Key Challenges and Future Directions of Dynamic Multi-objective Optimisation

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***Abstract*—Many real-world problems have more than one objective and are dynamic in nature, where either an objective function or constraint can vary over time. These problems are referred to as dynamic multi-objective optimisation problems (DMOOPs). A key challenge for dynamic multi-objective optimisation (DMOO) research is efficiently evaluating and analysing the performance of DMOO algorithms (DMOAs). This includes benchmarks, performance measures and the approach used to analyse the obtained results. Most research in recent years focussed on either dynamic single-objective or static multi-objective optimisation. In the field of DMOO, research focussed on unconstrained DMOOPs. A few papers have recently proposed constrained DMOOPs. Therefore, a key sub-challenge in DMOO is to have a standard benchmark suite that contains both unconstrained and constrained DMOOPs with various characteristics. In addition, the constraints used in the benchmarks should be guided by constraints that occur in real-world problems. Most approaches used to analyse the performance of DMOAs do not take into account how well a DMOA tracks the changing optimal solutions over time, i.e. how well it performs in each of the various environments. Furthermore, there are still certain DMOOPs that the proposed algorithms struggle to solve. Therefore, more research is required with regards to the development of algorithms that can solve DMOOPs efficiently. Another important aspect of DMOO is the decision making process that can either occur offline or interactively. This paper discusses these key challenges and progress that has been made to address these challenges. Furthermore, actions to deal with the outstanding issues are also proposed.**

I. Introduction

Even though many real-world problems are dynamic in nature and have multiple conflicting objectives, most research focusses on either dynamic single-objective optimisation (DSOO) or static multi-objective optimisation (MOO) [1], [2], [3], [4], [5], [6]. Algorithms solving DSOO problems (DSOOPs) have to overcome a loss of diversity and outdated memory [5]. Once the algorithm has converged to an optimal solution in an environment, the population's diversity has decreased and information about the best solution is based on the current environment. Therefore, if the environment changes, the population will struggle to find the optimal solution of the new environment. Multi-objective algorithms (MOAs) on the other hand are solving optimisation problems that do not have a single solution. Since some of the objectives are in conflict with one another, improving on one objective leads to worsening solutions for the other objective(s). Therefore, the goal of a MOA is to find the set of trade-off solutions that is as close as possible to the set of optimal trade-off solutions and that contains a diverse set of solutions [2]. The set of optimal trade-off solutions is referred to as the Pareto-optimal set (POS) in the decision variable space and as the Pareto-optimal front (POF) or Pareto frontier in the objective space.

Optimisation problems with more than one objective, with at least two objectives in conflict with one another, and where either the objectives or constraints (or both) change over time, are referred to as dynamic multi-objective optimisation problems (DMOOPs). Algorithms that solve DMOOPs have to achieve the goals of both DSOO algorithms and MOAs. Compared to the fields of DSOO and MOO, dynamic multi-objective optimisation (DMOO) is still new. A key challenge in DMOO is efficiently evaluating and analysing the performance of DMOO algorithms (DMOAs). Benchmarks are required to evaluate a DMOA on to determine whether the DMOA can successfully overcome specific difficulties, such as a disconnected POF, a non-linear POS, a deceptive or isolated optimum, spacing of solutions changing over time, various shapes of POFs, etc. Recently, Helbig and Engelbrecht discussed these challenges of DMOO [7]. In the paper they highlighted the lack of standard benchmark functions, lack of standard performance measures, and a lack of a standard way to compare the performance of DMOAs. Progress has been made with regards to theses challenges (refer to Section II), but further research is still required. Furthermore, research on DMOO mostly concentrated on unconstrained DMOOPs. Therefore, a key sub-challenge is to introduce benchmarks that contain constraints (both static and dynamic). This is necessary to apply the research to real-world problems. This

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paper discusses the progress that has been made with regards to these challenges of DMOO. It then focusses on research that is still required to adequately address these challenges.

The goal of a DMOA is to find a diverse set of solutions. However, at some point one solution from the set of solutions has to be chosen. The selection is normally based on the preferences of a decision maker. A solution can either be chosen offline (after the DMOOPs has been solved) or interactively (during the optimisation process). A key challenge of DMOO is to determine an efficient way to incorporate the preferences interactively to guide the search process [8].

Another key challenge is the development of new algorithms for DMOO that can solve DMOOPs efficiently. Algorithms that have been proposed for DMOO struggle to converge to the POF in fast changing environments and struggle to solve DMOOPs with non-linear POSs, where there are linkages or dependencies between the decision variables and where the POF is deceptive [9], [10].

