Challenges for Evolutionary Multiobjective Optimization Algorithms for Solving Variable-length Problems

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***Abstract*—In recent years, research interests have been paid in solving real-world optimization problems with variable-length representation. For population-based optimization algorithms, the challenge lies in maintaining diversity in sizes of solutions and in designing a suitable recombination operator for achieving an adequate diversity. In dealing with multiple conflicting objectives associated with a variable-length problem, the resulting multiple trade-off Pareto-optimal solutions may inherently have different variable sizes. In such a scenario, the fixed recombination and mutation operators may not be able to maintain large-sized solutions, thereby not finding the entire Pareto-optimal set. In this paper, we first construct multiobjective test problems with variable-length structures, and then analyze the difficulties of the constructed test problems by comparing the performance of three state-of-the-art multiobjective evolutionary algorithms. Our preliminary experimental results show that MOEA/D-M2M shows good potential in solving the multiobjective test problems with variable-length structures due to its diversity strategy along different search directions. Our correlation analysis on the Pareto-optimal solutions with variable sizes in the Pareto-optimal front indicates that mating restriction may be necessary in solving variable-length problem.**

I. Introduction

Over the past two decades, multiobjective evolutionary algorithms (MOEAs) have attracted a great research interest in the area of evolutionary computation and its application to industrial problems [1]. Compared with classical multiobjective methods, MOEAs have advantages to approximate the Pareto front by finding a set of representative solutions. For a multiobjective optimization problem, its objective space is often partially ordered. In the fitness assignment of MOEAs, a full order must be defined to rank different individuals in the evolving population. According to the schemes for fitness assignment, MOEAs can be categorized into three frameworks, i.e., Pareto-based MOEAs (e.g., NSGA-II [2] and SPEA2 [3]), decomposition-based MOEAs (e.g., MOGLS [4] and MOEA/D [5]), and indicator-based MOEAs (e.g., IBEA [6]). Apart from fitness assignment, the other important issue, i.e, diversity maintenance, has also been highly addressed in MOEAs.

Like many evolutionary algorithms for single objective optimization, the performances of MOEAs are also affected by the appropriate use of reproduction operators for generating new solutions. So far, many popular reproduction operators used in single objective EAs have also been extended to sample offspring solutions in MOEAs. For continuous MOPs, the most commonly-used reproduction operators are simulated binary crossover (SBX) and polynomial mutation [2]. The basic idea in these operators is to produce offspring solutions close to mating parents with higher probabilities. Over the past ten years, some recombination methods based on directional information, such as differential evolution (DE) [7] and particle swarm optimization (PSO) [8], have also been studied for solution generation in MOEAs. Moreover, probability methods based on Gaussian distributions, such as covariance matrix adaptation evolution strategy (CMAES) [9], were integrated into MOEAs [10], [11].

So far, a large number of MOEAs with different frameworks or reproduction operators have been proposed. According to no-free-lunch theory, there isn't a single MOEA that can have good performance on all MOPs with different difficulties. Therefore, the selection of frameworks and recombination operators must take problem difficulties into account. The typical difficulties in multiobjective optimization contain geometrical shape of Pareto front - PF (e.g., convexity and disconnection), geometrical shapes of Pareto set - PS (e.g, linearity and non-linearity), many local PFs, biased PFs, and so on. To study these difficulties, many benchmark multiobjective test problems have been constructed. The representative test problem sets are ZDT [12], DTLZ [13], LZ [7], UF [14], WFG [15], CPFT [16] and so on.

As reported in the literature [7], [13], NSGA-II has good performance in convergence on ZDT and DTLZ test problems with 2 or 3 objectives while MOEA/D is more effective on LZ and UF test problems with complicated PS. In NSGA-II, mating parents are selected from the current population based on the rank values and the crowding distance values. Any two nondominated solutions in different parts of PF have chance to recombine. In MOEA/D, only neighboring solutions are allowed to mate. Compared with NSGA-II, MOEA/D paid more attention on mating restriction. In fact, whether mating restriction is necessary in MOEAs is problem-dependent. When all solutions in the PS have very similar structures in the decision space (e.g., the PS of ZDT test problems), mating restriction is not very useful. When all solutions in the PS have different but highly correlated structures (e.g., the PS of LZ test problems), selecting similar mating parents should be encouraged in recombination.

Recently, some research interests have been paid on the optimization problems with variable-length representation [17], of which the number of decision variables is not fixed. To solve these problems, the major challenges for EAs lie in the design of their recombination operators. Since two mating parents may have different number of decision variables, the recombination between them can not be done as usual. In [17], some recombination operators for variable-length representation, such as spatial recombination, synapsing variable-length crossover, similar meta-variable recombination, and meta-variable insertion mutation, have been studied. However, it is not easy to extend these operators to solve different optimization problems with variable-length structure. In fact, the variable-length structure also exist in multiobjective optimization problems. Another study [18] has identified that NSGA-II is unable to find the entire Pareto-optimal set in a variable-length problem. In this paper, we study how to construct multiobjective test problems with variable-length structure. Some issues in MOEAs related to variable-length problems, such as diversity strategies and mating restriction, are also investigated.

