EliteNSGA-III: An Improved Evolutionary Many-Objective Optimization Algorithm

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Abstract— **Evolutionary algorithms are the most studied and successful population-based algorithms for solving single- and multi-objective optimization problems. However, many studies have shown that these algorithms fail to perform well when handling many-objective (more than three objectives) problems due to the loss of selection pressure to pull the population towards the Pareto front. As a result, there has been a number of efforts towards developing evolutionary algorithms that can successfully handle many-objective optimization problems without deteriorating the effect of evolutionary operators. A reference-point based NSGA-II (NSGA-III) is one such algorithm designed to deal with many-objective problems, where the diversity of the solution is guided by a number of well-spread reference points. However, NSGA-III still has difficulty preserving elite population as new solutions are generated. In this paper, we propose an improved NSGA-III algorithm, called EliteNSGA-III to improve the diversity and accuracy of the NSGA-III algorithm. EliteNSGA-III algorithm maintains an elite population archive to preserve previously generated elite solutions that would probably be eliminated by NSGA-III's selection procedure. The proposed EliteNSGA-III algorithm is applied to 11 many-objective test problems with three to 15 objectives. Experimental results show that the proposed EliteNSGA-III algorithm outperforms the NSGA-III algorithm in terms of diversity and accuracy of the obtained solutions, especially for test problems with higher objectives (greater than 15 objectives).**

Keywords— Evolutionary computation; large dimension; many-objective optimization; reference-point-based computation; non-dominated sorting, NSGA-III; EliteNSGA-III.

I. INTRODUCTION

Nowadays real-world applications are increasingly complex and more encompassing, in the sense that more decision variables are used to model complex situations and more input data and parameters are available to capture the complexity of the problems. Moreover, many of these problems involve optimizing a high number of objectives [1], [2].

Evolutionary multi-objective (EMO) algorithms such as NSGA-II, SPEA2, GDE3, MOEA/D, and others [3-6], have shown outstanding achievements in solving numerous economics, engineering and real-world scientific applications mainly involving 2 to 5 objectives. However, when solving problems involving higher number of objectives (usually more than 3 objectives, also known as many-objective optimization), a number of the evolutionary algorithms fail to find well-converged and well-diversified non-dominated solutions due to the loss of selection pressure in fitness evaluation [7]. In high-dimensional space the proportion of non-dominated individuals in a randomly generated initial population is often higher than 90% [8-10] and this will diminish the selection pressure considerably during the evolutionary process. Moreover, when the distance of nearly converged parent solutions is high, they will likely produce offspring solutions that are far from the true Pareto front [11, 12].

Since the Pareto-dominance schemes used in evolutionary many-objective algorithms failed to provide adequate selection pressure to guide the population towards the Pareto-optimal front, the focus has been shifted to improving the diversity-preserving schemes used in EMOs [13]. One such approach is to use a predefined multiple search directions [3] or predefined reference points [14-18] spanning the entire Pareto-optimal front to aid solutions towards targeted locations. NSGA-III [18] is one of the recent effective reference-point-based many-objective optimization algorithm whose population are guided by multiple predefined structured reference points to preserve the diversity of offspring solutions. Due to its power in guiding solutions to any predefined direction, NSGA-III has been gaining more acceptance in solving real-world many-objective optimization problems [19-21].

Since the introduction of NSGA-III, there have been a number of research studies toward improving the convergence and the overall performance of this algorithm. Yuan et al. [22] conducted an experimental investigation of variation operators in a NSGA-III. Their investigation concluded that NSGA-III performs better when selecting one of the three different variation operators, i.e., SBX, DE operator, and polynomial mutation randomly to produce an offspring solution.

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Yuan et al. [23] proposed an improved NSGA-III procedure, called θ-NSGA-III which aims to improve the convergence of NSGA-III in many-objective optimization. The θ-NSGA-III algorithm replaces the non-dominated sorting scheme utilized in NSGA-III by a θ-dominance scheme to rank solutions in the environmental selection phase to improve the convergence of NSGA-III. Although experimental results of θ-NSGA-III seem promising, it needs to incorporate a diversity enhancement strategy to improve the convergence and the diversity of obtained solutions.

Seada and Deb [24] proposed a unified evolutionary optimization algorithm, U-NSGA-III, for solving single-, multi- and many-objective problems. The proposed algorithm degenerates to an equivalent and efficient population-based optimization procedure just from the description of the number of specified objectives of a problem. Yuan et al. [19] introduced an extension to NSGA-III where dominance relationship is determined based only on constraint violation. In case of constraint violation, infeasible solutions are repaired by modifying the decision variables in feasible zone according to the violation amount.

This paper proposes a novel elite population archive-based NSGA-III (EliteNSGA-III) to mitigate the above-mentioned problem associated with many-objective optimization problems when the number of objectives is high. The rest of the paper is organized as follows. Section II provides the description of the NSGA-III algorithm. Section III provides the technical description of the proposed algorithm, called EliteNSGA-III. Section IV presents experimental studies on well-known many-objective test problems and finally, our conclusions are provided in Section V.

II. Introduction to NSGA-III

NSGA-III [18] is an extension of NSGA-II [5] designed for solving many-objective optimization problems. The fundamental components of NSGA-III are similar to NSGA-II algorithm, however, it has significant changes in its selection mechanism. In the original NSGA-II, the new population P_{t+1} is constructed from the combined population $R_t = P_t \cup Q_t$, in the order of their rankings. Let S_t be the population selected so far (including the last non-dominated front F_l). If the size of S_t is greater than the population size N, then the best members in F_l with the largest crowding distance values are selected. However, in NSGA-III the best members from the last non-dominated front F_l are selected based on the supplied reference points. In the original NSGA-III study, they have used Das and Dennis' [25] procedure to create these structured reference points.

First, each objective's values are adaptively normalized based on members of S_t. Then, reference lines corresponding to each reference point on the hyper-plane are constructed by joining the reference point with the origin. Thereafter, all population members of S_t and F_l are associated with a reference point whose reference line is closest to a population member in the normalized objective space. Then, the number of population members from $P_{t+1} = S_t$ that are associated with each reference point are counted. If there is a reference point with no member associated with it and one or more members of F_l associated with the reference point, then the one having the shortest perpendicular distance from the reference line is added to P_{t+1}. However if all reference points are associated with at least one population member, then a randomly chosen member from F_l is added to P_{t+1}. This procedure is repeated until the desired population size is achieved. Algorithm 1 without the gray shaded lines describes the NSGA-III algorithm.

