Interpretable-AI Policies using Evolutionary Nonlinear Decision Trees for Discrete Action Systems

Yashesh Dhebar¹, Kalyanmoy Deb¹, Subramanya Nageshrao², Ling Zhu², and Dimitar Filev²

¹ Michigan State University, East Lansing, MI, 48824 USA
² Ford Motor Company, Detroit, USA

dhebarya,kdeb}@msu.com, {snageshr,lzhu40,dfilev}@ford.com

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Abstract

Black-box artificial intelligence (AI) induction methods such as deep reinforcement learning (DRL) are increasingly being used to find optimal policies for a given control task. Although policies represented using a black-box AI are capable of efficiently executing the underlying control task and achieving optimal closed-loop performance – controlling the agent from initial time step until the successful termination of an episode, the developed control rules are often complex and neither interpretable nor explainable. In this paper, we use a recently proposed nonlinear decision-tree (NLDT) approach to find a hierarchical set of control rules in an attempt to maximize the open-loop performance for approximating and explaining the pre-trained black-box DRL (oracle) agent using the labelled state-action dataset. Recent advances in nonlinear optimization approaches using evolutionary computation facilitates finding a hierarchical set of nonlinear control rules as a function of state variables using a computationally fast bilevel optimization procedure at each node of the proposed NLDT. Additionally, we propose a re-optimization procedure for enhancing closed-loop performance of an already derived NLDT. We evaluate our proposed methodologies (open and closed-loop NLDTs) on four different control problems having two to four discrete actions. In all these problems our proposed approach is able to find simple and interpretable rules involving one to four non-linear terms per rule, while simultaneously achieving on par closed-loop performance when compared to a trained black-box DRL agent. The obtained results are inspiring as they suggest the replacement of complicated black-box DRL policies involving thousands of parameters (making them non-interpretable) with simple interpretable policies. Results are encouraging and motivating to pursue further applications of proposed approach in solving more complex control tasks.

1 Introduction

Control system problems are increasingly being solved by using modern reinforcement learning (RL) and other machine learning (ML) methods to find an autonomous agent (or controller) to provide an optimal action $A_t$ for every state variable combination $S_t$ in a given environment at every time step $t$. Execution of the output action $A_t$ takes the object to the next state $S_{t+1}$ in the environment and the process is repeated until a termination criteria is met. The mapping between input state $S_t$ and output action $A_t$ is usually captured through an artificial intelligence (AI) method. In the RL literature, this mapping is referred to as policy $\pi(S) : \mathbb{S} \rightarrow \mathbb{A}$, where $\mathbb{S}$ is the state space and $\mathbb{A}$ is the action space. Sufficient literature exists in efficient training of these RL policies [Schulman et al. 2015, 2017, Lillicrap et al. 2015, Mnih et al. 2016]. While these methods are efficient at training the AI policies for a given control system task, the developed AI policies, captured through complicated networks, are complex and non-interpretable.

Interpretability of AI policies is important to a human mind due to several reasons: (i) they help provide a better insight and knowledge to the working principles of the derived policies, (ii) they can be easily deployed with a low fidelity hardware, (iii) they may also allow an easier way to extend the control policies for more complex versions of the problem. While defining interpretability is a subjective matter, a number of past efforts have attempted to find interpretable AI policies with limited success.

In the remainder of this paper, we first present the main motivation behind finding interpretable policies in Section 2. A few past studies in arriving at interpretable AI policies is presented in Section 3. In Section 4 we review a recently proposed nonlinear decision-tree (NLDT) approach in the context of arriving at interpretable AI policies. The overall open-loop and closed-loop NLDT policy generation methods are described in Section 5. Results on four control system problems are presented in Section 6. Finally, conclusions and future studies are presented in Section 7. Supplementary document provides further details.

2 Motivation for the Study

Various data analysis tasks, such as classification, controller design, regression, image processing, etc., are increasingly being solved using artificial intelligence (AI) methods. These are done, not because they are new and interesting, but because they have been demonstrated to solve complex data analysis tasks without much change in their usual frameworks. With more such studies over the past few decades, they are faced with a huge challenge. Achieving a high-accuracy solution does not necessarily satisfy a curious domain expert, particularly if the solution is not interpretable or explainable. A technique (whether AI-based or otherwise)
to handle data well is no more enough, researchers now demand an explanation of why and how they work.

Consider the MountainCar control system problem, which has been extensively studied using various AI methods (Sutton 1996; Peters, Mülling, and Altun 2010; Smart and Kaelbling 2000). The problem has two state variables (position $x_t$ along $x$-axis and velocity $v_t$ along positive $x$-axis) at every time instant $t$ which would describe the state of the car at $t$. Based on the state vector $S_t = (x_t, v_t)$, a policy $\pi(S)$ must decide on one of the three actions $A_t$: decelerate ($A_t = 0$) along positive $x$-axis with a pre-defined value $-\alpha$, do nothing ($A_t = 1$), or accelerate ($A_t = 2$) with $\alpha$ in positive $x$-axis direction. The goal of the control policy $\pi(S)$ is to take the under-powered car (it does not have enough fuel to directly climb the mountain and reach the destination) over the right hump in a maximum of 200 time steps starting anywhere at the trough of the landscape. Physical laws of motion are applied and a policy $\pi(S)$ has been trained to solve the problem. The RL produces a black-box policy $\pi_{oracle}(S)$ for which an action $A_t \in [0, 1, 2]$ will be produced for a given input $S_t = (x_t, v_t) \in \mathbb{R}^2$. Figure 1a shows the state-action combinations obtained from 92 independent successful trajectories (amounting to total of 10,000 time steps) leading to achieving the goal using a pre-trained deterministic black-box policy $\pi_{oracle}$. The $x$-location of the car and its velocity can be obtained from a point on the 2D plot. The color of the point $S_t = (x_t, v_t)$ indicates the action $A_t$ suggested by the oracle policy $\pi_{oracle}$ ($A_t = 0$: blue, $A_t = 1$: orange, and $A_t = 2$: green). If a user is now interested in understanding how the policy $\pi_{oracle}$ chooses a correct $A_t$ for a given $S_t$, one way to achieve this would be through an interpretable policy function $\pi_{int}(S_t)$ as follows:

$$\pi_{int}(S_t) = \begin{cases} 0, & \text{if } \phi_0(S_t) \text{ is true,} \\ 1, & \text{if } \phi_1(S_t) \text{ is true,} \\ 2, & \text{if } \phi_2(S_t) \text{ is true,} \end{cases}$$

where $\phi_i(S_t) : \mathbb{R}^2 \to \{0, 1\}$ is a Boolean function which partitions the state space $S$ into two sub-domains based on its output value and for a given state $S_t$, exactly one of $\phi_i(S_t)$ is true, thereby making the policy $\pi_{int}$ deterministic. If we re-look at Figure 1a we notice that the three actions are quite mixed at the bottom part of the $x$-$v$ plot (state space). Thus, the partitioning Boolean functions $\phi_i$ need to be quite complex in order to have $\phi_0(S_t) = true$ for all blue points, $\phi_1(S_t) = true$ for all orange points and $\phi_2(S_t) = true$ for all green points.

What we address in this study is an attempt to find an approximated policy function $\pi_{int}(S_t)$ which may not explain all 100% time instance data corresponding to the oracle black-box policy $\pi_{oracle}(S_t)$ (Figure 1a), but it is fairly interpretable to explain close to 100% data. Consider the state-action plot in Figure 1b which is generated with a simple and interpretable policy $\pi_{int}(S_t) = \{i | \phi_i(S_t) \text{ is true, } i = 1, 2, 3\}$ obtained by our proposed procedure as shown below

$$\begin{align*}
\phi_0(S_t) &= -\psi_1(S_t), \\
\phi_1(S_t) &= (\psi_1(S_t) \land -\psi_2(S_t)), \\
\phi_2(S_t) &= (\psi_1(S_t) \land \psi_2(S_t)),
\end{align*}$$

where $\psi_1(S_t) = |0.96 - 0.63|\hat{x}_t^2 + 0.28|\hat{v}_t - 0.22\hat{x}_t\hat{v}_t| \leq 0.36$, and $\psi_2(S_t) = |1.39 - 0.28\hat{x}_t^2 - 0.30\hat{v}_t^2| \leq 0.53$. Here, $\hat{x}_t$ and $\hat{v}_t$ are normalized state variables (see Supplementary document for details). The action $A_t$, predicted using the above policy does not match the output of $\pi_{oracle}$ at some states (about 8.1%), but from our experiments we observe that it is still able to drive the mountain-car to the destination goal located on the right hill in 99.8% episodes.