The rest of this paper is organized as follows. In Section II, the background on the recombination strategies in two MOEAs will be reviewed. Then, the construction of the multiobjective test problems with variable-length structure will be presented in Section III. In the following section, some experimental results on the performance of MOEAs on five multiobjective test problems with variable-length structure are reported and discussed. Section V analyzes the landscape of the MOP with variable-length PF structure. The final section concludes the paper.

II. Multiobjective Optimization and Methods

In this section, we first introduce the Pareto-optimality in multiobjective optimization and two classical multiobjective methods. Then, two commonly-used MOEAs, i.e., NSGA-II and MOEA/D, are reviewed.

A. Pareto-Optimality

In the case of m objectives, a continuous multiobjective optimization problem can be formulated as:

$$\text{minimize}_{x\in\Omega} F(x) = (f_1(x), \ldots, f_m(x)) \tag{1}$$

where $x = (x_1, \ldots, x_n)$ is a decision vector in the feasible region $\Omega \subset R^n$. F is a vector of m objective functions.

A solution $y \in \Omega$ is said to dominate $z \in \Omega$ if $F(y) < F(z)$ and $F(y) \neq F(z)$. A solution $x^* \in \Omega$ is said to be Pareto-optimal if $F(x^*)$ is not dominated by the objective vector of any other solutions in Ω. In the decision space, the set of all Pareto-optimal solutions is called Pareto set. The set of the objective vectors of all solutions in the Pareto set is called Pareto front.

B. Classical Multiobjective Methods

The basic idea in classical multiobjective methods is to convert a MOP into a single objective optimization problem. The most well-known classical methods are the weighted sum method and the weighted Tchebycheff method. The formulations of these two methods can be written as follows:

- the weighted sum method: All objective functions are linearly aggregated into a scalar function
$$\min g^{ws}(x|\lambda) = \sum_{i=1}^{m} \lambda_i f_i(x)$$
with $\lambda_i \geq 0, i = 1, \ldots, m$ and $\sum_{i=1}^{m} \lambda_i = 1$.
- the weighted Tchebycheff method: A max-min rule is used to convert a MOP into a single objective optimization problem in a nonlinear way:
$$\min g^{tch}(x|\lambda) = \max_{i\in\{1,\ldots,m\}} \lambda_i |f_i(x) - z_i|$$
where $(z_1, \ldots, z_m)$ is a reference point. $(\lambda_1, \ldots, \lambda_m)$ is the same as in the weighted sum method.

Note that the weighted sum method is not suitable to find the nonconvex parts of PF. In contrast, the weighted Tchebycheff method can deal with both the convex PF and the nonconvex PF.

C. Multiobjective Evolutionary Algorithms

Over the past twenty years, evolutionary algorithms have became the dominant methods for multiobjective optimization. Among all multiobjective evolutionary algorithms, NSGA-II and MOEA/D have attracted much attention in recent a few years.

- NSGA-II [2]:
  NSGA-II is the extensive version of NSGA . It uses Pareto dominance to classify the population into a number of nondominated fronts. Within each front, none of the solutions is dominated by the others. In NSGA-II, mating parents are selected via tournament selection with two kinds of preference.
  - The solutions in the front closer to the PF are always preferred in selection.
  - When two solutions belong to the same front, one of them in the sparse area along the front is more likely to become mating parents.

  To estimate the density value of solutions in the front nearest to the PF, the crowding distance method is used in NSGA-II. Moreover, any two solutions with good quality in the population may have chance to recombine.

This strategy is good for the exploration of search space. However, no mating restriction is considered in NSGA-II. Therefore, its ability on the exploitation of search space is weak in some senses.
- MOEA/D [5]:
Similar to the classical multiobjective methods, MOEA/D needs to optimize some single objective scalar subproblems. The performance of MOEA/D depends on a number of issues, such as problem decomposition, mating selection, population replacement, archiving assistance, local search, recombination operator, and so on. In the first version of MOEA/D, the mating solutions are only selected from local neighborhood. Therefore, mating restriction is emphasized during the whole search procedure. This strategy is good for the exploitation of search space. To increase the ability of MOEA/D for the exploration of search space, selecting dissimilar mating parents should be encouraged.

III. Multiobjective Optimization with Variable-length Structures

In this section, we first introduce the background on the variable-length problems. Then, the details on constructing multiobjective test problems with variable-length structure along PF are given.