Algorithm 1: EliteNSGA-III Procedure

P_0 = InitializePopulation() %uniform random
Z^r = GenerateReferencePoints()
$E_0 = \emptyset$ % $|E_0| = |Z^r|$
$\mu_0 = \infty$ % $|\mu_0| = |E_0|$
$[E_{t+1}, \mu_{t+1}]$ = `updateElite`(Z^r, P_0, E_0, μ_0)
while termination criteria is not met **do**
  $S_t = \emptyset, i = 1$
  Q_t = `Recombination + Mutation`(P_t, E_t)
  $R_t = P_t \cup Q_t$
  $(F1, F2, \ldots)$ = `Non-dominated-sort`(R_t)
  repeat
  $S_t = S_t \cup Fi$ and $i = i + 1$
  until $|S_t| \geq N$
  Last front to be included: $F_l = F_i$
  if $|S_t| = N$ **then**
    $P_{t+1} = S_t$, break
  else
    $P_{t+1} = \cup_{j=1}^{l-1} F_j$
    Points to be chosen from F_l: $K = N - |P_{t+1}|$
    Normalize objectives `Normalize`$(f^n, S_t, Z^r, Z^s, Z^a)$
    Associate each member $\boldsymbol{s}$ of S_t with a reference point: $[\pi(\boldsymbol{s}), d(\boldsymbol{s})]$ = `Associate`(S_t, Z^r) % $\pi(\boldsymbol{s})$: closest reference point, *d:* distance between **s** and π(**s**)
    Compute niche count of reference point $j \in \rho_j = Z^r : \sum_{s \in S_t/F_l}((\pi(s) = j\, ?\, 1 : 0)$
    Choose K members one at a time from F_l to construct P_{t+1}: `Niching`$(K, \rho_j, \pi, d, Z^r, F_l, P_{t+1})$
  end if
  Update elite archive and their distance to ideal point $[E_{t+1}, \mu_{t+1}]$ = `updateElite`$(Z^r, P_{t+1}, E_t, \mu_t)$
end while

III. Proposed Algorithm: EliteNSGA-III

The framework of the proposed EliteNSGA-III algorithm is similar to NSGA-III; however, EliteNSGA-III introduces two mechanisms to improve the performance of NSGA-III. First, we introduce an elite archive to preserve elite members of the population which may have otherwise been eliminated by the NSGA-III selection procedure. Second, we introduce new parent selection mechanism to improve the diversity of parent population. Algorithm 1 summarizes the proposed EliteNSGA-III algorithm. The following subsections explain the newly added segments of the proposed algorithm.

A. Elite Population Archive

The use of external archive to preserve obtained solutions in each generation is not new - in the past, elite-preserving external archives in NSGA [26] and NSGA-II [27] have shown respectable results compared to parent algorithms. The shaded areas of Algorithm 1 below are the contribution of the current work; only the updateElite procedure is explained here (explanation as per Fig.1). Assuming P_t is the current population which is depicted by the bold dots on Fig.1. Let $E_t(i)$ be the elite member associated with reference point r_i. Also, let π_i be a set of current solutions as well as previously archived elite member solution associated with r_i. At every generation, $E_{t+1}(i)$ is updated according to the following:

$$E_{t+1}(i) \ = min_{j=1}^{|\pi_i|} d(\pi_{i,j}) \qquad (1)$$

Where $d(\pi_{i,j})$ is the Euclidean distance between the j^{th} member solution and the origin. Note that when no new solution is associated with r_i, the previously archived elite solution is retained. For further explanation refer to Algorithm 2.

Algorithm 2: updateElite (Z^r, P_t, E_t, μ_t) Procedure

Input: References points on normalized hyper-plane Z^r: each $\mathbf{z} \in Z^r$ identified by its location $1\ldots|Z^r|$, Parent population P_t, elite population E_t, elite population distance to ideal point μ_t
Output: E_{t+1}, μ_{t_+1}

Associate each member $\boldsymbol{p}$ of P_t with a reference point: $[\pi(\boldsymbol{p}), d(\boldsymbol{p})]$ = `Associate`(P_t, Z^r) % $\pi(\boldsymbol{p})$: closest reference point, *d:* distance between $\boldsymbol{p}$ and $\pi(\boldsymbol{p})$
for $i \ = \ \boldsymbol{p} \ \in P_t$
  Find the location of the reference point associated with $\boldsymbol{p}$: loc = location $(\pi(\boldsymbol{p}))$
  Calculate the Euclidian distance between $f(p)$ and the ideal point: $\mu'_t(loc) = d(p, z^{min})$
  if $\mu'_t(loc) < \mu_t(loc)$ **then**
    $\mu_{t+1}(loc) = \mu'_t(loc)$
    $E_{t+1}(loc) = \boldsymbol{p}$
  else
  $\mu_{t+1}(loc) = \mu_t(loc)$
  $E_{t+1}(loc) = E_t(loc)$
  end if
end for

The main advantage of this elite archive is the preservation of elite members of the population which may have otherwise been eliminated by the NSGA-III selection procedure. As a result, the diversity of an offspring population is improved. Fig. 1 illustrates how this preservation occurs. In Fig.1, the black dots represent P_t while the white dots represent Q_t, the offspring population. In this example, the population size is six. Since solutions a, c, e, f, and g have the lowest distance in their respective reference point location, they will be archived in E_t. According to NSGA-III, since solution f is not in rank 1 and rank 1 contains more than six solution members, solution f will be eliminated and consequently won't be part of P_{t+1}. This renders the solution set in P_{t+1} less diverse than that of P_t. However, since solution member f was preserved in E_t and no better solution exists in its respective reference point location, solution member f will also be part of E_{t+1}. Note that other elite members will get replaced with a better solution (if any exists) which results in the preservation of solutions that are well-spread and has better convergence.