Importantly, the policies are simplistic and amenable to an easier understanding of the relationships between $x_t$ and $v_t$ to make a near perfect control. Since the explanation process used the data from $\pi_{oracle}$ as the universal truth, the derived relationships will also provide an explanation of the working of the black-box policy $\pi_{oracle}$. A more gross approximation to Figure 1a by more simplified relationships ($\phi_i$) may reduce the overall open-loop accuracy of matching the output of $\pi_{oracle}$. Hence, a balance between a good interpretability and a high open-loop accuracy in searching for Boolean functions $\phi_i(S_t)$ becomes an important matter for such an interpretable AI-policy development study.

In this paper, we focus on developing a search procedure for arriving at the $\psi$-functions (see Eq. 2) for discrete action systems. The structure of the policy $\pi_{int}(S_t)$ shown in Eq. 1 resembles a decision tree (DT), but unlike a standard DT, it involves a nonlinear function at every non-leaf node, requiring an efficient nonlinear optimization method to arrive at reasonably succinct and accurate functionals. The procedure we propose here is generic and is independent of the AI method used to develop the black-box policy $\pi_{oracle}$.

3 Related Past Studies

In (Noothigattu et al. 2018), an interpretable orchestrator is developed to choose from two RL-policies $\pi_C$ for maximizing reward and $\pi_R$ for maximizing an ethical consideration. The orchestrator is dependent on only one of the state-variables and despite being interpretable, the policies: $\pi_C$ and $\pi_R$ are still black-box and convoluted. (Maes et al. 2012) constructs a set of interpretable index based policies and uses multi-arm bandit procedure to select a high performing index based policy. The search space of interpretable policies is much smaller and the procedure suggested for finding an interpretable policy is computationally heavy, taking about hours to several days of computational time on simple control problems. In (Hein, Udluft, and Runkler 2018), genetic programming (GP) is used to obtain interpretable policies on control tasks involving continuous actions space through model-based policy learning. However
the interpretability was not captured in the design of the fitness function and a large archive was created passively to store every policy for each complexity encountered during the evolutionary search. A linear decision tree (DT) based model is used in Liu et al. (2018) to approximate the Q-values of trained neural network. In that work, the split in DT occurs based on only one feature, and at each terminal node the Q-function is fitted using a linear model on all features. Verma et al. (2018) uses a program sketch S to define the domain of interpretable policies c. Interpretable policies are found using a trained black-box oracle cN as a reference by first conducting a local search in the sketch space S to mimic the behaviour of the oracle cN and then fine-tuning the policy parameters through online Bayesian optimization. The bias towards generating interpretable programs is done through controlled initialization and local search rather than explicitly capturing interpretability as one of the fitness measure. Particle swarm optimization (Kennedy and Eberhart 1995) is used to generate interpretable fuzzy rule set in (Hein et al. 2017) and is demonstrated on classic control problems involving continuous actions. Works on DT (Breiman 2017) based policies through imitation learning has been carried out in (Ross, Gordon, and Bagnell 2011), (Bastani, Pu, and Solar-Lezama 2018) extends this to utilize Q-values and eventually render DT policies involving < 1,000 nodes on some toy games and CartPole environment with an ultimate aim to have the induced policies verifiable. (Bastani, Kim, and Bastani 2017) used axis-aligned DTs to develop interpretable models for black-box classifiers and RL-policies. They first derive a distribution function P by fitting the training data through axis-aligned Gaussian distributions. P is then used to compute the loss function for splitting the data in the DT. Vandewiele et al. (2016) attempts to generate interpretable DTs from an ensemble using a genetic algorithm. In Ernst, Geurts, and Wehenkel (2005), regression trees are derived using classical methods such as CART (Breiman 2017) and Kd-tree (Bentley 1975) to model Q-function through supervised training on batch of experiences and comparative study is made with ensemble techniques. In Silva et al. (2020), a gradient based approach is developed to train the DT of pre-fixed topology involving linear split rules. These rules are later simplified to allow only one feature per split node and resulting DTs are pruned to generate simplified rule-set.

While the above methods attempt to generate an interpretable policy, the search process does not use complexity of policy in the objective function, instead, they rely on the initializing the search with certain interpretable policies. In our approach described below, we build an efficient search algorithm to directly find interpretable policies using recent advances in nonlinear optimization.

4 Nonlinear Decision Tree (NLDT) Approach

In this study, we use a direct mathematical rule generation approach (presented in Eq. 2) using a nonlinear decision tree (NLDT) approach (Dhebar and Deb 2020), which we briefly describe here. The intention is to model the interpretable policy πint to approximate and explain the pre-trained black-box policy πoracle using the labelled state-action data generated using πoracle. Decision trees are considered a popular choice due to their interpretability aspects. They are intuitive and each decision can be easily interpreted. However, in a general scenario, regular decision trees come up with a complicated topology since the rules at each conditional node can assume only axis parallel structure x_i ≤ τ to make a split. On the other end, single rule based classifiers like support vector machines (SVMs) have just one rule but its complicated and highly nonlinear. Keeping these two extremes in mind, we develop a nonlinear decision tree framework where each conditional node can assume a nonlinear functional form while the tree is allowed to grow by recursively splitting the data in conditional nodes, similar to the procedure used to induce regular decision trees. In our case of replicating a policy πoracle, the conditional node captures a nonlinear control logic and the terminal leaf nodes indicate the action. This is schematically shown in Figure 2.

In the binary-split NLDT, used in this study, a conditional node is allowed to have exactly two splits as shown in Figure 2. The non-linear split rule f(x) at each conditional node is expressed as a weighted sum of power laws:

\[
f(x) = \begin{cases} 
\sum_{i=1}^{p} w_i B_i + \theta_1, & \text{if } m = 0, \\
\sum_{i=1}^{p} w_i B_i + \theta_1 - |\theta_2|, & \text{if } m = 1, 
\end{cases}
\]  

(3)

where power-laws B_i are given as B_i = \prod_{j=1}^{d} x_{ij}^{b_{ij}} and m indicates if an absolute operator should be present in the rule or not. In Section 5.1 we discuss procedures to derive values of exponents b_{ij}, weights w_i, and biases \theta_i.

5 Overall Approach

The overall approach is illustrated in Figure 3. First, a dedicated black-box policy πoracle is trained from the actual environment/physics of the problem. This aspect is not the focus of this paper. Next, the trained policy πoracle (Block 1 in the figure) is used to generate labelled training and testing datasets of state-action pairs from different time steps. We generate two types of training datasets: Regular – as they are recorded from multiple episodes, and Balanced – selected from multiple episodes to have almost equal number of states for each action, where an episode is a complete simulation of controlling an object with a policy over multiple time steps. Third, the labelled training dataset (Block 2) is used to find the NLDT (Block 3) using the recursive bilevel evolutionary algorithm described in Section 5.1. We call this an open-loop NLDT (or, NLDT_{OL}), since it is derived from a labelled state-action dataset generated from πoracle, without using any overall reward or any final goal.
objective in its search process, which is typically a case while doing reinforcement learning. Use of labelled state-action data in supervised manner allows a faster search of NLDT even with a large dataset as compared to constructing the NLDT from scratch through reinforcement learning by interacting with the environment to maximize the cumulative rewards (Verma et al., 2018). Next, in an effort to make the overall NLDT interpretable while simultaneously ensuring better closed-loop performance, we prune the NLDT by taking only the top part of NLDT\textsubscript{OL} (we call NLDT\textsubscript{OL*} in Block 4) and re-optimize all non-linear rules within it for the weights and biases using an efficient evolutionary optimization procedure to obtain final NLDT\textsuperscript{*} (Block 5). The re-optimization is done here with closed-loop objectives, such as the cumulative reward function or closed-loop completion rate. We briefly discuss the open-loop training procedure of inducing NLDT\textsubscript{OL} and the closed-loop training procedure to generate NLDT\textsuperscript{*} in next sections.