A. Background

In traditional optimality theory, the length of the decision vector is always assumed to be fixed. However, the length of decision vector is not necessarily fixed in many real optimization problems, such as sensor coverage problem [19], wind farm layout [20], and composite laminate stacking problem [21]. In these problems, the optimal number of components in the decision vector is unknown. When the number of decision variables is large, the classical fixed-length EAs are not efficient for solving the variable-length optimization problems. The main difficulty in EAs is the lack of efficient recombination operators for variable-length representation. Over the past few years, some efforts have been made to overcome this difficulty. In [17], a class of EAs for variable-length optimization, called metameric genetic algorithms (MGAs), were studied. Compared with the conventional EAs, the major changes in MGAs lie in the design of reproduction operators for variable-length structure, such as spatial recombination, synapsing variable-length crossover, similar metavariable recombination, as well as metavariable insertion mutation.

B. Problem Construction

The majority of problem difficulties in single optimization can be extended to multiobjective optimization. So far, no efforts have been devoted to variable-length structures in multiobjective optimization. In this work, we focus on the variable-length structure in the PF, where the Pareto solutions at different parts of PF may have different sizes. In the following, we give a general formulation on the multiobjective optimization problems with variable-length structure.

$$\min_{x,y_{1:L(x)}} f_i(x,y) = \alpha_i(x) + g_i(x,y,L(x)), i = 1,\dots,m \quad (2)$$

- The decision vector is $\bar{x} = (x,y) \in [0,1]^{m-1+L(x)}$ with $(m-1)$ fixed-length position variables
$$x = (x_1,\dots,x_{m-1}) \in [0,1]^{m-1}$$
and $L(x)$ variable-length distance variables
$$y = (y_1,\dots,y_{L(x)}) \in [0,1]^{L(x)}.$$
Note that the variable length $L(x)$ is not greater than N - the maximal number of components in y.
- The PF of (2) consists of the nondominated members in the following set:
$$\left\{\alpha(x) | x \in [0,1]^{m-1}\right\}.$$
Here, $\alpha(x) = (\alpha_1(x),\dots,\alpha_m(x))$ is a vector of m shape functions.
- The PS of (2) is of the form:
$$\{(x,y)|\alpha(x) \in \text{PF and } g_i(x,y,L(x)) = 0, i = 1,\dots,m\}$$
where the distance function $g_i(x,y,L(x))$ is a mapping from $R^{m-1+L(x)}$ to $\{0\} \cup R^+$.

In this work, we mainly focus on the construction of multiobjective test problems with variable-length PF in the case of 2-3 objectives. Note that the difficulties of Problem (2) mainly depend on the definitions of the variable length $L(x)$ and the distance function g_i.

1) Bi-objective Test Problems ($m = 2$): To produce very simple PF shape, we use the following shape functions in this work.

$$\begin{bmatrix} \alpha_1(x) \\ \alpha_2(x) \end{bmatrix} = \begin{bmatrix} x \\ 1-x \end{bmatrix} \quad (3)$$

In this case, the PF of (2) is a line segment between $(0,1)$ and $(1,0)$.

The distance functions $g_i, i = 1,\dots,m$ are defined as follows:

$$g_i(x,y,L(x)) = \sum_{j=1}^{L(x)} \left(y_j - \sin\left(\frac{L(x)}{2N} \times \pi\right)\right)^2 \quad (4)$$

where
$$L(x) = 1 + H(x).$$

Note that $H(x) \in \{0,\dots,N-1\}$.

To construct a test problem with the variable length $(m-1+L(x))$, we define $H(x)$ in the following geometrical manner:

$$H(x) = \left\lfloor \frac{\theta(x)}{\theta_{max}} \times (N-1) \right\rfloor \quad (5)$$

where two angles $\theta(x)$ and θ_{max} are needed. The PF parts with small values of θ are easier in convergence than those with large values of θ. In the following, we give three examples on the formulations of $\theta(x)$.

- Example 2-1:
  Let $\alpha(x) = (\alpha_1(x), \alpha_2(x))$, $v = (0, 1)$, and $\theta_{max} = \frac{\pi}{2}$, then

$$\begin{aligned} \theta(x) &= \arccos\left(\frac{<\alpha(x), v>}{\|\alpha(x)\| \cdot \|v\|}\right) \\ &= \arccos\left(\frac{1}{\sqrt{\alpha_1(x)^2 + \alpha_2(x)^2}}\right) \end{aligned} \quad (6)$$

  The geometrical illustration of $\theta(x)$ can be found in Fig.1.