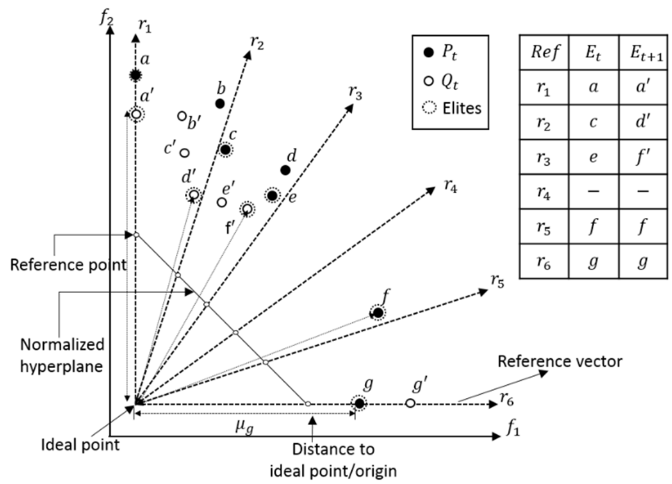


Fig. 1. Illustration of an elite population mechanism used in EliteNSGA-III.

Algorithm 3: Recombination + Mutation (P_t, E_t) Procedure

Input: Parent population P_t, elite population E_t
Output: Q_t

$Q_t = \emptyset$
s_P = size (P_t)
s_E = size (E_t)
for $i = 1 \ to \ s_P/2$
  $r_1 = rand(1, s_P)$
  $r_2 = rand(1, s_P)$, $r_1 \neq r_2$
  $r_3 = rand(1, s_E)$, $E(r_3) \neq null$
  $r_4 = rand(1, s_E)$, $E(r_4) \neq null$ and $r_3 \neq r_4$
  $p_1 = rand < 0.5 \ ? \ P_t(r_1) : E_t(r_3)$
$p_2 = rand < 0.5 \ ? \ P_t(r_2) : E_t(r_4)$
$[o_1, o_2]$ = `crossover + mutation` (p_1, p_2)
$Q_t = Q_t \cup [o_1, o_2]$
end for

B. Parent Selection Procedure

The other contribution of EliteNSGA-III is in parent diversity improvement. In NSGA-III, early generations of the population are associated with few of the supplied reference points. Subsequently, when selecting parent population, there is a likelihood of selecting parents associated with the same reference point. This might lead to the generation of offspring solutions that are close to their parents resulting in a reduction of diversity.

In EliteNSGA-III, and as explained by Algorithm 3, parent populations are selected with equal probability from both the current population and the elite archive. Also, since there is at most one elite member representing a reference point, the likelihood of selecting parents associated with the same reference point is minimized - hence improving diversity.

IV. Experimental Setup and Results

In this section, we describe the test problems used, parameter settings, and simulation results of EliteNSGA-III on 3- to 15-objective optimization problems.

A. Test Problems

In order to test the quality of the proposed algorithm, we have used 11 many-objective benchmark test problems. The first sets of these test problems are the DTLZ (DTLZ1 – DTLZ4) family of test problems introduced by Deb et al. [28]. The number of variables are $(M + k - 1)$, where M is the number of objectives and $k = 5$ for DTLZ1, while $k = 10$ for DTLZ2, DTLZ3, and DTLZ4. The corresponding Pareto-optimal fronts lie in $f_i \in [0, 0.5]$ for the DTLZ1 problem and in $f_i \in [0, 1]$ for other DTLZ problems. The DTLZ1 problem has a linear Pareto-optimal front and DTLZ2 to DTLZ4 problems have concave Pareto-optimal fronts.

The second sets of test problems utilized in this study are the WFG (WFG1, WFG2, and WFG4 – WFG7) family of test problems introduced by Huband et al. [29]. The number of position parameters is set to $k = M$, where M is the number of objectives and the number of distance parameters is set to $l = 3$ for all dimensions. The WFG1 has a mixed Pareto-optimal front, WFG2 problem has a convex Pareto-optimal front and WFG4 to WFG7 problems have concave Pareto-optimal fronts. The Pareto-optimal fronts for WFG test problems used in this work lie in $f_i \in [0, 2i]$. Table I presents detailed characteristics of the test problems used in this study.

The last test problem used in this study is a convex DTLZ2 problem. The construction of the convex DTLZ2 is similar to DTLZ2, however the convex DTLZ2 test problem powers the objective values of DTLZ2 $(1 \ldots (M - 1))$ by 4 and squares the M^{th} objective value. The corresponding Pareto-optimal fronts for convex DTZ2 lie in $f_i \in [0, 1]$.

TABLE I. Test Problems

Problem	*Characteristics*
DTLZ1	Linear, Multimodal
DTLZ2	Concave
Convex DTLZ2	Convex
DTLZ3	Concave, Multimodal
DTLZ4	Concave, Biased
WFG1	Convex, Mixed, Biased
WFG2	Convex, Disconnected, Multimodal
WFG4	Concave, Multimodal
WFG5	Concave
WFG6	Concave
WFG7	Concave, Biased

B. Parameter and Experimental Settings

The NSGA-III algorithm has four control parameters: SBX probability (p_c), Polynomial mutation (p_m), Crossover Distribution Index (η_c), and Mutation Distribution Index (η_m) which need to be tuned by the user. Similar to NSGA-III, EliteNSGA-III does not require any new parameter other than the above-mentioned genetic algorithm related parameters used in NSGA-III. In order to maintain a consistent and fair comparison, the parameter settings of NSGA-III and EliteNSGA-III are kept the same for all experiments. However EliteNSGA-III requires a dependent variable (archive size) to maintain the elite population throughout the generations. The archive size is set to H; where H is the number of reference points used and it is directly related to the desired number of trade-off points. In all experiments the archive size is kept the same as the number of reference points used for each test problems. Table II presents parameter settings used by NSGA-III and EliteNSGA-III algorithms.

TABLE II. NSGA-III and EliteNSGA-III Parameter Settings. n is the Number of Variables.

Parameters	*NSGA-III*	*EliteNSGA-III*
SBX probability (p_c)	0.9	0.9
Polynomial mutation (p_m)	$1/n$	$1/n$
Crossover Distribution Index (η_c)	30	30
Mutation Distribution Index (η_m)	20	20

Table III shows the number of reference points (H), the population size (N), and the number of inner and outer divisions used for different dimensions of test problems. These values are similar to the values used in the original NSAG-III study which included test problems with up to 15 objectives.