5.1 Open-loop Training
A labelled state-action dataset is first created using a pre-trained black-box policy \(\pi_{oracle}\). Since we are dealing with discrete-action control problems, the underlying imitation task of replicating the behavior of \(\pi_{oracle}\) using the labelled state-action data translates to a classification problem. We train NLDT discussed in Section 4 to fit the state-action data through supervised learning. Nonlinear split-rule \(f(x)\) at each conditional node (Figure 2 and Eq. 3) is derived using a dedicated bilevel optimization algorithm, where the upper level searches the template of the non-linear rule and the corresponding lower level focuses at estimating optimal values of weights/coefficients for optimal split of data present in the conditional node. The optimization formulation for deriving a non-linear split rule \(f(x)\) (Eq. 3) at a given conditional node is given below:

Minimize \(F_{U}(B, m, w^{*}, \theta^{*})\), subject to \((w^{*}, \theta^{*}) \in \arg\min \{F_{L}(w, \theta)|_{(B, m)}\} \leq \tau_{I}, \quad F_{L}(w, \theta)|_{(B, m)} \leq \tau_{I}, \quad -1 \leq w_{i} \leq 1, \forall i, \theta \in [-1, 1]^{m+1}, \quad m \in \{0, 1\}, \quad b_{ij} \in \mathbb{Z}, \quad \mathcal{Z}\) is a set of exponents allowed to limit the complexity of the derived rule structure. In thus study, we use \(\mathcal{Z} = \{-3, -2, -1, 0, 1, 2, 3\}\). The objective \(F_{U}\) quantifies the complexity of the non-linear rule by enumerating the number of terms present in the equation of the rule \(f(x)\) as shown below:

\[
F_{U}(B, m, w^{*}, \theta^{*}) = \sum_{i=1}^{p} \sum_{j=1}^{d} g(b_{ij}),
\]

where \(g(\alpha) = 1\), if \(\alpha \neq 0\), zero otherwise, \(m\) indicates the presence or absence of a modulus operator and \(w\) and \(\theta\) encode rule weights \(w_{i}\) and biases \(\theta_{j}\) respectively. The lower level objective function \(F_{L}\) quantifies the net impurity of child nodes resulting from the split. Impurity \(I\) of a node \(P\) is computed using a Gini-score: \(\text{Gini}(P) = 1 - \sum_{i} \left(\frac{N_{i}}{N}\right)^{2}\), where \(N\) is the total number of points present in the node and \(N_{i}\) represents number of points belonging to class \(i\). Datapoints present in node \(P\) gets distributed into two non-overlapping subsets based on their split function value. Datapoints with \(f(x) \leq 0\) go to the left child node \(L\) and rest go to the right child node \(R\). The lower level objective function \(F_{L}\) which quantifies the quality of this split is then given by

\[
F_{L}(w, \theta)|_{(B, m)} = \left(\frac{N_{L}}{N} \text{Gini}(L) + \frac{N_{R}}{N} \text{Gini}(R)\right)_{(w, \theta, B, m)}.
\]

The \(\tau_{I}\) parameter in Eq. 4 represents maximum allowable net-impurity (Eq. 6) of child nodes. The resulting child nodes obtained after the split undergo another split and the process continues until one of the termination criteria is met.

We use a bilevel-optimization algorithm (Sinha, Malo, and Deb, 2018) to derive split-rule \(f_{i}(x)\) at \(i\)-th conditional node in NLDT. The upper level of the optimization navigates through the domain of discrete exponents \(b_{ij}\) to prescribe the structure of the rule. Then, the lower level optimization finds optimal values of weights \(w_{i}\) and biases \(\theta_{j}\) of the rule structure to make the overall NLDT search efficient. After the entire NLDT is found, in this study, a pruning and tree simplification strategy (see Supplementary Document for more details) is applied to reduce the size of NLDT in an

Figure 3: A schematic of the proposed overall approach.
effort to improve on the interpretability of the overall rule-sets. This entire process of inducing NLDT from the labelled state-action data results into the open-loop interpretable tree NLDT_{OL}. NLDT_{OL} can then be used to explain the behavior of the oracle policy \( \pi_{\text{oracle}} \). We will see in Section 6 that despite being not 100% accurate in imitating \( \pi_{\text{oracle}} \), NLDT_{OL} manages to achieve respectable closed-loop performance with 100% completion rate and a high cumulative reward value. Next, we discuss the closed-loop training procedure to obtain NLDT*.

5.2 Closed-loop Training

The intention behind the closed-loop training is to enhance the closed-loop performance of the interpretable NLDT. It will be discussed in Section 6 that while closed-loop performance of NLDT_{OL} is at par with \( \pi_{\text{oracle}} \) on control tasks involving two to three discrete actions, like CartPole and MountainCar, the NLDT_{OL} struggles to autonomously control the agent for control problems such as LunarLander having more states and actions. In closed-loop training, we fine-tune and re-optimize the weights \( W \) and biases \( \Theta \) of an entire NLDT_{OL} (or pruned NLDT_{OL}, i.e. NLDT_{OL}(\pi) - block 4 in Figure 3) to maximize its closed-loop fitness \( F_{CL} \), which is expressed as the average of the cumulative reward collected on \( M \) episodes:

Maximize \( F_{CL}(W, \Theta) = \frac{1}{M} \sum_{i=1}^{M} R_e(W, \Theta), \)

Subject to \( W \in [-1, 1]^{n_w}, \Theta \in [-1, 1]^{n_\theta}, \)

where \( n_w \) and \( n_\theta \) are total number of weights and biases appearing in entire NLDT and \( M = 20 \) in our case.

6 Results

In this section, we present results obtained by using our approach for control tasks on four problems: (i) CartPole, (ii) CarFollowing, (iii) MountainCar, and (iv) LunarLander. The first two problems have two discrete actions, third problem has three discrete actions, and the fourth problem has four discrete actions. The open-loop statistics are reported using the scores of training and testing accuracy on labelled state-action data generated from \( \pi_{\text{oracle}} \). For quantifying the closed-loop performance, we use two metrics: (i) Completion Rate which gives a measure on the number of episodes which are successfully completed, and (ii) Cumulative Reward which quantifies how well an episode is executed. For each problem, 10 runs of open-loop training are executed using 10,000 training datapoints. Open-loop statistics obtained from these 10 independent runs of 10,000 training and 10,000 test data each are reported. We choose the median performing NLDT_{OL} for closed-loop analysis. We run 50 batches with 100 episodes each and report statistics of completion-rate and cumulative reward.

6.1 CartPole Problem

This problem comprises of four state variables and is controlled using two actions - move left and move right with an objective to stabilize an inverted pendulum on a cart (see Supplementary Document for more details). We conduct an ablation study to show the effect of training-data size on the open-loop and closed-loop performance of NLDT_{OL}. The results for this study are shown in Table 1. It is observed that NLDT_{OL} trained with at least 5,000 data points shows a robust open-loop performance. The obtained NLDT_{OL} has a about two rules with on an average three terms in the derived policy function. Interestingly, the same NLDT (without closed-loop training) also produces 100% closed-loop performance by achieving the maximum cumulative reward value of 200.

6.2 CarFollowing Problem

We have developed a discretized version of the car following problem discussed in (Nageshrao, Costa, and Filev 2019), wherein the task is to follow the car in the front which moves with a random acceleration profile (between \(-1 m/s^2\) and \(+1 m/s^2\)) and maintain a safe distance of \( d_{safe} = 30 m \) from it. The rear car is controlled using two discrete acceleration values of \(+1 m/s^2\) (action 0) and \(-1 m/s^2\) (action 1). The car-chase episode terminates when the relative distance \( d_{rel} = x_{front} - x_{rel} \) is either zero (i.e. collision case) or is greater than 150 m. At the start of the simulation, both the cars start with the initial velocity of zero. A DNN policy for CarFollowing problem was obtained using a double Q-learning algorithm (Van Hasselt, Guez, and Silver 2015). The reward function for the CarFollowing problem is shown in the Supplementary document, indicating that a relative distance close to 30 m produces the highest reward. It is to note here that unlike the CartPole control problem, where the dynamics of the system was deterministic, the dynamics of the CarFollowing problem is not deterministic due to the random acceleration profile with which the car in the front moves. This randomness introduced by the unpredictable behaviour of the front car makes this problem more challenging.