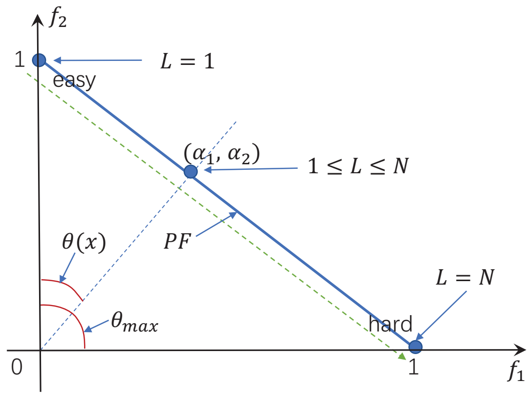

Fig. 1: The variable-length PF with the easy part near the direction $(0, 1)$ and the hard part near the direction $(1, 0)$.

- Example 2-2:

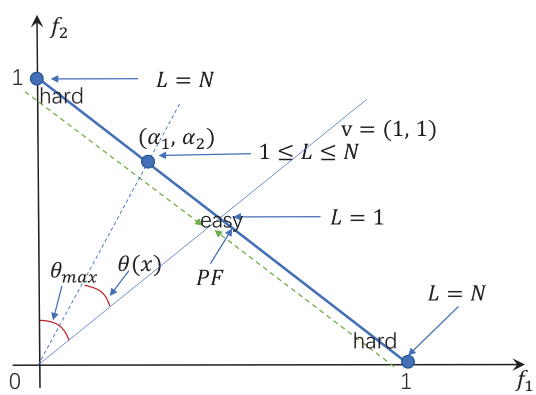

Fig. 2: The variable-length PF with one easy part near the direction $(1, 1)$, and two hard parts near the directions - $(1, 0), (0, 1)$.

  Let $\alpha(x) = (\alpha_1(x), \alpha_2(x))$, $v = (1, 1)$, and $\theta_{max} = \frac{\pi}{4}$, then

$$\begin{aligned} \theta(x) &= \arccos\left(\frac{<\alpha(x), v>}{\|\alpha(x)\| \cdot \|v\|}\right) \quad (7) \\ &= \arccos\left(\frac{1}{\sqrt{\alpha_1(x)^2 + \alpha_2(x)^2}\sqrt{2}}\right) \quad (8) \end{aligned}$$

  The geometrical illustration of $\theta(x)$ can be found in Fig.2.

- Example 2-3:

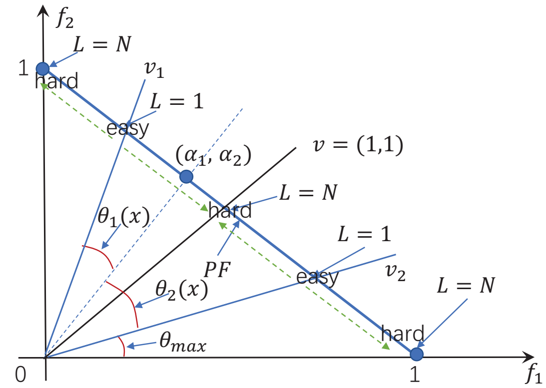

Fig. 3: The variable-length PF with two easy parts near two directions (v_1 and v_2) as well as three hard parts near three directions - $(0, 1), (1, 1), (1, 0)$.

  Let $\alpha(x) = (\alpha_1(x), \alpha_2(x))$, $\theta_{max} = \frac{\pi}{8}$, and

$$v_1 = \left(\frac{\sqrt{2-\sqrt{2}}}{4}, \frac{\sqrt{2+\sqrt{2}}}{4}\right)$$
$$v_2 = \left(\frac{\sqrt{2+\sqrt{2}}}{4}, \frac{\sqrt{2-\sqrt{2}}}{4}\right)$$

  then

$$\theta(x) = \min\{\theta_1(x), \theta_2(x)\} \quad (9)$$

  with

$$\theta_i(x) = \arccos\left(\frac{<\alpha(x), v_i>}{\|\alpha(x)\| \cdot \|v_i\|}\right), i = 1, 2$$

  The geometrical illustration of $\theta_1(x)$ and $\theta_2(x)$ can be found in Fig.3.

2) Three-objective Test Problems ($m = 3$): In this work, we use the following shape functions for constructing three-objective test problems:

$$\begin{bmatrix} \alpha_1(x) \\ \alpha_2(x) \\ \alpha_3(x) \end{bmatrix} = \begin{bmatrix} x_1(1 - x_2) \\ x_1 x_2 \\ 1 - x_1 \end{bmatrix} \quad (10)$$

The PF determined by the above shape functions is a unit simplex in $[0, 1]^3$. The distance functions are the same as in the equation (4) for constructing two-objective test problems. In the following, we discuss two examples on the formulations of $L(x) = 1 + H(x)$ using the similar method based on angles, where $H(x)$ is the same as in equation (5).