TABLE III. Number of Reference Points and Population Sizes Used in NSGA-III and EliteNSGA-III Algorithms.

Number of Objectives (M)	*Divisions*		*Reference Points(H)*	*Population Size (N)*
	Outer	*Inner*		
3	12	0	91	92
5	6	0	210	212
8	3	2	156	156
10	3	2	275	276
15	2	1	135	136

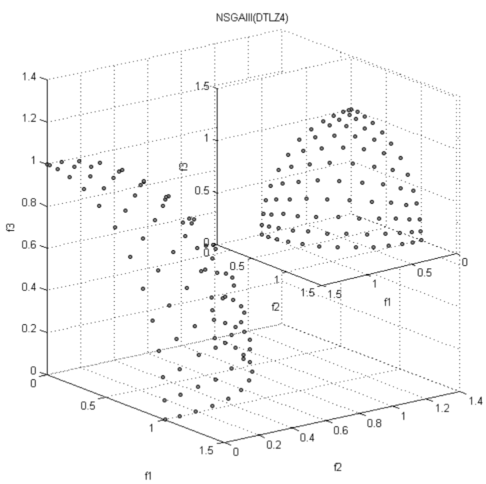
To evaluate the performance of the proposed algorithm, we have used the inverse generational distance (IGD) metric, which is capable of measuring the convergence and the diversity of the obtained Pareto-optimal solutions. The IGD measure has been predominantly used to evaluate the performance of evolutionary many-objective problems [30], [8], [31], [18], [32]. The IGD metric measures the distances between each solution composing the Pareto-optimal front and the obtained solution. The IGD metric is defined as follows:

$$IGD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \qquad (2)$$

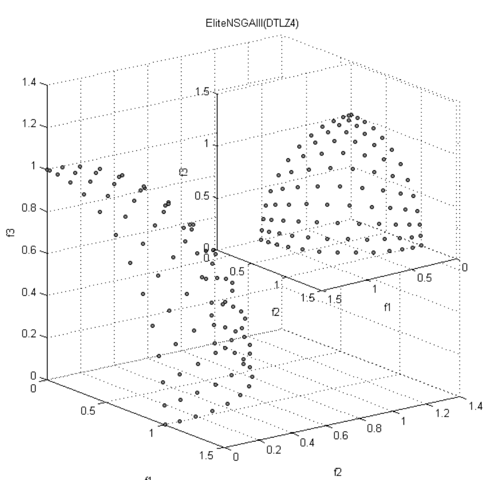
Where n refers to the number of solutions in the Pareto-optimal front, and d_i refers to the Euclidean distance (measured in objective space) between each point of the Pareto-optimal front (reference Pareto front) and the nearest member of obtained solution. In this study, the reference Pareto front is constructed by joining all results of all the executions and then selecting the non-dominated solutions (Note: the jMetal framework is used for all experimentation conducted in this study [33]). Furthermore, all algorithms were executed 20 times independently and the best, the worst, the median, and the average results of each algorithm are recorded. Additionally, the Wilcoxon's signed rank statistical test [34] is conducted at a 5% significance level in order to evaluate the statistical significance of the obtained results.

C. Experimental Results and Discussion

Two sets of experiments have been conducted to evaluate the performance of the proposed algorithm. The first set of experiments are designed to investigate the performance of the proposed algorithm for 3- to 15-objective DTLZ (DTLZ1, DTLZ2, DTLZ3, DTLZ4, and convex DTLZ2) and WFG (WFG1 to WFG7, except WFG3) test problems. The second set of experiments investigate the influence of elite archive for the proposed algorithm by examining the distribution of obtained solutions throughout the generation.



(a) NSGA-III



(b) EliteNSGA-III

Fig. 2. Obtained solutions by NSGA-III and EliteNSGA-III for DTLZ4 test problem.

1) Performance Based on IGD Metric: The first experiment investigates the performance of EliteNSGA-III on problems having linear or concave Pareto-optimal fronts for 3- to 15-objectives DTLZ (DTLZ1 to DTLZ4) problems. Table IV provides the best, median, worst, average IGD values of EliteNSGA-III and NSGA-III algorithms for DTLZ test problems. From this we can see that the performance of EliteNSGA-III is significantly better that NSGA-III for most of the test problems. Fig. 2 show the obtained Pareto fronts by NSGA-III and EliteNSGA-III algorithms for the three-objective DTLZ4 test problem. These diagrams are associated with the median value of IGD performance after 250 generations. It is clear that both algorithms are able to find uniformly distributed solutions for these test problems. However, as the dimension of the test problem increases, EliteNSGA-III is able to perform significantly better in terms of the IGD metric. From Fig. 3 we can clearly see that NSGA-III is not able to find uniformly distributed sets of solution for the 15-objective DTLZ1 problem.

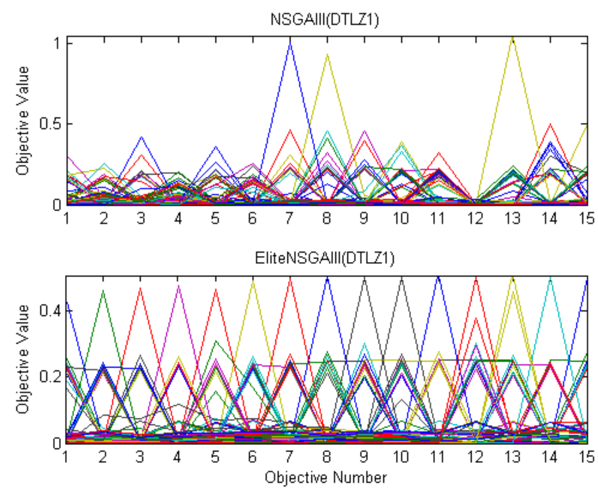
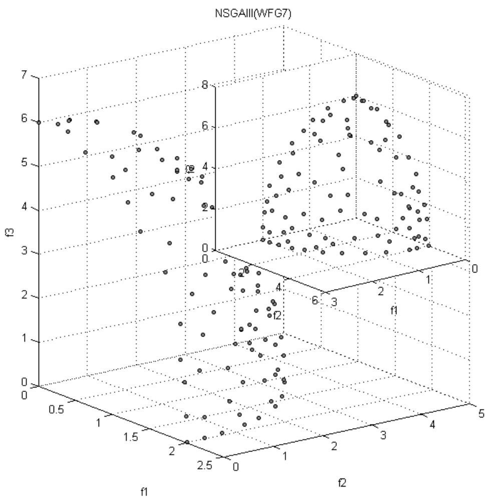