Results for the CarFollowing problem are shown in Table 2. An average open-loop accuracy of 96.53% is achieved with at most three rules, each having 3.28 terms on an average. For this problem, we apply the closed-loop re-optimization (Blocks 4 and 5 to produce Block 6 in Figure 3) on the entire NLDT_{OL}. As shown Table 3, NLDT* is able to achieve better closed-loop performances (100% completion rate and slightly better average cumulative reward). Figure 4 shows that NLDT* adheres the 30 m gap between the cars more closely than original DNN or NLDT_{OL}.

Results of NLDT’s performance on problems with two discrete actions (Tables 1, 2 and 3) indicate that despite having a noticeable mismatch with the open-loop output of the oracle black-box policy \( \pi_{\text{oracle}} \), the closed-loop performance of NLDT is at par or at times better than \( \pi_{\text{oracle}} \). This observation suggests that certain state-action pairs are not of crucial importance when it comes to executing the closed-loop control and, therefore, errors made in predicting these state-action events do not affect and deteriorate the closed-loop performance.
Table 1: Effect of training data size to approximate performance of NLDT\textsubscript{OL} on CartPole problem.

<table>
<thead>
<tr>
<th>Training Data Size</th>
<th>Training Accuracy</th>
<th>Test. Accuracy (Open-loop)</th>
<th># Rules</th>
<th>Rule Length</th>
<th>Cumulative Reward, Max=200</th>
<th>Compl. Rate (Closed-loop)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>97.00 ± 1.55</td>
<td>82.79 ± 2.40</td>
<td>1.50 ± 0.50</td>
<td>3.30 ± 0.93</td>
<td>199.73 ± 0.32</td>
<td>95.00 ± 5.10</td>
</tr>
<tr>
<td>500</td>
<td>95.54 ± 1.53</td>
<td>79.66 ± 3.10</td>
<td>1.90 ± 0.54</td>
<td>3.88 ± 0.60</td>
<td>175.38 ± 2.61</td>
<td>51.00 ± 5.10</td>
</tr>
<tr>
<td>1,000</td>
<td>91.90 ± 0.87</td>
<td>90.59 ± 1.87</td>
<td>1.80 ± 0.40</td>
<td>4.05 ± 1.04</td>
<td>200.00 ± 0.00</td>
<td>100 ± 0.00</td>
</tr>
<tr>
<td>5,000</td>
<td>92.07 ± 1.28</td>
<td>92.02 ± 1.27</td>
<td>1.70 ± 0.46</td>
<td>4.25 ± 0.90</td>
<td>200.00 ± 0.00</td>
<td>100 ± 0.00</td>
</tr>
<tr>
<td>10,000</td>
<td>91.86 ± 1.25</td>
<td>92.05 ± 1.10</td>
<td>1.30 ± 0.46</td>
<td>4.45 ± 1.56</td>
<td>200.00 ± 0.00</td>
<td>100 ± 0.00</td>
</tr>
</tbody>
</table>

Table 2: Results on CarFollowing problem.

<table>
<thead>
<tr>
<th>Train. Acc.</th>
<th>Test. Acc.</th>
<th>Depth</th>
<th># Rules</th>
<th>Rule Length</th>
<th>Compl. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>96.41 ± 1.97</td>
<td>96.53 ± 1.90</td>
<td>1.90 ± 0.30</td>
<td>2.40 ± 0.66</td>
<td>3.28 ± 0.65</td>
<td>100 ± 0.00</td>
</tr>
</tbody>
</table>

Table 3: Closed-loop performance analysis after re-optimizing NLDT for CarFollowing problem (k = 10\textsuperscript{3}).

<table>
<thead>
<tr>
<th>AI</th>
<th>Cumulative Reward</th>
<th>Compl. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>DNN</td>
<td>174.16k ± 173.75k</td>
<td>100 ± 0.00</td>
</tr>
<tr>
<td>NLDT\textsubscript{OL}</td>
<td>174.15k ± 173.87k</td>
<td>100 ± 0.00</td>
</tr>
<tr>
<td>NLDT*</td>
<td>179.76k ± 179.71k</td>
<td>100 ± 0.00</td>
</tr>
</tbody>
</table>

6.3 MountainCar Problem

This problem comprises two state-variables to capture x position and velocity of the car. The task is to use there actions and drive the under-powered car to the destination (see Supplementary Document for more details).

Compilation of results of the NLDT\textsubscript{OL} induced using training data sets comprising of different data distributions (regular and balanced) is presented in Table 4. A state-action plot obtained using π\textsubscript{oracle} and one of the NLDT policy corresponding to the first row of Table 4 is provided in Figures 1a and 1b respectively. It is observed that about 8% mismatch in the open-loop performance (i.e. testing accuracy in Table 4) comes from the lower-left region of state-action plot (Figures 1a and 1b) due to highly non-linear nature of π\textsubscript{oracle}. Despite having this mismatch, our interpretable NDLT policy is able to achieve close to 100% closed-loop control performance with an average of 2.4 rules having 2.97 terms. Also, NLDT trained on balanced dataset

Table 4: Results on MountainCar problem.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg.</td>
<td>91.28 ± 0.57</td>
<td>91.18 ± 0.35</td>
<td>2.00 ± 0.00</td>
<td>2.40 ± 0.49</td>
<td>2.97 ± 0.41</td>
<td>99.00 ± 1.71</td>
</tr>
<tr>
<td>Bal.</td>
<td>81.45 ± 7.36</td>
<td>87.23 ± 1.10</td>
<td>1.90 ± 0.60</td>
<td>2.80 ± 0.60</td>
<td>3.07 ± 0.42</td>
<td>100 ± 0.00</td>
</tr>
</tbody>
</table>

(2nd row of Table 4) is able to achieve 100% closed-loop performance and involves about three control rules with an average 1.67 terms in each rule.

6.4 LunarLander Problem

The task in this problem is to control the lunar-lander using four discrete actions and successfully land it on the lunar terrain. The state of the lunar-lander is expressed with eight state variables, of which six are continuous, and two are categorical. More details for this problem are provided in the Supplementary document.

Table 5: NLDT\textsubscript{OL} with depths 3 and 6 for LunarLander.

<table>
<thead>
<tr>
<th>Data Depth</th>
<th>Train. Acc.</th>
<th>Test. Acc.</th>
<th># Rules</th>
<th>Rule Length</th>
<th>Compl. Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reg.</td>
<td>79.17 ± 1.78</td>
<td>76.36 ± 3.36</td>
<td>5.60 ± 0.49</td>
<td>5.59 ± 0.75</td>
<td>14.00 ± 5.93</td>
</tr>
<tr>
<td>Bal.</td>
<td>69.83 ± 2.82</td>
<td>66.58 ± 2.03</td>
<td>4.40 ± 0.66</td>
<td>5.79 ± 1.31</td>
<td>42.00 ± 4.40</td>
</tr>
<tr>
<td>Reg.</td>
<td>87.43 ± 0.65</td>
<td>81.74 ± 0.91</td>
<td>34.70 ± 2.83</td>
<td>4.94 ± 0.34</td>
<td>48.00 ± 2.77</td>
</tr>
<tr>
<td>Bal.</td>
<td>81.74 ± 1.78</td>
<td>71.52 ± 1.24</td>
<td>25.70 ± 5.83</td>
<td>5.17 ± 0.37</td>
<td>93.00 ± 3.30</td>
</tr>
</tbody>
</table>

Table 5 provides the compilation of results obtained using NLDT\textsubscript{OL}. In this problem, while a better open-loop performance occurs for regular dataset, a better closed-loop performance is observed when the NLDT open-loop training is done on the balanced dataset. Also, NLDT\textsubscript{OL} with depth three are not adequate to achieve high closed-loop performance. The best performance is observed using balanced dataset where NLDT\textsubscript{OL} achieves 93% episode completion rate. A specific NLDT\textsubscript{OL} with 26 rules each having about 4.15 terms is shown in the Supplementary Document.
Table 6: Closed-loop performance on LunarLander problem with and without re-optimization on 26-rule NLDT\textsubscript{OL}. Number of rules are specified in brackets for each NLDT and total parameters for the DNN is marked.