- Example 3-1:
  Let $\alpha(x) = (\alpha_1(x), \alpha_2(x), \alpha_3(x))$, $\theta_{max} = \arcsin\frac{\sqrt{3}}{3}$, and then compute three angles shown in Fig.4 as follows:

$$\theta_1(x) = \arcsin\left(\frac{\alpha_1(x)}{\sqrt{\alpha_1^2+\alpha_2^2+\alpha_3^2}}\right)$$
$$\theta_2(x) = \arcsin\left(\frac{\alpha_2(x)}{\sqrt{\alpha_1^2+\alpha_2^2+\alpha_3^2}}\right)$$
$$\theta_3(x) = \arcsin\left(\frac{\alpha_3(x)}{\sqrt{\alpha_1^2+\alpha_2^2+\alpha_3^2}}\right)$$

  Then, the minimal angle from $\alpha(x)$ to three coordinate planes (i.e., $f_2 - f_3$, $f_1 - f_3$, and $f_1 - f_2$) is given by:

$$\theta(x) = \min\{\theta_1(x), \theta_2(x), \theta_3(x)\} \quad (11)$$

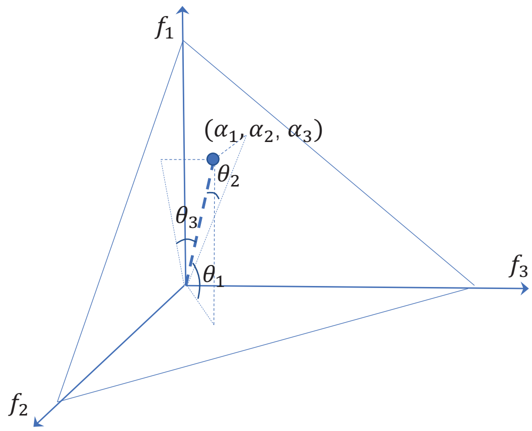

Fig. 4: The angles computed in Example 3-1.

It is easy to prove that $H(x)$ has the minimal value zero when $\theta(x)$ reaches its minimal value zero. In this case, $\alpha(x)$ is on the boundary of the simplex PF. On the other hand, when $\theta(x)$ reaches its maximal value θ_{max}, $H(x)$ has the maximal value $N-1$. That is, $\alpha(x)$ is located at the center of the simplex PF. Therefore, the test problem in this example has hard part in the center of the simplex PF and easy parts on the boundary of PF.

- Example 3-2

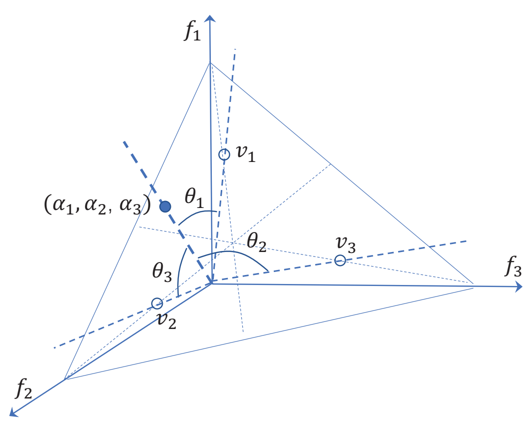

Fig. 5: The angles computed in Example 3-2.

Let $v_1 = (\frac{2}{3}, \frac{1}{6}, \frac{1}{6})$, $v_2 = (\frac{1}{6}, \frac{2}{3}, \frac{1}{6})$, $v_3 = (\frac{1}{6}, \frac{1}{6}, \frac{2}{3})$, $\theta_{max} = \frac{\sqrt{6}}{3}$
and

$$\theta_1 = \arccos\left(\frac{<\alpha, v_1>}{\|\alpha\|\|v_1\|}\right)$$
$$\theta_2 = \arccos\left(\frac{<\alpha, v_2>}{\|\alpha\|\|v_2\|}\right)$$
$$\theta_3 = \arccos\left(\frac{<\alpha, v_3>}{\|\alpha\|\|v_3\|}\right)$$

Again, $\theta(x)$ is defined as the minimum of the above three angles. That is,

$$\theta(x) = \min\{\theta_1(x), \theta_2(x), \theta_3(x)\} \quad (12)$$

The geometrical illustration of $(\theta_1(x), \theta_2(x), \theta_3(x))$ can be found in Fig.5.

Based on five examples described above, we can construct three bi-objective test problems and two three-objective test problems with variable-length PF structures named MOVL1-MOVL5 in order.

IV. Computational Experiments

In this section, we mainly discuss the performance of three well-known MOEAs on five multiobjective test problems with variable-length PF structures.

A. Experimental Settings

The maximal number of variables is set to 30 for three bi-objective test problems MOVL1-MOVL3 (i.e., $N = 29$) and 15 for two three-objective test problems MOVL4-MOVL5 (i.e., $N = 14$). To verify the difficulty of these test problems, we used three well-known MOEAs, i.e., NSGA-II, MOEA/D-DE, and MOEA/D-M2M [22], to approximate the variable-length PFs in our experiments. In all three algorithms, the population size is set to 100 for two objectives and 300 for three objectives. The total number of function evaluations is set to 100,000 for two objectives and 200,000 for three objectives. The neighborhood size in MOEA/D-DE is one tenth of the population size.