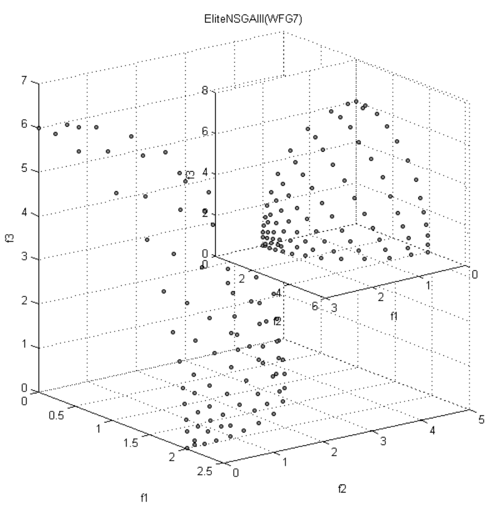
Fig. 3. Value path plot comparison of the obtained solutions by NSGA-III and EliteNSGA-III for 15-objective DTLZ1 test problem.

The second experiment investigates the performance of EliteNSGA-III on problems having linear or concave Pareto-optimal fronts for 3- to 15-objectives WFG (WFG4 to WFG7) problems. Table V shows the IGD metric values for EliteNSGA-III are significantly better than the NSGA-III algorithm, especially when the dimension of the test problems approaches 15 objectives. Fig. 4 shows the obtained Pareto fronts by NSGA-III and EliteNSGA-III algorithms for three-objective WFG7 test problem. These diagrams are associated with the median value of IGD performance after 400 generations. From this figures we can see that the EliteNSGA-III algorithm is able to find well-distributed solutions along the Pareto-optimal front.

The third experiment investigates the performance of the EliteNSGA-III algorithm on problems having convex or mixed Pareto-optimal fronts for 3- to 15-objectives DTLZ (convex DTLZ2) and WFG (WFG1 and WFG2) problems. Similar to the above results, EliteNSGA-III is able to significantly outperform NSGA-III in almost all instances except for the 3-objective WFG1 test problem. Table 3 provides IGD metric values of each compared algorithm for convex DTLZ1, WFG1 and WFG2 problems having 3 to 15 objectives. Fig. 5 illustrates the distribution of the obtained solution for NSGA-III and EliteNSGA-III algorithms for three-objective convex DTLZ2 test problem.



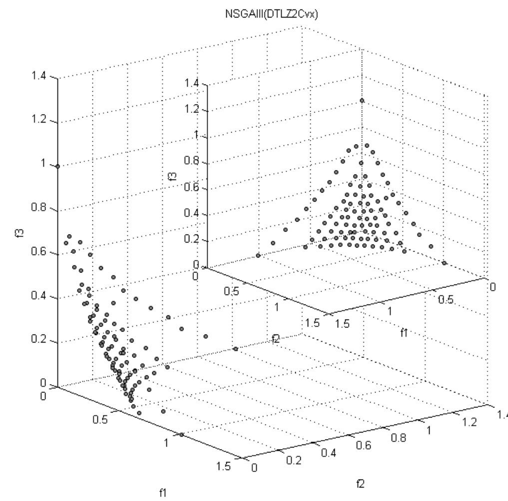
(a) NSGA-III



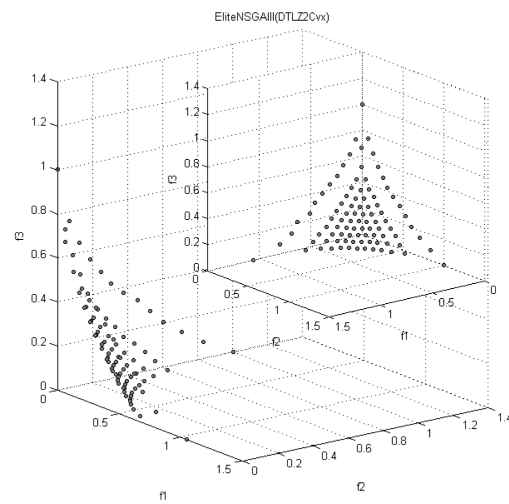
(b) EliteNSGA-III

Fig. 4. Obtained solutions by NSGA-III and EliteNSGA-III for WFG7 test problem.

2) *Influence of EliteNSGA-III Archive:* The main goal of the NSGA-III algorithm is to generate well-converged and well-distributed sets of solutions over multiple runs. However, as the number of objectives increase, NSGA-III loses its power to archive this goal. In this section we investigate the influence of an elite population archive on the distribution of obtained solutions throughout the generation. Since NSGA-III continuously try to find one population member corresponding to each supplied reference point close to the Pareto-optimal front, we investigate the number of reference points associated with at least one solution member in every generation. Since the number of reference points are equivalent to the population size, a well-distributed solution set should have one member associated with one reference point.



(a) NSGA-III



(b) EliteNSGA-III

Fig. 5. Obtained solutions by NSGA-III and EliteNSGA-III for convex DTLZ2 test problem.

Tables VII to IX present how many times (in terms of percentage) the target algorithm reached its goal for the 3- to 15-objective test problems listed in table I over 20 runs. The goal is to associate 90% (for 3- to 8-objective test problems) and 80% (for 10- and 15-objective test problems) of the supplied reference points with at least one population member before the maximum generation is reached. The overall results show that as the number of objectives increase, the NSGA-III algorithm has difficulty maintaining well-distributed solutions throughout the generations.

TABLE IV. BEST, MEDIAN, WORST, AND AVERAGE IGD VALUES FOR ELITENSGA-III AND NSGA-III ON M-OBJECTIVE DTLZ TEST PROBLEMS. BEST PERFORMING ALGORITHM IS SHOWN IN BOLD. GRAY SHADE INDICATES A SIGNIFICANCE LEVEL OF 0.05.