<table>
<thead>
<tr>
<th>Re-Opt.</th>
<th>NLDT-2 (2)</th>
<th>NLDT-3 (4)</th>
<th>NLDT-4 (7)</th>
<th>NLDT-5 (13)</th>
<th>NLDT-6 (26)</th>
<th>DNN (4,996)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cumulative Reward</td>
<td>Completion Rate</td>
<td>Cumulative Reward</td>
<td>Completion Rate</td>
<td>Cumulative Reward</td>
<td>Completion Rate</td>
</tr>
<tr>
<td>Before</td>
<td>$-1675.77 \pm 164.29$</td>
<td>$42.96 \pm 13.83$</td>
<td>$54.24 \pm 27.44$</td>
<td>$56.16 \pm 23.50$</td>
<td>$169.43 \pm 23.96$</td>
<td>247.27 $\pm$ 3.90</td>
</tr>
<tr>
<td>After</td>
<td>$-133.95 \pm 2.51$</td>
<td>$231.42 \pm 17.95$</td>
<td>234.98 $\pm$ 22.25</td>
<td>182.87 $\pm$ 21.92</td>
<td>214.94 $\pm$ 17.31</td>
<td>94.00 $\pm$ 1.96</td>
</tr>
<tr>
<td>Before</td>
<td>0.00 $\pm$ 0.00</td>
<td>51.00 $\pm$ 3.26</td>
<td>82.00 $\pm$ 9.80</td>
<td>79.00 $\pm$ 7.66</td>
<td>93.00 $\pm$ 3.30</td>
<td>94.00 $\pm$ 4.45</td>
</tr>
<tr>
<td>After</td>
<td>48.00 $\pm$ 7.38</td>
<td>96.00 $\pm$ 2.77</td>
<td>99.00 $\pm$ 1.71</td>
<td>93.00 $\pm$ 7.59</td>
<td>94.00 $\pm$ 4.45</td>
<td>94.00 $\pm$ 1.96</td>
</tr>
</tbody>
</table>

It is understandable that a complex control task involving many state variables cannot be simplified or made interpretable with just one or two control rules. Next, we use a part of the NLDT\textsubscript{OL} from the root node to obtain the pruned NLDT\textsuperscript{*}\textsubscript{OL} (step ‘B’ in Figure 3) and re-optimize all weights ($W$) and biases ($\Theta$) using the procedure discussed in Section 5.2 (shown by orange box in Figure 3) to find closed-loop NLDT\textsuperscript{*}. Table 6 shows that for the pruned NLDT\textsuperscript{3} which comprises of the top three layers and involves only four rules of original 26-rule NLDT\textsubscript{OL} (i.e. NLDT-6), the closed-loop performance increases from 51% to 96% (NLDT\textsuperscript{*} results in Table 6) after re-optimizing its weights and biases with closed-loop training. The resulting NLDT with its associated four rules are shown in Figure 5.

Figure 5: Final NLDT\textsuperscript{*}-3 for LunarLander problem. $\hat{x}_i$ is a normalized state variable (see Supplementary Document).

As shown in Table 6, the NLDT\textsuperscript{*} with just two rules (NLDT-2) is too simplistic and does not recover well after re-optimization. However, the NLDT\textsuperscript{*}'s with four and seven rules achieve a near 100% closed-loop performance. Clearly, an NLDT\textsuperscript{*} with more rules (NLDT-5 and NLDT-6) are not worth considering since both closed-loop performances and the size of rule-sets are worse than NLDT\textsuperscript{*}-4. Note that DNN produces a better reward, but not enough completion rate, and the policy is more complex with 4,996 parameters.

7 Conclusions

In this paper, we have proposed a two-step strategy to arrive at hierarchical and interpretable rulesets using a nonlinear decision tree (NLDT) concept to facilitate an explanation of the working principles of AI-based policies. The NLDT training phases use recent advances in nonlinear optimization to focus its search on rule structure and details describing weights and biases of the rules by using a bilevel optimization algorithm. Starting with an open-loop training, which is relatively fast but uses only time-instant state-action data, we have proposed a final closed-loop training phase in which the complete or a part of the open-loop NLDT is re-optimized for weights and biases using complete episode data. Results on four popular discrete action problems have amply demonstrated the usefulness of the proposed overall approach.

This proof-of-principle study encourages us to pursue a number of further studies. First, the scalability of the interpretable NLDT approach to large-dimensional state-action space problems must now be explored. A previous study on NLDT (Dhebar and Deb 2020) on binary classification of dominated versus non-dominated data in multi-objective problems was successfully extended to 500-variable problems. While it is encouraging, the use of customization methods for initialization and genetic operators using problem heuristics and/or recently proposed innovation methods (Deb and Srinivasan 2006) in the upper level problem can be tried. Second, this study has used a computationally fast open-loop accuracy measure as the fitness for evolution of the NLDT\textsubscript{OL}. This is because, in general, an NLDT\textsubscript{OL} with a high open-loop accuracy is likely to achieve a high closed-loop performance. However, we have observed here that a high closed-loop performance is achievable with a NLDT\textsubscript{OL} having somewhat degraded open-loop performance, but re-optimized using closed-loop performance metrics. Thus, a method to identify the crucial (open-loop) states from the AI-based controller data set that improves the closed-loop performance would be another interesting step for deriving NLDT\textsubscript{OL}. This may eliminate the need for re-optimization through closed-loop training. Third, a more comprehensive study using closed-loop performance and respective complexity as two conflicting objectives for a bi-objective NLDT search would produce multiple trade-off control rule-sets. Such a study can, not only make the whole search process faster due to the expected similarities among multiple policies, they will also enable users to choose a single policy solution from a set of accuracy-complexity trade-off solutions.
References


A Additional Information about the Proposed Method

A.1 Data Normalization

First, we provide the exact normalization of state variables performed before the open-loop learning task is executed. Before training and inducing the non-linear decision tree (NLDT), features in the dataset are normalized using the following equation:

\[ \hat{x}_i = 1 + \frac{(x_i - x_i^{\text{min}})}{(x_i^{\text{max}} - x_i^{\text{min}})}, \]

where \( x_i \) is the original value of the \( i \)-th feature, \( \hat{x}_i \) is the normalized value of the \( i \)-th feature, \( x_i^{\text{min}} \) and \( x_i^{\text{max}} \) are minimum and maximum value of \( i \)-th feature as observed in the training dataset. This normalization will make every feature \( x_i \) to lie within \([1, 2]\). This is done to ensure that \( x_i = 0 \) is avoided to not cause a division by zero error.

A.2 Pruning and Tree Simplification for NLDT<sub>OL</sub>

Next, we discuss the pruning process performed to the NLDT<sub>OL</sub> to keep it within a reasonable depth and also achieve a reasonable open-loop accuracy. The NLDT representing our interpretable AI is induced using successive hierarchical splitting algorithm. A dedicated bilevel approach is used to derive the split rule for each conditional node, i.e., if a child node created after the split is still impure (with its impurity \( I > \tau_I \)), it is subjected to further split. Initially, we allow the tree to grow to a pre-specified maximum depth of \( d_{\text{max}} \). The resulting tree is fairly complicated with about hundreds of split nodes. Thus, we simplify this tree further to lower depths and remove redundant splits by pruning them. Lower depth trees are relatively simpler than the full grown depth \( d_{\text{max}} \) tree and also have better generalizability.

A.3 Reduction of NLDT<sub>OL</sub> for Closed-loop Optimization

If the obtained NLDT<sub>OL</sub> has a reasonable number of rules (say, less than five) and each rule has a reasonable complexity (with fewer terms in each rule), the overall NLDT<sub>OL</sub> may be acceptable and a closed-loop optimization may still be performed to obtain a fine-tuned NLDT* with better closed-loop performance.

However, in complex problems, the obtained NLDT<sub>OL</sub> may have many rules (> 5), whereby making the NLDT<sub>OL</sub> somewhat un-interpretable, despite the interpretable structure of each rule. The sheer number of rules will make the whole solution difficult to comprehend. To alleviate, we propose to reduce the size of NLDT<sub>OL</sub> so that five or fewer rules are retained from the root of NLDT<sub>OL</sub>. Since it is not known before the closed-loop analysis how many rules would produce an acceptable closed-loop performance, we propose to choose a few dissections of the NLDT<sub>OL</sub> from the root node, so that they contain 1-5 rules. Then, the closed-loop optimization can be performed to each dissection and a size-accuracy trade-off can be obtained. Based on this analysis, a final solution can then be chosen. We have illustrated this analysis for the LunarLander environment in the main paper in Table 6. Some instances of the dissection approach is illustrated in Figure Appendix A.7 of this document.