Note that different solutions may have different sizes in representation due to the variety of $L(x)$. To produce offspring solutions in the same was as in many existing MOEAs, we store all solutions in a set of fixed-length vectors. The length of these vectors equals to the maximal length - $(m-1+N)$. In all three algorithms, DE and polynomial mutation are applied to produce offspring solutions component by component as done in [7]. Note that the recombination between two solutions with different sizes might not be helpful since some components in the 'shorter' solution don't have any values.

B. Experimental Results

Fig.7 shows the distributions of the final populations found in one run of MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL1. From this figure, we can see that all three algorithms have difficulties to approximate the PF part near $(1, 0)$. The main reason is that the Pareto optimal solutions near $(1, 0)$ have large values of $L(x)$. Therefore, the distance function g_i in MOVL1 involves more terms in these solutions. In contrast, the Pareto optimal solutions near $(0, 1)$ can be easily found due to the small values of $L(x)$ in these solutions. Fig.8 plots the final populations found in one run of all three algorithms on MOVL2. This result shows that the populations of MOEA/D-DE and NSGA-II only approximate a small part of PF in the middle of PF while MOEA/D-M2M is well-converged toward the whole PF. The good performance of MOEA/D-M2M might be due to its diversity strategy on angle-based update. Fig.9 depicts the final populations found in one run of MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL3. This result indicates that all three algorithms have very poor convergence toward the whole PF and only approximate two small parts of PF, which have small values of $L(x)$.

Fig.6 shows the performance of NSGA-II on MOVL4 with easy-to-find region in the middle of the three-dimensional efficient front. The boundary region involves solutions having a large number of variables, hence is difficult to find and maintain by a standard EMO algorithm, such as NSGA-II. This phenomenon was also reported in another study [18], in which shorter genetic programs represented the middle of the efficient front and larger GPs represented the boundary region.

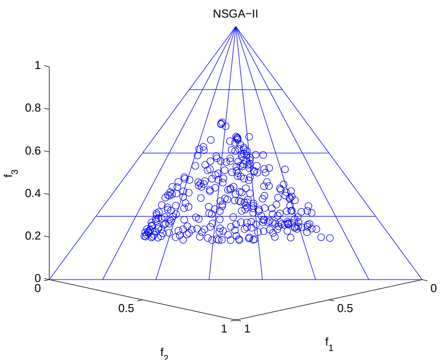

Fig. 6: The final population found by NSGA-II on a variant of MOVL4 with $N = 28$ and $H(x) = \left\lfloor \left(1 - \frac{\theta(x)}{\theta_{max}}\right) \times (N-1)) \right\rfloor$.

Fig.10 plots the final populations found in one run of MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL4. It is easy to observe from this figure that (i) the final populations found by both MOEA/D-DE and NSGA-II have an empty hole in the center; and (2) MOEA/D-M2M is the only algorithm that has the ability to approximate the whole PF. In Fig.11, the final populations found in one run of three algorithms on MOVL5 are shown. Among three algorithms, MOEA/D-DE is the only one that performs poorly in diversity.

Overall, the performance of MOEA/D-DE is worse than the other two algorithms on all five test problems. Among three algorithms, MOEA/D-M2M perform better than the other two algorithms. These results indicate that maintaining the population diversity along different search directions is helpful in searching hard PF parts.

V. Correlation Analysis on Variable-length Along PF

In this section, we analyze the correlation between different solutions in the PF with variable-length structure according to their difference in objective space and the change on the distance function. In this work, we only discuss the landscape of MOVL1. In our analysis, a naive recombination between any two solutions (x^1, y^1) and (x^2, y^2) in the PF of MOVL1 is considered. The change on the distance function g_i is evaluated when replacing the components in the shorter solution by the corresponding components in the longer solution. We denote L_1 and L_2 as the length values of y^1 and y^2, respectively.

In Fig.12, the correlation between $\Delta L = |L_1 - L_2|$ and Δg_i from 500 pairs of (x^1, y^1) and (x^2, y^2) are plotted. From this result, we can observe that modifying the short-length solution by copying the components from the long-length solution will not change too much regarding the distance function g_i if two solutions have close values of $L(x)$. However, when ΔL is large, the deterioration of g_i in offspring solutions is very obvious. From our analysis, it is reasonable to believe that mating restriction should be highly addressed in recombination.

VI. Conclusions

In this paper, we have studied the construction of evolutionary multiobjective optimization (EMO) problems with variable-length structures along the Pareto-optimal front. To bring out the challenges faced by an EMO algorithm, we have constructed five test problems for which the Pareto-optimal solutions are constituted with different variable sizes. Our experimental results have shown that the variable-length structures in multiobjective optimization mainly cause difficulties in maintaining diversity of unequal sized solutions in the usual EMO populations.