Problem	*M*	*Max Gen*	*EliteNSGA-III*	*NSGA-III*
DTLZ1	3	400	6.80E-04	**6.75E-04**
			7.26E-04	1.06E-03
			8.23E-04	5.24E-03
			7.31E-04	1.26E-03
	5	1000	7.57E-04	**7.50E-04**
			9.81E-04	1.20E-03
			3.92E-03	9.16E-03
			1.61E-03	2.51E-03
	8	1500	1.84E-03	**1.83E-03**
			3.36E-03	9.94E-03
			5.82E-03	1.11E-02
			3.14E-03	8.01E-03
	10	2000	1.55E-03	**1.55E-03**
			2.89E-03	6.73E-03
			4.21E-03	8.63E-03
			2.75E-03	5.20E-03
	15	2500	**2.13E-03**	2.72E-03
			3.50E-03	1.00E-02
			7.90E-03	1.14E-02
			3.58E-03	8.63E-03
DTLZ2	3	250	**3.05E-04**	3.07E-04
			3.09E-04	3.33E-04
			3.16E-04	1.50E-03
			3.10E-04	4.02E-04
	5	500	**2.72E-04**	2.78E-04
			2.76E-04	2.96E-04
			4.26E-03	**3.80E-04**
			4.76E-04	**3.01E-04**
	8	750	**2.32E-03**	2.36E-03
			2.33E-03	8.08E-03
			1.01E-02	1.06E-02
			4.25E-03	6.59E-03
	10	1000	1.70E-03	**1.70E-03**
			1.71E-03	6.64E-03
			1.01E-02	**8.67E-03**
			3.71E-03	5.42E-03
	15	1250	**3.58E-03**	8.04E-03
			9.98E-03	**8.13E-03**
			1.52E-02	**9.04E-03**
			9.26E-03	**8.44E-03**
DTLZ3	3	250	**1.04E-03**	1.05E-03
			1.05E-03	1.87E-03
			1.16E-03	7.81E-03
			1.06E-03	2.51E-03
	5	500	**9.59E-04**	9.84E-04
			9.71E-04	4.35E-03
			7.83E-03	8.57E-03
			1.31E-03	4.36E-03
	8	750	**1.32E-03**	2.42E-03
			2.22E-03	5.12E-03
			1.31E-02	**7.61E-03**
			3.43E-03	4.75E-03
	10	1000	**1.45E-03**	2.51E-03
			3.04E-03	5.39E-03
			6.21E-03	1.23E-02
			3.30E-03	5.36E-03
	15	1250	**2.57E-03**	2.94E-03
			3.99E-03	**3.64E-03**
			4.20E-03	1.08E-01
			3.66E-03	9.72E-03
DTLZ4	3	250	**6.88E-04**	6.93E-04
			1.13E-03	1.26E-03
			4.93E-03	1.75E-02
			2.13E-03	5.55E-03
	5	500	**8.97E-04**	9.07E-04
			1.01E-03	1.49E-03
			8.56E-03	**6.01E-03**
			2.00E-03	**1.68E-03**
	8	750	**1.41E-03**	1.53E-03
			1.42E-03	4.29E-03
			1.29E-02	**1.06E-02**
			2.73E-03	4.30E-03
	10	1000	**8.45E-04**	8.87E-04
			8.47E-04	1.28E-03
			1.01E-02	**1.01E-02**
			2.51E-03	**2.37E-03**
	15	1250	**2.25E-03**	3.50E-03
			9.85E-03	**6.09E-03**
			1.63E-02	**9.38E-03**
			9.30E-03	**6.21E-03**

TABLE V. BEST, MEDIAN, WORST, AND AVERAGE IGD VALUES FOR NSGA-III AND ELITENSGA-III ON M-OBJECTIVE WFG TEST PROBLEMS. BEST PERFORMING ALGORITHM IS SHOWN IN BOLD. GRAY SHADE INDICATES A SIGNIFICANCE LEVEL OF 0.05.

Problem	*M*	*Max Gen*	*EliteNSGA-III*	*NSGA-III*
WFG4	3	400	**1.16E-03**	1.25E-03
			1.21E-03	1.33E-03
			2.37E-03	**1.41E-03**
			1.29E-03	1.33E-03
	5	750	**1.67E-03**	1.68E-03
			1.70E-03	1.75E-03
			6.72E-03	**1.83E-03**
			2.24E-03	**1.75E-03**
	8	1500	**4.20E-03**	5.23E-03
			4.36E-03	7.57E-03
			8.75E-03	1.25E-02
			5.72E-03	7.71E-03
	10	2000	**3.34E-03**	4.19E-03
			3.56E-03	5.43E-03
			6.57E-03	7.67E-03
			4.27E-03	5.62E-03
	15	3000	**5.52E-03**	9.35E-03
			9.75E-03	1.26E-02
			1.18E-02	1.57E-02
			8.67E-03	1.26E-02
WFG5	3	400	**8.85E-04**	1.04E-03
			9.02E-04	1.09E-03
			1.02E-03	1.23E-03
			9.11E-04	1.10E-03
	5	750	**1.44E-03**	1.56E-03
			1.48E-03	1.60E-03
			2.08E-03	5.37E-03
			1.59E-03	1.79E-03
	8	1500	**3.18E-03**	3.63E-03
			3.28E-03	6.04E-03
			9.50E-03	1.30E-02
			3.84E-03	6.84E-03
	10	2000	**2.79E-03**	2.84E-03
			3.07E-03	4.34E-03
			6.73E-03	7.03E-03
			3.90E-03	4.52E-03
	15	3000	**3.84E-03**	8.28E-03
			7.85E-03	1.16E-02
			1.08E-02	1.32E-02
			7.14E-03	1.15E-02
WFG6	3	400	**1.65E-03**	1.73E-03
			1.69E-03	1.82E-03
			2.43E-03	**2.07E-03**
			1.75E-03	1.85E-03
	5	750	1.97E-03	**1.96E-03**
			2.04E-03	**2.03E-03**
			4.18E-03	**2.12E-03**
			2.44E-03	**2.03E-03**
	8	1500	**4.56E-03**	6.12E-03
			4.74E-03	9.75E-03
			1.20E-02	1.25E-02
			6.70E-03	9.53E-03
	10	2000	**3.92E-03**	4.85E-03
			4.11E-03	7.57E-03
			1.14E-02	**1.01E-02**
			5.44E-03	7.36E-03
	15	3000	**8.47E-03**	1.09E-02
			1.49E-02	1.21E-02
			1.64E-02	2.02E-02
			1.38E-02	1.28E-02
WFG7	3	400	**1.23E-03**	1.36E-03
			1.28E-03	1.44E-03
			1.31E-03	1.52E-03
			1.27E-03	1.43E-03
	5	750	**1.72E-03**	1.73E-03
			1.77E-03	1.78E-03
			6.02E-03	**1.87E-03**
			2.61E-03	**1.79E-03**
	8	1500	**3.73E-03**	5.73E-03
			4.38E-03	7.95E-03
			7.57E-03	1.64E-02
			5.04E-03	8.45E-03
	10	2000	**3.23E-03**	3.76E-03
			4.71E-03	5.97E-03
			6.00E-03	7.51E-03
			4.69E-03	5.79E-03
	15	3000	**4.48E-03**	6.27E-03
			7.62E-03	1.07E-02
			9.07E-03	1.50E-02
			7.42E-03	1.08E-02