A.4 Differences between Open-loop and Closed-loop Searches

The overall search procedure described in Figure 3 in the main paper clearly indicated that it is a two-step optimization procedure. In the first optimization procedure, an open-loop NLDT (NLDT<sub>OL</sub>) is evolved using a bilevel optimization approach applied recursively to derive split-rule \( f(x) \) at each conditional node. Here, each training datapoint consists of a time-instant state-action pair obtained using oracle policy \( \pi_{\text{oracle}} \). One of the objective function of the overall bilevel algorithm is the minimization of the weighted Gini-score (\( F_L \), Eq. 6 in main paper), which quantifies the purity of nodes created after the split. This measure can also serve as a proxy to indicate the error between predicted action and the AI-model action. For a node \( P \), the Gini-score is computed as

\[ \text{Gini}(P) = 1 - \sum_{i=1}^{c} \left( \frac{N_i}{N} \right)^2, \]

where \( N \) is the total number of datapoints in node \( P \) and \( N_i \) is the number of datapoints present in node \( P \) which belongs to action-\( i \). As can be seen from Eq. [9] the computation of Gini-score is computationally cheap and fast. This eventually makes the computation of \( F_L \) (Eq. 6 in main paper) to be cheap and fast, a feature which is desired for any bilevel-algorithm since for each solution member in the upper-level of the search, a dedicated full run of lower-level optimization performed and if the lower-level objective function is computationally taxing then it will make the overall bilevel algorithm extremely slow. Additionally, it is to note here that every rule structure \( (f_j(x)) \) starting from the root node \( (j = 0) \) is optimized independently by using a subset of the training data dictated by the completed NLDT thus far. Nowhere in the development of the NLDT<sub>OL</sub>, any closed-loop evaluation function (such as, a cumulative reward function of completing the task, or success rate of completion) is used in the optimization process. The structure of the NLDT<sub>OL</sub> and structure of every rule (with its mathematical structure and coefficients/biases associated with each rule) are evolved. Due to the vastness of the search space of this optimization task, we developed a computationally efficient bilevel optimization procedure composing of a computationally cheap and fast lower-level objective. The two levels allow the structure of each rule and the associated coefficients and biases to be learnt in a hierarchical manner. This is also
possible due to recent advances in nonlinear optimization using hybrid evolutionary and point-based local search algorithms (Dhebar and Deb 2020).

On the contrary, the closed-loop optimization restricts its search to a fixed NLDT structure (which is either identical to NLDT\(_{\text{OL}}\) or a part of it from the root node, as illustrated in Figure A.7, but modifies the coefficients and biases of all rules simultaneously in order to come up with a better closed-loop performance. Here, an entire episode (a series of time-instance state-action pairs from start (\(t = 0\)) to finish (\(t = T\)) can be viewed as a single datapoint. As an objective function, the average of cumulative-reward collected across 20 episodes, each with a random starting state \(S_0\) is used to make a better evaluation of the resulting NLDT. Due to this aspect, the computational burden is more, but the search process stays in a single level. We employ an efficient real-parameter genetic algorithm with standard parameter settings (Deb and Agrawal 1995; Deb 2005). To make the search more efficient, we include the NLDT\(_{\text{OL}}\) (or its part, as the case may be) in the initial population of solutions for the closed-loop search.

The differences between the two optimization tasks are summarized in Table A.1. As discussed, both optimization tasks have their role in the overall process. While evaluation of a solution in the open-loop optimization is computationally quicker, it does not use a whole episode in its evaluation process to provide how the resulting rule or NLDT perform on the overall task. The goal here is to maximize the state-action match with the true action as prescribed by \(\pi_{\text{oracle}}\). This task builds a complete NLDT structure from nothing by finding an optimized rule for every conditional node. The use of a bilevel optimization, therefore, is needed. On the other hand, keeping a part (or whole) of the NLDT\(_{\text{OL}}\) structure fixed, the closed-loop optimization fine-tunes all associated rules to maximize the cumulative reward \(R_{\text{total}}\). A closed-loop optimization alone on episodic time-instance data to estimate \(R_{\text{total}}\) will not be computationally tractable in complex problems.

B Problems Used in the Study

In this section, we provide a detail description of the four environments used in this study.

B.1 CartPole Environment

The CartPole problem comprises of four state variables: 1) \(x\)-position (\(x \rightarrow x_0\)), velocity in +ve \(x\) direction (\(v \rightarrow v_1\)), angular position from vertical (\(\theta \rightarrow x_2\)) and angular velocity (\(\omega \rightarrow x_3\)) and is controlled by applying force towards left (Action 0) or right (Action 1) to the cart (Figure A.1a). The objective is to balance the inverted pendulum (i.e. \(-24\text{ deg} \leq \theta \leq 24\text{ deg}\)) while also ensuring that the cart doesn’t fall off from the platform (i.e. \(-4.8 \leq x \leq 4.8\)). For every time step, a reward value of 1 is received while \(\theta\) is within \(\pm24\text{ deg}\). The maximum episode length is set to 200 time steps. A deep neural network (DNN) controller is trained on the CartPole environment using the PPO algorithm (Schulman et al. 2017).

B.2 CarFollowing Environment

As mentioned in the main paper, we have developed a discretized version of the car following problem discussed in (Nageshrao, Costa, and Filev 2019) (illustrated in Figure A.1b), wherein the task is to follow the car in the front which moves with a random acceleration profile (between \(-1m/s^2\) and \(+1m/s^2\)) and maintain a safe distance of \(d_{\text{safe}} = 30\text{m}\) from it. The rear car is controlled using two discrete acceleration values of \(+1m/s^2\) (Action 0) and \(-1m/s^2\) (Action 1). The car-chase episode terminates when the relative distance \(d_{\text{rel}} = x_{\text{front}} - x_{\text{rel}}\) is either zero (i.e. collision case) or is greater than 150 m. At the start of the simulation, both the cars start with the initial velocity of zero. A DNN policy for CarFollowing problem was obtained using a double Q-learning algorithm (Van Hasselt, Guez, and Silver 2015). The reward function for the CarFollowing problem is shown in Figure A.2, indicating that a relative distance close to 30 m produces the highest reward.

Figure A.2: Reward function for CarFollowing environment.

It is to note here that unlike the CartPole control problem, where the dynamics of the system was deterministic, the dynamics of the CarFollowing problem is not deterministic due to the random acceleration profile with which the car in the front moves. This randomness introduced by the unpredictable behaviour of the front car makes this problem more challenging.
### Table A.1: Differences between open-loop and closed-loop optimization problems.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Open-loop Optimization</th>
<th>Closed-loop Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goal</td>
<td>Find each rule-structure ( f_j(x) ) one at a time from root node ( j = 0 )</td>
<td>Find overall NLDT simultaneously</td>
</tr>
<tr>
<td>Variables</td>
<td>Nonlinear structure ( B_{ij} ) for ( i )-th term for every ( j )-th rule, coefficients ( w_{ij} ), and biases ( \theta_j )</td>
<td>Coefficients ( w_{ij} ) and biases ( \theta_j ) for all rules ( (j) ) in the NLDT</td>
</tr>
<tr>
<td>Each training data</td>
<td>State-action pair ((x^t-a^t)) for each time-instance ( t )</td>
<td>Randomly initialized ( M ) Episodes comprising of state-action-reward triplets ((x^t-a^t-r_t, \ t = 1, \ldots, T)) for each simulation</td>
</tr>
<tr>
<td>Objective function</td>
<td>Weighted Gini-score (mismatch in actions)</td>
<td>Average cumulative reward value</td>
</tr>
<tr>
<td>Optimization method</td>
<td>Bilevel optimization: Upper-level by customized evolutionary algorithm and lower-level by regression</td>
<td>Single-level genetic algorithm</td>
</tr>
<tr>
<td>Termination condition</td>
<td>Upper level (Change in fitness &lt; 0.01% for consecutive 5 generations in Upper level GA, with maximum 100 generations). Lower level (Change in fitness &lt; 0.01% for consecutive 5 generations, with maximum 50 generations).</td>
<td>30 generations</td>
</tr>
<tr>
<td>Outcome</td>
<td>NLDT(_{OL})</td>
<td>NLDT(_*)</td>
</tr>
</tbody>
</table>

### B.3 MountainCar Environment

A car starts somewhere near the bottom of the valley and the goal of the task is to reach the flag post located on the right up-hill with non-negative velocity (Figure A.3). The fuel is not enough to directly climb the hill and hence a control strategy needs to be devised to move car back (left up-hill), leverage the potential energy and then accelerate it to eventually reach the flag-post within 200 time steps. The car receives the reward value of \(-1\) for each time step, until it reaches the flag-post where the reward value is zero. The car is controlled using three actions: accelerate left (Action 0), do nothing (Action 1) and accelerate right (Action 2) by observing its state which is given by two state-variables: \( x \rightarrow x_0 \) and \( velocity \ v \rightarrow v_1 \). We use the SARSA algorithm [Rummery and Niranjan 1994] with tile encoding to derive the black-box AI controller, which is represented in form of a tensor, has a total of 151,941 elements.