In this paper, we have not suggested any procedure to remedy the above difficulties associated with variable length multi-objective optimization problems. We are currently pursuing methods for this purpose. The key difficulties for dealing with the variable-length structure in the PF still lies in the design of the recombination operator. Our landscape analysis on the PF with variable-length structure shows that mating similar solutions will not greatly deteriorate the solution quality. In our future work, we will consider mating restriction in the design of recombination operators in solving the second type of variable-length problems. Also, keeping different operator probabilities for different sized individuals can also make differential treatment to solutions, thereby hopefully leading to better performance.

This paper has raised an important issue in EMO research and application. In most EMO applications, the entire Pareto-optimal set consists of solutions having an identical number of variables. However, in dealing with variable-length problems, Pareto-optimal solutions may consist of differing variable sizes. To find and maintain large-sized Pareto-optimal structures with small-sized Pareto-optimal structures in the same population generation after generation, more careful EMO operators (such as, special size-based niching methods and parent-specific recombination and mutation operators) must be used. In this paper, we have demonstrated this aspect by designing five test problems, which have provided difficulties to standard EMO algorithms. Further studies are now needed to fully understand the challenges and remedies for handling challenges inherent to variable-length problems.

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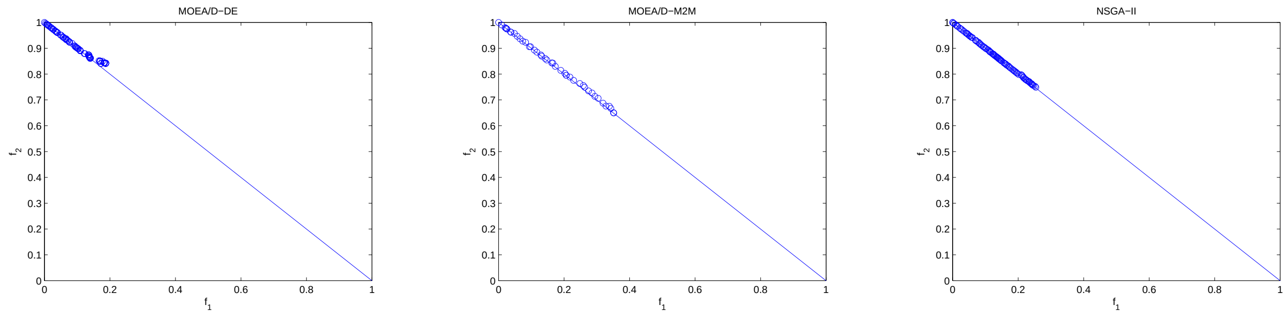

Fig. 7: The final populations found by MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL1.

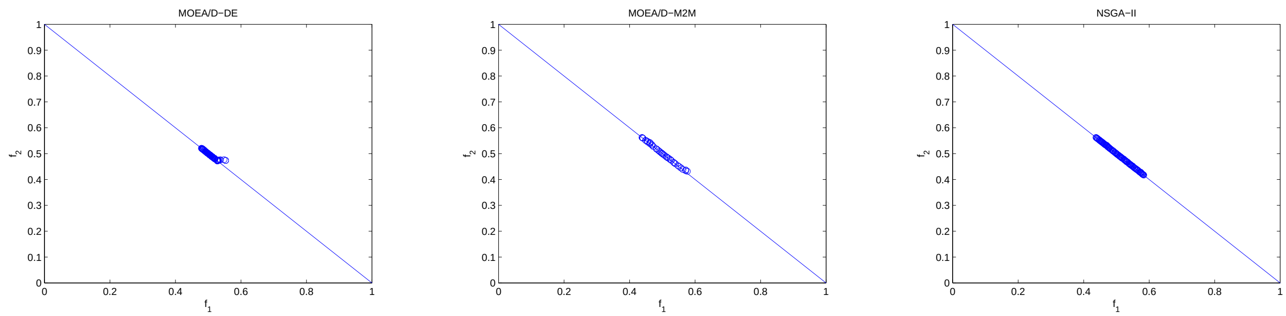

Fig. 8: The final populations found by MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL2.