TABLE VI. BEST, MEDIAN, WORST, AND AVERAGE IGD VALUES FOR NSGA-III AND ELITENSGA-III ON M-OBJECTIVE CONVEX DTLZ2, WFG1, AND WFG2 TEST PROBLEMS. BEST PERFORMING ALGORITHM IS SHOWN IN BOLD. GRAY SHADE INDICATES A SIGNIFICANCE LEVEL OF 0.05.

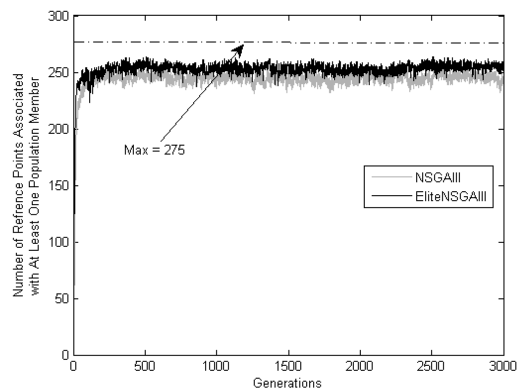
Problem	*M*	*Max Gen*	*EliteNSGA-III*	*NSGA-III*
DTLZ2Cvx	3	250	**5.48E-04**	**5.72E-04**
			5.62E-04	**7.83E-04**
			5.96E-04	**2.04E-03**
			5.64E-04	**9.22E-04**
	5	750	**4.91E-04**	6.68E-04
			5.03E-04	7.66E-04
			1.33E-03	1.38E-03
			5.44E-04	8.44E-04
	8	2000	**1.42E-03**	2.08E-03
			1.69E-03	3.39E-03
			3.48E-03	5.39E-03
			2.06E-03	3.19E-03
	10	4000	**1.24E-03**	2.12E-03
			2.14E-03	2.38E-03
			2.78E-03	3.90E-03
			2.11E-03	2.69E-03
	15	4500	1.76E-03	**1.75E-03**
			2.54E-03	**2.24E-03**
			3.84E-03	**3.73E-03**
			2.56E-03	**2.33E-03**
WFG1	3	250	4.32E-03	**1.56E-03**
			9.03E-03	**3.85E-03**
			1.21E-02	**6.99E-03**
			9.09E-03	**4.02E-03**
	5	500	1.93E-03	**1.88E-03**
			3.77E-03	**3.44E-03**
			6.77E-03	7.15E-03
			3.61E-03	4.00E-03
	8	750	**2.04E-03**	2.53E-03
			2.28E-03	3.15E-03
			1.51E-02	**7.29E-03**
			2.92E-03	3.27E-03
	10	1000	**1.22E-03**	1.65E-03
			1.38E-03	1.90E-03
			1.56E-03	2.34E-03
			1.38E-03	1.93E-03
	15	1250	**3.05E-03**	3.33E-03
			3.21E-03	3.55E-03
			3.88E-03	3.92E-03
			3.26E-03	3.55E-03
WFG2	3	250	**9.77E-04**	1.02E-03
			1.06E-03	1.13E-03
			1.25E-03	1.35E-03
			1.06E-03	1.15E-03
	5	500	**9.17E-04**	1.12E-03
			9.67E-04	1.81E-03
			2.29E-03	2.56E-03
			1.03E-03	1.86E-03
	8	750	**1.67E-03**	1.92E-03
			1.81E-03	2.47E-03
			3.20E-03	4.21E-03
			1.97E-03	2.68E-03
	10	1000	**1.08E-03**	1.35E-03
			1.25E-03	1.61E-03
			1.86E-03	2.32E-03
			1.34E-03	1.69E-03
	15	1250	**1.93E-03**	2.44E-03
			2.60E-03	2.86E-03
			3.53E-03	4.84E-03
			2.55E-03	3.07E-03

However, from table VII we see that EliteNSGA-III is able to reach its goal 95% of the time as the dimension of DTLZ4 test problem reached 15 objectives. Moreover, for the WFG, with concave Pareto-optimal from test problems (WFG4 to WFG7), EliteNSGA-III is able to reach its goal 100% of the time as the WFG6 test problem dimension reached 15 objectives.

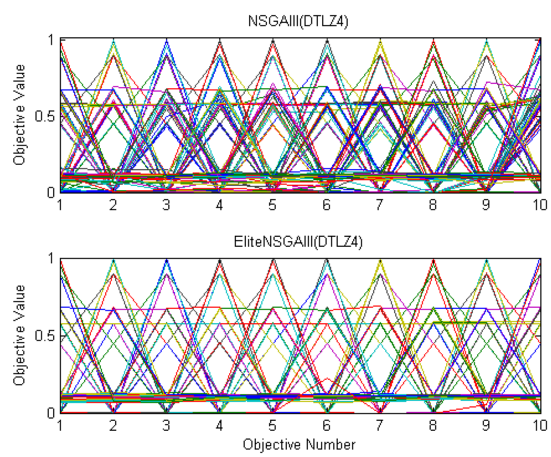
Figs. 6 and 7 show the correlation between the number of reference points associated with at least one population member at each generation and the quality of the obtained Pareto-optimal solution set by the target algorithm.