**Figure A.3: MountainCar Environment.**

### B.4 LunarLander Environment

This problem is motivated form a classic problem of design of a rocket-controller. Here, the state of the lunar-lander is expressed with eight state variables, of which six can assume continuous real values, while the rest two are categorical, and can assume a Boolean value (Figure A.1c). The first six state variables indicate the \((x, y)\) position, and velocity and angular orientation and angular velocity of the lunar-lander. The two Boolean state variables provides the indication regarding the left-leg and right-leg contact of lunar-lander with the ground terrain. The lunar-lander is controlled using four actions: Action 0 \( \rightarrow \) do nothing, Action 1 \( \rightarrow \) fire left engine, Action 2 \( \rightarrow \) fire main engine and Action 3 \( \rightarrow \) fire right engine. The black-box DNN based controller for this problem is trained using the PPO algorithm (Schulman et al. 2017) and involves two hidden layers of 64 nodes.

### C Additional Results

Here, we present the additional results and one of the final NLDT\(_*\) obtained by our overall approach. The parameter settings used to train NDLTs (and other black-box AI agents) are provided in Section E.

### C.1 CartPole Problem

The NLDT\(_{OL}\) obtained for the CartPole environment is shown in Figure A.4 in terms of normalized state variable vector \( \hat{x} \).

**Figure A.4: CartPole NLDT\(_{OL}\) induced using 10,000 training samples. It is 91.45% accurate on the training dataset but has 100% closed loop performance. Normalization constants are: \( x_{\min} = [-0.91, -0.43, -0.05, -0.40], x_{\max} = [1.37, 0.88, 0.10, 0.45] \).**

The respective policy is stated as follows:
if \[-0.18\hat{x}_0\hat{x}_2^2 - 0.63\hat{x}_3^2 + 0.67 \leq 0\]
then
  | Action = 0
else
  | Action = 1

A little manipulation will reveal that for a correct control strategy, Action 0 must be invoked if following condition is true:
\[2.39 \leq \left( \frac{x_0}{x_2^2} + \frac{3.50}{x_3^2} \right) \leq 5.06,\]
otherwise, Action 1 must be invoked. First, notice that the above policy does not require the current velocity \((\hat{v}_i)\) to determine the left or right action movement. Second, for small values of angular position \((x_2 \approx 1)\) and angular velocity \((x_3 \approx 1)\), meaning that the pole is falling towards left, the above condition is always true. That is, the cart should be pushed towards left, thereby trying to stabilize the pole to vertical position. On the other hand, if the pole is falling towards right (large values of \(x_2 \approx 2\) and \(x_3 \approx 2\), the term in bracket will be smaller than 2.39 for all \(x_0 \in [1, 2]\), and the above policy suggests that Action 1 (push the cart towards right) must be invoked. When the pole is falling right, a push of the cart towards right helps to stabilize the pole towards its vertical position. These extreme case analyses are intuitive and our policy can be explained for its proper working, but what our NLDT approach is able to find is a precise rule for all situations of the state variables to control the Cart-Pole to a stable configuration, mainly using the AI-blackbox data.

C.2 CarFollowing Problem

The NLDT\textsubscript{OL} obtained for the CarFollowing problem is shown in Figure A.5. The rule-set is provided in its natural if-then-else form, as follows:

Recall that the physical meaning of state variables is: \(x_0 \rightarrow d_{rel}\) (relative distance between front car and rear car), \(x_1 \rightarrow v_{rel}\) (relative velocity between front car and rear car) and \(x_2 \rightarrow a\) (acceleration value \((-1\ or\ +1\ \text{m/s}^2)\) of the previous time step). Action = 1 stands for acceleration and Action = 0 denotes deceleration of the rear car in the next time step.

From the first rule (Node 0), it is clear that if the rear car is close to the front car \((\hat{x}_0 \approx 1)\), the root function \(f_0(x)\) is never going to be positive for any relative velocity or previous acceleration of the rear car (both \(\hat{x}_1\) and \(\hat{x}_2\) lying in \([1,2]\)). Thus, Node 4 (Action = 1, indicating acceleration of the rear car in the next time step) will never be invoked when the rear car is too close to the front car. Thus for \(\hat{x}_0 \approx 1\), the control always passes to Node 1. A little analysis will also reveal that for \(\hat{x}_0 \approx 1\), the rule \(f_1(x) > 0\) for any relative velocity \(\hat{x}_1\) in \([1, 2]\). This means that when the two cars are relatively close, only Node 3 gets fired to decelerate (Action = 0) the rear car. This policy is intuitively correct, as the only way to increase the gap between the cars is for the controlled rear car to be decelerating.

However, when the rear car is far way maintaining a distance of about \(x_0 \text{max} = 30\ m\) for which \(\hat{x}_0 \approx 2\), Action 1 (Node 4) gets fired if \(\hat{x}_1 > 1.829\sqrt{x_2}\). If the rear car was decelerating in the previous time step (meaning \(\hat{x}_2 = 1\)), the obtained NLDT* recommends that the rear car should accelerate if \(\hat{x}_1 \in [1.829, 2]\), or when the magnitude of the relative velocity is small, or when \(x_1 \in [-0.776, 0.700] \text{m/s}\). This will help maintain the requisite distance between the cars. On the other hand, if the rear car was already accelerating in the previous time step (\(\hat{x}_2 = 2\)), Node 4 does not fire, as \(\hat{x}_1\) can never be more than 1.829\sqrt{2} and the control goes to Node 1 for another check. Thus, the rule in Node 0 makes a fine balance of the rear car’s movement to keep it a safe distance away from the front car, based on the relative velocity, position, and previous acceleration status. When the control comes to Node 1, Action 1 (acceleration) is invoked if \(\hat{x}_1 > 0.96/(0.58\hat{x}_0 - 1)\). For \(\hat{x}_0 \approx 2\), this happens when \(\hat{x}_1 > 1.817\) (meaning that when the magnitude of the relative velocity is small, or \(x_1 \in [-0.879, 0.700] \text{m/s}\), the rear car should accelerate in the next time step. For all other negative but large relative velocities \(x_1 \in [-7.930, 0.879] \text{m/s}\), meaning the rear car is rushing to catch up the front car, the rear car should decelerate in the next time step. From the AI-blackbox data, our proposed methodology is able to create a simple decision tree with two nonlinear rules to make a precise balance of movement of the rear car and also allowing us to understand the behavior of a balanced control strategy.

C.3 MountainCar Problem

The NLDT\textsubscript{OL} obtained for the MountainCar problem is shown below in Figure A.6.

Respective rules are stated in if-then-else statements:
mains intact.

Closed-loop training through closed-loop training (Sec-

the table for a comparison. It can be noticed that the re-

F \text{ (Depth 3) are also shown in }

NLDT*-3, which is shown in Figure A.8. This NLDT also

posed approach, we perform another run of the open-loop

loop training is shown in Table 6 of the main paper. The main

sized NLDTs (such as, NLDT-5, NLDT-4, NLDT-3, NLDT-

machine to evaluate population members in parallel manner.