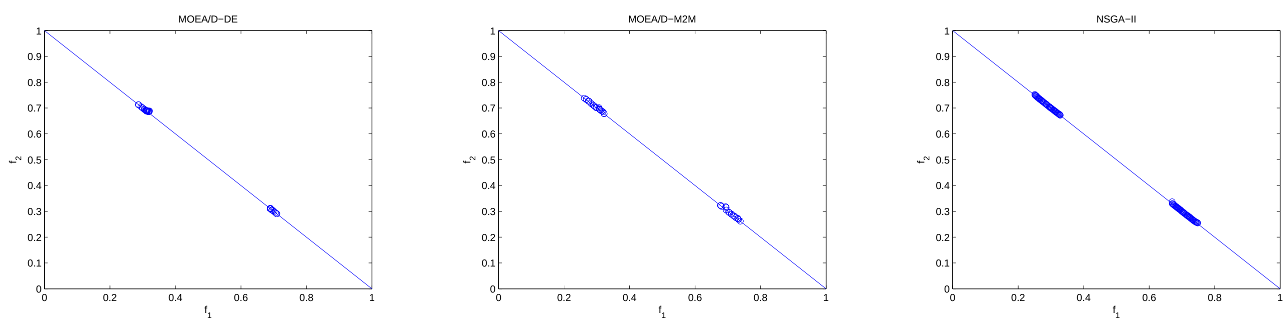

Fig. 9: The final populations found by MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL3.

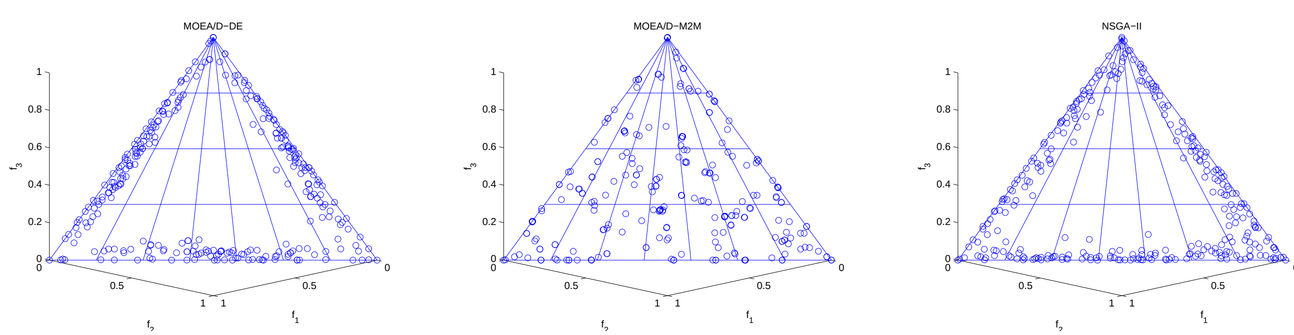

Fig. 10: The final populations found by MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL4.

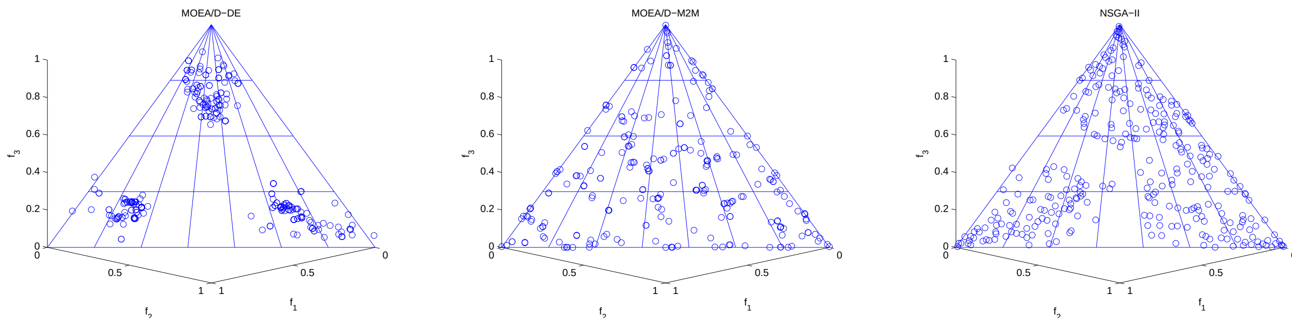

Fig. 11: The final populations found by MOEA/D-DE, MOEA/D-M2M, and NSGA-II on MOVL5.

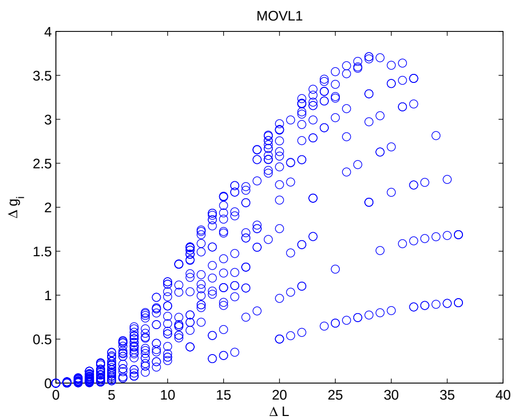

Fig. 12: The correlation between the changes of L and the changes of g_i for MOVL1 with $N = 29$.

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