An emerging research field in computational intelligence (CI) is many-objective optimisation, where an optimisation problem has more than three objectives. It should be noted that when referring to DMOO in this paper, it includes both the traditional DMOOPs with two or three objectives, as well as many-objective dynamic problems with more than three objectives.

The rest of the paper's layout is as follows: Section II discusses the challenges of DMOO. It highlights which aspects of these challenges have been addressed and which challenges remain to be solved. Future directions of DMOO are discussed in Section III. Finally, conclusions are drawn in Section IV.

II. Key Challenges of DMOO

This section discusses challenges in the field of DMOO. Section II-A discusses the key challenge of efficiently evaluating DMOAs. Section II-B discusses the challenge of decision making when solving DMOOPs. The other key challenge of developing DMOAs that can efficiently solve DMOOPs is discussed in Section II-C.

A. Evaluating DMOAs

In [7] Helbig and Engelbrecht discuss challenges in the field of DMOO as observed in 2013. The paper lists a lack of standard benchmark functions, a lack of standard performance measures, a lack of a standard approach to compare the performance of DMOAs and issues with performance measures that were used at that time as the main challenges. These challenges were partially addressed in the following publications: [10], [11], [12], [13]. Outstanding issues with benchmarks are discussed in Section II-A1. Section II-A2 discusses outstanding issues with performance measures.

1) Benchmarks: This section discusses outstanding issues with benchmarks, namely constructing constrained DMOOPs and DMOOPs with more than three objectives. It also discusses the development of a standard benchmark function suite.

Constrained DMOOPs

Most research in DMOO focussed on unconstrained DMOOPs [10]. Recently Wei and Wang [14] proposed a particle swarm optimisation (PSO) DMOA that uses a hyper rectangle search to predict solutions of the next time step. The DMOA was evaluated on unconstrained DMOOPs, as well as two constrained DMOOPs. Wei and Jia [15] evaluated a DMOA on constrained DMOOPs. The constrained DMOOPs were constructed by modifying constrained multi-objective optimisation problems (MOOPs). These publications are the first attempts on developing constrained DMOOPs and DMOAs that can solve constrained DMOOPs. However, more research is required to propose a set of benchmark functions that will adequately evaluate DMOAs to determine whether they can solve problems with different types of constraints.

Since most real-world DMOOPs contain constraints (both static and dynamic), DMOAs should be able to deal with constraints before they can be successfully applied to real-world problems. The types of constraints selected for the benchmarks functions should be guided by real-world DMOOPs. This will ensure that the constraints are typical of those that occur in real-world problems. However, care should be taken to not arbitrarily add constraints. The effect of the added constraints on the fitness landscape of the sub-objectives and the POF should be taken into account.

The following real-world DMOOPs were recently proposed to evaluate a DMOA on:

- Planning of mineral processing under varied equipment capability [16]
- Dynamic control of sustained overvoltage during a power system restoration process [17] and optimisation of power supply for magnesia grain manufacturing [18]
- Downlink power allocation in distributed satellite system [19]
- The production of high-grade polysilicon at a chemical vapor deposition reactor [20] and optimisation of chemical processes [21]
- Decision support system for manufacturing industry [22]
- Dynamic job-shop scheduling [23]
- Earthworks that involve the leveling or shaping of a target area through the moving or processing of the ground surface, i.e. optimisation of a production line [24]
- Dynamic vehicle routing [25] and the dynamic travelling salesman problem (TSP) [26]
- Optimization of quality of service for cloud-based dynamic data-driven application systems [27]
- Online path planning for unmanned aerial vehicles [28]
- Optimization of Inventory Management [29]

These problems can be used as a starting point to determine the various types of constraints that occur in real-world problems. The real-world problems should be analyzed to identify the fitness landscape, POS and POF characteristics, as well as the impact of the constraints. This will ensure that more than one problem with the same or similar characteristics are not added to the benchmark suite discussed below. In addition, these characteristics of the real-world problems should also be correlated with those of the artificial benchmark problems.

Uncertainty-based DMOOPs
Another aspect that should be considered when solving DMOOPs, is uncertainty that can be present in either the decision variables or the objectives of the DMOOPs. Limited research has been conducted on uncertainty-based DMOAs. Soroudi and Afrasiab [30] proposed a stochastic dynamic multi-objective model for distribution networks. The uncertainties were taken into account with scenario modelling, and a scenario-reduction technique is used to reduce the computational cost of the model. A binary PSO is used to solve the optimisation problem, after which a fuzzy method is applied to select the optimal solution based on the preferences of the decision maker (refer to Section II-B). Uncertainty is present in some of the real-world problems and therefore more research is required to develop DMOAs that can efficiently manage uncertainties.

