change in design variables on torque

pulsations is explained in the following points.

- Increase in magnet width (x_3) and angle between the magnets (x_4) increases magnet pole arc, which reduces torque pulsations.
- While larger slot cross-section increases average torque, it also increases saturation in stator teeth and yoke due to increased magnetic flux density, leading to more harmonics and pulsations. Since more choice of slot width (x_8) is available than slot height (x_7) for non-dominated solutions, with the variable ranges used for optimization, stator teeth are more prone to saturation than stator yoke.
- Additionally, an increase in bridge height (x_5) reduces the saturation close to rotor OD, reducing the saturation harmonics. Thus, a gradual increase in bridge height is related to a gradual reduction in pulsations.
- The height and width of slot opening, x_9 and x_{10} respectively, which are responsible for slot harmonics, have relatively large ranges to be optimal. However, since the absolute variation in their ranges is small, it is advisable to study the effect of manufacturing uncertainties on slot harmonics.

D. Selection of Preferred Solutions

The selection of an electric machine design depends on the type of application and the desired performance. A popular approach uses a scalarized function for optimization, which yields a single optimal solution at the end of an optimization run. However, selecting weights for a scalarized function is difficult and may result in a biased search if not appropriately defined. Scalarization also prevents the possibility of analyzing trade-offs offered by Pareto-optimal solutions. Furthermore, including all the performance criteria during optimization

TABLE V
PERFORMANCE COMPARISON OF SELECTED SOLUTIONS.

Solution	Avg torque(Nm)	Pulsations(Nm)	THDV(%)	MUF(Nm/mm ³)	F-BEMF(V)
A	260.915	51.766	15.029	0.029	244.436
B	257.397	40.734	13.806	0.031	235.219
C	218.209	10.679	11.180	0.029	223.650
Reference	214.776	36.184	14.409	0.033	209.262

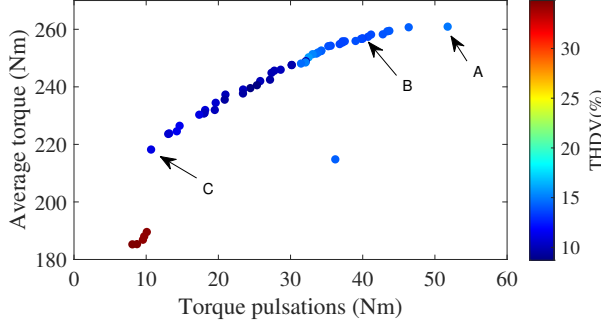


Fig. 7. *THDV* of non-dominated solutions from all 10 runs combined.

makes the task extremely time-consuming and likely produces a Pareto-front converged to sub-optimal regions.

In this work we have used a domain specific a posteriori multi-criteria decision-making (MCDM) method to select preferred solutions which involves machine expertise. Apart from the objective functions defined in (1) we have used the following performance measures, important to all PMSMs, to select preferred solutions from the non-dominated set.

- Total harmonic distortion of no-load back emf (*THDV*)
- Peak of fundamental of back emf (*F-BEMF*)
- Magnet utilization factor (*MUF*)

For electric machine design, a low value of *THDV* is desired since it is a direct measure of noise, vibration, and harshness (NVH) during the operation of the electric machine. Similarly, an increase in *F-BEMF* increases the average torque but reduces the maximum speed that the machine can achieve, thus, presenting a trade-off. On the other hand, a higher value of *MUF* is desired, where *MUF* is defined as the ratio of average torque to PM volume. Since PM material is expensive, a higher value of *MUF* means the cost of the machine is reduced.

Fig. 7 shows *THDV* for non-dominated solutions from all ten optimization runs combined, along with reference design in the objective space. Although solutions lying in the bottom region of Pareto-front have the least torque pulsation, they have the highest *THDV* (more than 30%), and must be avoided during selection. The remaining non-dominated solutions, which have similar *THDV*, are called good solutions. We have selected three solutions, A, B, and C, from these good non-dominated solutions, which are highlighted in Fig. 7 and presented in Table V. The basis of the selection of these designs is as follows.