C.4 LunarLander Problem

One of the NLDT\textsubscript{OL}S induced using the open-loop super-

6 and it involves a total of 26 rules. The figure also

shows how this 26-rule NLDT\textsubscript{OL} can be pruned to smaller

sized NLDTs (such as, NLDT-5, NLDT-4, NLDT-3, NLDT-

2) starting from the root node. A compilation of results cor-

responding to these trees regarding their closed-loop perfor-

mance before and after re-optimizing them using the closed-

loop training is shown in Table 6 of the main paper. The main

paper has also presented a four-rule NLDT*-3 obtained by a

closed-loop training of the above NLDT-3.

To demonstrate the efficacy and repeatability of our pro-

posed approach, we perform another run of the open-loop

and closed-loop training and obtain a slightly different

rule-sets (i.e. before applying re-optimization and af-

ter applying the re-optimization) is shown in the video at –

https://youtu.be/DByYWTQ6X3E. It can be observed in the

video that the closed-loop control executed using the Depth-

3 NLDT\textsuperscript{(P)}\textsubscript{OL} comprising of rules directly obtained from the

open-loop training (i.e. without any re-optimization) makes

the LunarLander comes close to the target nicely, but hovers

above the land and does not land it in most occasions,

thereby terminating an episode after the flight-time runs out.

On the other hand, the Depth-3 NLDT* comprising of

rule-sets obtained after re-optimization through closed-loop

training is able to successfully come close to the landing

base and lands the LunarLander. A comparison between the

oracle DNN and NLDT* is also provided at the end of the

video. DNN is able to execute the control task, but in some

cases it is not able to land the LunarLander properly and has

about 5000 parameters. On the other hand NDLT* has only

4 simple non-linear rules and is able to execute the control

taks efficiently.

D Computing Infrastructure

For open-loop training, 10 runs are performed in parallel

using Python’s multiprocessing module on a 56 cores Intel(R) Xeon(R) CPU E5-2697 v3 @ 2.60GHz. For closed-

loop training, only single run is performed. We distribute

the population pool across 50 cores of the above mentioned

machine to evaluate population members in parallel manner.

E Parameter Settings

E.1 Open-loop Training

For NLDT open-loop training, we used the default param-

eter setting as prescribed in (Dhebar and Deb 2020) apart

from the population size for upper-level GA which in our

case, we have set it to 10 for all problems. The lower level

optimization was done using the implementation of a real

coded genetic algorithm (RGA) from a Python package pymoo:

Multi-objective Optimization in Python (Blank and

Deb 2020).

E.2 Closed-loop Training

We use RGA implementation from pymoo (Blank and

Deb 2020) to do the closed-loop training. The parameter setting

we used is mentioned below

- Population size: different for NLDTs of different depths. See Table A.3 for details.
- Initialization: Random and seeded with one population member with the coefficients and bias values correspond-

ing to the parent NLDT\textsubscript{OL}.
- Crossover: Simulated Binary Crossover (Deb and Agrawal 1995), $\eta_c = 3$ and $p_c = 0.9$.
Figure A.7: NLDT-6 (with 26 rules) and other lower depth NLDTs for the LunarLander problem. Lower depth NLDTs are extracted from the depth-6 NLDT. Each node has an associated node-id (on top) and a node-class (mentioned in bottom within parenthesis). Table 6 in main paper provides results on closed-loop performance obtained using these trees before and after applying re-optimization on rule-sets using the closed-loop training procedure.

Figure A.8: Topology of Depth-3 NLDT obtained from a different run on the LunarLander problem. The equations corresponding the conditional-nodes before and after re-optimization are provided in Table A.2.

- Mutation: Polynomial mutation (Deb, Sindhya, and Okabe 2007). \( \eta_m = 5 \) and \( p_m = 1/n_{\text{vars}} \).
- Selection: Binary tournament selection (Goldberg and Deb 1991).

E.3 Black-box RL Algorithms

CartPole and LunarLander Problems: For CartPole and LunarLander problem, we use an implementation of the proximal policy gradient algorithm (PPO) (Schulman et al. 2017) from https://github.com/nikhilbarhate99/PPO-PyTorch with its default parameter setting other than maximum episodes, which in our case is set to 2000.

MountainCar Problem: We use an implementation of SARSA algorithm (Rummery and Niranjan 1994) based on tile encoding (Sutton 1996) from https://github.com/amohamed11/OpenAIGym-Solutions with its default parameter setting.

CarFollowing Problem: We implemented double deep Q-learning algorithm (Van Hasselt, Guez, and Silver 2015) using Pytorch. Following parameter setting was used

- Maximum episodes = 400
- Batch Size = 32
Table A.2: NLDT rules before and after the closed-loop training for LunarLander problem, for which NLDT* is shown in Figure A.8 Video at https://youtu.be/DByYWTQ6X3E shows the simulation output of the performance of NLDTs with rule-sets mentioned in this table. Respective minimum and maximum state variables are $x^{\text{min}} = [-0.38, -0.08, -0.80, -0.88, -0.42, -0.85, 0.00, 0.00]$, $x^{\text{max}} = [0.46, 1.52, 0.80, 0.50, 0.43, 0.95, 1.00, 1.00]$, respectively.

<table>
<thead>
<tr>
<th>Node</th>
<th>Rules before Re-optimization (Depth-3 NLDT$_{OL}^{(P)}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-0.23x_0x_2^{-1}x_6^{-1}x_7^{-1} - 1.00x_1^{-1}x_6 - 0.79x_0^{-1}x_1^{-1}x_6^2 + 0.83$</td>
</tr>
<tr>
<td>1</td>
<td>$0.17x_2^{-1} - 0.64x_3x_7^{-1} + 0.90x_1^{-2}x_6^{-2}x_7^{-3} + 0.29$</td>
</tr>
<tr>
<td>2</td>
<td>$0.82x_7^{-1} + 0.52x_0^{-1}x_4x_6^{-1} - 0.59x_4^{-1} - 0.95$</td>
</tr>
<tr>
<td>6</td>
<td>$-0.16x_4^{-3}x_6^{-3}x_7 - 0.86x_0x_5^{-1}x_6^{-3} + 1.00x_4x_6^{-1} - 0.70$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Node</th>
<th>Rules after Re-optimization (Depth-3 NLDT*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$-0.39x_0x_2^{-1}x_6^{-1}x_7^{-1} - 0.96x_1^{-1}x_6 - 0.12x_0^{-1}x_1^{-1}x_6^2 + 0.89$</td>
</tr>
<tr>
<td>1</td>
<td>$0.17x_2^{-1} - 0.78x_3x_7^{-1} + 0.90x_1^{-2}x_6^{-2}x_7^{-3} + 0.35$</td>
</tr>
<tr>
<td>2</td>
<td>$0.82x_7^{-1} + 0.52x_0^{-1}x_4x_6^{-1} - 0.59x_4^{-1} - 0.96$</td>
</tr>
<tr>
<td>6</td>
<td>$- (1.3 \times 10^{-3}) x_4^{-3}x_6^{-3}x_7 - 0.86x_0x_5^{-1}x_6^{-3} + 0.65x_4x_6^{-1} - 0.42$</td>
</tr>
</tbody>
</table>

Table A.3: Population size for Closed-loop training

<table>
<thead>
<tr>
<th>Depth</th>
<th>Population Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 3$</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>75</td>
</tr>
<tr>
<td>5</td>
<td>100</td>
</tr>
<tr>
<td>6</td>
<td>150</td>
</tr>
</tbody>
</table>

- Learning Rate = 0.01
- $\epsilon$ (for greedy policy) = 0.9
- Discount factor ($\gamma$) = 0.9
- Target-net update frequency = 100
- Replay Memory Capacity = 2000
- Number of hidden layers = 2 with ReLU activation functions.
- Number of hidden nodes per hidden layer = 50.

F Summary of this Document

This supplementary document has provided additional information and details about the proposed approach. Four problems used in the study have been described in more detail. Results have been discussed by presenting the obtained nonlinear rules. For the CartPole and CarFollowing problems, a detailed analysis of the obtained rule-sets has been provided for an intuitive reasoning of the working of the control policies. For the LunarLander problem, details about the pruning of NLDT$_{OL}$ for the final closed-loop training have been illustrated. The main paper and this supplementary document have amply demonstrated the power of finding interpretable rule-sets for a number of discrete-action based control system problems.