Dynamic Many-objective Problems
An emerging field in CI is many-objective optimisation problems [31], [32]. The main challenge when solving these problems is that the non-domination concept of MOO can no longer be used to determine the quality of solutions. The more objectives the problem has, the more non-dominated solutions there are. To the best of the authors' knowledge, no work on dynamic many-objective optimisation has been published. Limited scalable DMOOPs exist. Therefore, scalable benchmarks would have to be proposed for dynamic many-objective optimisation. Recently, Jian and Yang [33] proposed a scalable DMOOP suite by using scalable MOOPs and adding various dynamic changes to these problems.

Constructing a Benchmark Suite
It is important that both real-world problems and benchmarks are used to evaluate DMOAs. Using only real-world problems as benchmark functions will enable researchers to evaluate a DMOA to determine whether it can successfully solve constrained DMOOPs. However, one real-world problem may contain a set of difficulties (such as a change in the spacing of solutions over time, a deceptive POF, a disconnected POF, etc.). Therefore, it will be difficult to understand and analyse the performance of the DMOA for each of these difficulties. However, benchmark functions test the performance of the DMOA for a specific type of problem with specific difficulties. Therefore, it is easier to analyse the performance of the DMOA when benchmarks are selected with various characteristics.

Helbig and Engelbrecht proposed characteristics that a benchmark function suite should have [10]. However, benchmarks with more diverse characteristics are still required. The current DMOOPs all have a fixed time interval between changes, i.e. a change occurs every n iterations. However, DMOAs should also be evaluated on benchmarks with adaptive time intervals between environment changes, i.e. benchmarks where the change can occur at different time intervals during the run. Recently, Biswas *et al.* [34] proposed a benchmark generator for DMOO. It constructs DMOOPs with various characteristics by changing the POS or the POF. The POS is shifted horizontally, vertically, horizontally and vertically. A polynomial POS is adapted by changing the polynomial order or shifting the POS. The POF is adapted over time by changing the shape or curvature or shifting the POF vertically, horizontally or diagonally.

An updated set of benchmark functions, that incorparates these new types of benchmarks, should be used to evaluate DMOAs on. More work is required to establish a standard set of benchmarks for the field. However, when designing benchmark problems the complexity of the fitness landscape should also be considered, and not only the different characteristics of the POS or POF. One approach is to investigate fitness landscape analysis of the sub-objectives and to ensure that the sub-objectives exhibit different fitness landscapes.

2) Performance Measures and Performance Analysis: Helbig and Engelbrecht discussed issues with performance measures used for DMOO [11], [12]. These issues include the effect of outliers and issues the occur when a DMOA loses track of the changing POF. Most of the papers in static MOO report performance measure values through an average value of the measure, as well as the standard deviation. However, that approach does not give an indication of how well the DMOA performed in the various environments and whether it could successfully track the changing POF and/or POS over time. If a DMOA performed really well in some of the environments, and not so well in other environments, the average value will not enable analysis of the DMOA's tracking ability. Therefore, a different approach is required. A wins-losses approach was proposed [13] to quantify how well a DMOA performed in each environment in comparison with the other DMOAs it is compared against. However, most reviewers still want to see the performance measure's average and standard deviation value. Therefore, a discussion is required in the field to establish the best approach to report on the performance of the DMOAs.

In addition, a challenge that remains is to standardise a DMOOP benchmark suite, performance measures and the evaluation and analysis process to facilitate cross-comparisons of DMOAs. This will enable researchers to compare their DMOA against existing DMOAs without having to re-implement the other DMOAs.

B. Decision Making

Since a DMOOP does not have a single solution (due to the conflicting objectives), an approach is required to select the optimal solution from the found POS. Selecting a solution can occur either offline or interactively. With the offline approach, the optimisation process is solved and the decision maker selects a solution after the optimisation process is completed. When solving a DMOOP, the decision maker will typically select a solution for every environment, i.e. the iteration just before a change in each environment occurs.

However, some real-world problems, such as control problems, require decision making during the optimisation process, and in some cases after each time step. This requirement for frequent decision making is very time consuming. Only limited research has been conducted on approaches to incorporate decision making or preferences of the decision maker into the DMOA [35], [36], [37]. Therefore, more research is required to develop a fast decision making process for DMOO. Another approach to incorparate the preferences of the decision maker is to move from a global DMOA to a local DMOA through decision making. In this case, the DMOO process is started with the goal of finding the entire POS. Then, based on preference information, the focus in the Pareto-optimal region is dictated by the decision maker. Therefore, more research is required to determine the best approach to make this transition from global to local optimisation.