- Solution A — maximum average torque

- Solution B — maximum *MUF*
- Solution C — minimum pulsation and *F-BEMF*

Fig. 8 shows the magnetic flux density plot of the selected solutions, A, B, and C. Solutions A and B require more magnet volume for producing more average torque, while solution C has a significantly larger magnet pole arc leading to a reduction in torque pulsations. Solutions A and B also suffer from more saturation in stator teeth and yoke than C because of the larger slot cross-section area, as discussed in the previous subsection. Additionally, although solution A produces maximum average torque, solution B outperforms A in every other performance index, with less than 2% difference in torque production capability (Table V). On the other hand, solution C outperforms both solutions A and B when pulsations, *THDV*, and *F-BEMF* are compared, but C has the smallest average torque out of the three solutions. Furthermore, it can be shown that with a 7% increment in *F-BEMF*, the maximum speed of the machine reduces approximately by 10%. Fig. 9 shows the torque-speed curve for solutions B and C and demonstrates the reduction in torque producing capability in high-speed regions for higher *F-BEMF*. Thus, we have two choices for the selection of a design based on the following performance requirements.

- Solution B, when the focus is on increasing torque production capability and reduction of cost of machine.
- Solution C, when more emphasis is placed on reduction of pulsations and *THDV* along with a higher speed range.

Choosing between B and C would require further preference information. However, this paper has shown how a computational procedure can first find feasible and implementable solutions with trade-offs and pick a handful of preferred solutions based on the technical aspirations.

V. CONCLUSION

In this work, a repair operator, an efficient method to handle inexpensive constraints, is introduced. An improvement in the quality of Pareto-optimal solutions for electric machine design is demonstrated when the repair operator is integrated with multi-objective optimization algorithm NSGA-II. The improvement is achieved in the diversity and convergence of Pareto-optimal solutions while the computational resources are limited. Proposed repair operator can also be tried with other EMO methods. An MCDM approach focused on reducing machine cost and noise, vibration, and harshness (NVH), with an improvement in the speed range of the electric machine is also presented. The presented a posteriori selection approach identifies the trade-off offered by different Pareto-optimal so-

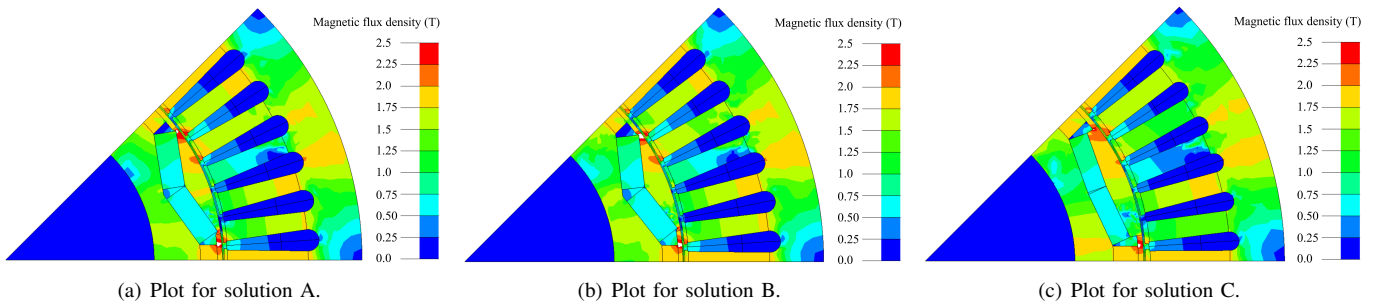


Fig. 8. Magnetic flux density plots highlighting variation in material saturation for non-dominated solutions A, B, and C.

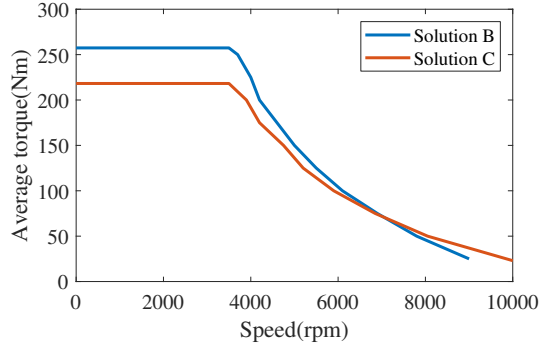


Fig. 9. Torque-speed curve for solutions B and C. Solution A was found not competitive to B and C in terms of all five performance metrics (Table V).

lutions and facilitates selecting a handful of optimized electric machine designs.

The selection of preferred designs is entirely dependent on the required performance at the user end. A more comprehensive optimization problem will also involve constraints on the thermal and structural performance of the machine. Future extensions of this work will include solving a multi-physics optimization and decision-making problem. Since more criteria will increase optimization run-time, a surrogate-based extension of the current approach will be required.

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