C. Developing New DMOAs

A comprehensive overview of algorithms proposed for DMOO can be found in [6]. The algorithms can be divided into the following main categories:

- Evolutionary algorithms (EAs)
- PSOs
- New CI algorithms proposed for DMOO, such as membrane computing
- Prediction-based algorithms
- Algorithms that convert a DMOOP into a DSOOP

Most of these algorithms struggle to converge to the POF in fast changing environments and struggle to solve DMOOPs with non-linear POSs, where there are linkages or dependencies between the decision variables and where the POF is deceptive [9], [10]. In addition, most of the proposed algorithms were evaluated on only a few DMOOPs, and normally on DMOOPs that are easy to solve. Furthermore, to the best of the authors' knowledge, no algorithms for dynamic many-objective optimisation have been proposed yet. Therefore, there is a need for the development of algorithms that can solve DMOOPs efficiently. In addition, a comprehensive study should be done to compare the performance of DMOAs that have been proposed.

III. Future Directions of DMOO

This section discusses the future directions of DMOO. Section III-A discusses the course of action that is required to address the key challenges of DMOO. The potential impact of these actions are discussed in Section III-B.

A. Addressing Challenges

The key challenge of standardising the evaluation of DMOAs incorporate the sub-challenges of developing a standard benchmark suite, performance measures and an approach to analyse the performance of the DMOAs. This challenge should be addressed by a task force. A standard set of benchmarks, performance measures and approach to analyse the performance of DMOAs should be developed by the task force. The benchmarks should include DMOOPs with up to three objectives, as well as DMOOPs with more than three objectives. Once it has been developed, the results of DMOAs that were evalauted on these benchmarks should be freely available for other researchers to use. Therefore, the results should be stored in a central location that is easily accessible.

In addition, interest and participation from the community have to be increased. This can be achieved through the organization of special sessions, tutorials and competitions at major conferences in the field. This will lead to the development of new algorithms to solve DMOOPs.

B. Potential Impact

Increasing interest in DMOO will lead to more researchers working in the field. In addition, by focussing on constrained DMOOPs, algorithms can be applied to real-world problems. This will lead to more interaction between industry players and researchers. Lessons learned from solving constrained MOOPs and constrained DSOOs should be investigated and where appropriate, applied to algorithms that solve constrained DMOOPs. Similary, algorithms developed for static many-objective optimisation should be investigated and lessons learned applied when developing new DMOAs.

IV. Conclusion

Most of the research in recent years focussed on either static MOO or DSOO. However, many real-world problems are dynamic in nature with multiple objectives. In addition, most research in DMOO focussed on unconstrained DMOOPs. A key challenge in DMOO is to standardise the benchmark functions, performance measures and approach to analyse the results when evaluating the performance of a DMOA. A key sub-challenge is incorporating both static and dynamic constraints into the benchmark problems. In addition, the selected constraints should be guided by real-world problems, but also by considering the complexity of the fitness landscape and how the constraints affect the fitness landscape of the sub-objectives and the POF. This will lay the foundation for applying DMOAs to real-world DMOOPs. A standard set of benchmark functions should be developed taking into account proposed benchmarks in DMOO, DSOO and static MOO, as well as real-world problems. In addition, some of the benchmarks should be scalable to enable the evaluation of DMOAs on dynamic many-objective optimisation problems. Approaches used to analyse the performance of MOAs do not provide sufficient information on the tracking ability of a DMOA. Therefore, a different approach should be followed to quantify how well the DMOA tracked the changing POF and/or POS over time.

In addition, the community's interest in DMOO should be increased by organizing special sessions, tutorials and competitions at the major conferences in the field. The first competition on DMOO took place at CEC 2015 and the first tutorial on DMOO at SSCI 2015. These first two events laid the foundation, but future events are required that incorporates new advances in the field.

As more researchers become involved in the field of DMOO, the second challenge will be addressed, namely the development of DMOAs that can efficiently solve a wide range of DMOOPs. The third key challenge is addressing decision making in DMOO. Research into the development of DMOAs that can efficiently incorporate the decision maker's preferences into the search process in an interactive manner is also required. This challenge links closely to the second key challenge, and should be taken into account when new DMOAs are developed.

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