TABLE VII. THE PERCENTAGE OF RUNS FOR WHICH THE TARGET ALGORITHM IS ABLE TO ASSOCIATE 90% (FOR 3- TO 8-OBJECTIVE) AND 80% (FOR 10- AND 15-OBJECTIVE DTLZ1 TO DTLZ4 TEST PROBLEMS) OF THE SUPPLIED REFERENCE POINTS WITH AT LEAST ONE POPULATION MEMBER BEFORE THE MAXIMUM GENERATION IS REACHED.

Problem	*M*	*Max Gen*	*EliteNSGA-III*	*NSGA-III*
DTLZ1	3	1000	100%	55%
	5	1500	90%	30%
	8	2000	40%	20%
	10	3000	30%	35%
	15	5000	15%	0%
DTLZ2	3	1000	100%	100%
	5	1500	90%	100%
	8	2000	45%	60%
	10	3000	70%	15%
	15	5000	80%	0%
DTLZ3	3	1000	100%	65%
	5	1500	100%	40%
	8	2000	45%	0%
	10	3000	30%	5%
	15	5000	40%	0%
DTLZ4	3	1000	95%	50%
	5	1500	100%	50%
	8	2000	80%	45%
	10	3000	95%	85%
	15	5000	95%	45%



(a) DTLZ4 (10 Objectives)



(b) DTLZ4 (10 Objectives)

Fig. 6. Comparison of the number of reference points associated with at least one population member after each generation and value path plot for the obtained solutions by NSGA-III and EliteNSGA-III for DTLZ4 test problems with 10 objectives. Figure (a) show the average value of 20 independent runs.

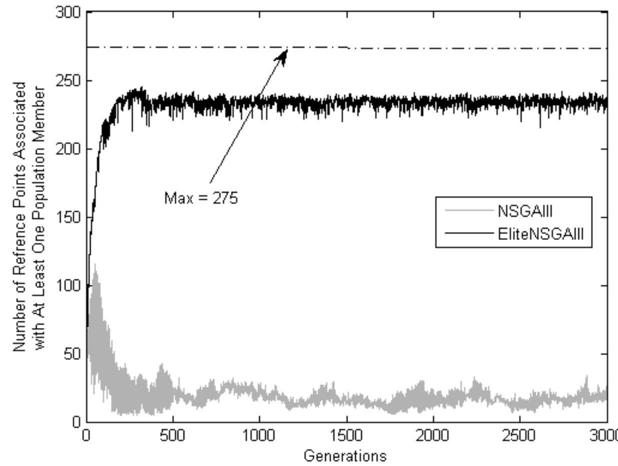
We can see that when the number of reference points associated to at least one population member is high, this leads to achieving well-distributed Pareto-optimal front. From Fig. 6 we see that both algorithms were able to diversify the population members and hence they are able to find well-distributed solutions to the 10-objective DTLZ4 problem. From Fig. 7 we see that NSGA-III is not able to generate well-distributed solutions throughout the generations and resulting poorly-distributed sets of solutions for the 10-objective WFG6 problem. However, EliteNSGA-III is able to find well-distributed solutions over $f_i \in [0, 2i]$ for all ten objectives and a trade-off among them can be seen from the value path plots in Fig 7.

TABLE VIII. THE PERCENTAGE OF RUNS FOR WHICH THE TARGET ALGORITHM IS ABLE TO ASSOCIATE 90% (FOR 3 TO 8 OBJECTIVES) AND 80% (FOR 10- AND 15-OBJECTIVE CONVEX DTLZ2, WFG1, AND WFG2 TEST PROBLEMS) OF THE SUPPLIED REFERENCE POINTS WITH AT LEAST ONE POPULATION MEMBER BEFORE THE MAXIMUM GENERATION IS REACHED.

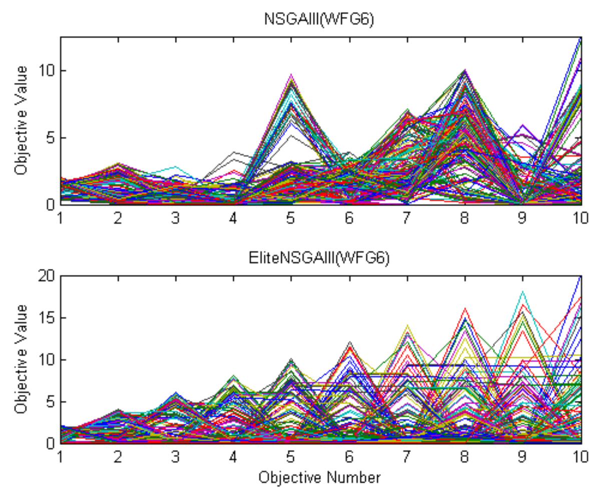
Problem	***M***	***Max Gen***	***EliteNSGA-III***	***NSGA-III***
	3	1000	100%	100%
	5	1500	100%	40%
DTLZ2Cvx	8	2000	65%	40%
	10	3000	40%	5%
	15	5000	40%	0%
	3	1000	10%	30%
	5	1500	0%	0%
WFG1	8	2000	0%	0%
	10	3000	0%	0%
	15	5000	0%	0%
	3	1000	100%	90%
	5	1500	100%	5%
WFG2	8	2000	90%	0%
	10	3000	90%	0%
	15	5000	70%	0%

TABLE IX. THE PERCENTAGE OF RUNS FOR WHICH THE TARGET ALGORITHM IS ABLE TO ASSOCIATE 90% (FOR 3 TO 8 OBJECTIVES) AND 80% (FOR 10- AND 15-OBJECTIVE WFG4 TO WFG7 TEST PROBLEMS) OF THE SUPPLIED REFERENCE POINTS WITH AT LEAST ONE POPULATION MEMBER BEFORE THE MAXIMUM GENERATION IS REACHED.

Problem	***M***	***Max Gen***	***EliteNSGA-III***	***NSGA-III***
	3	1000	100%	100%
	5	1500	85%	40%
WFG4	8	2000	70%	0%
	10	3000	90%	0%
	15	5000	95%	0%
	3	1000	100%	100%
	5	1500	100%	25%
WFG5	8	2000	95%	0%
	10	3000	100%	25%
	15	5000	95%	10%
	3	1000	100%	100%
	5	1500	95%	0%
WFG6	8	2000	75%	0%
	10	3000	85%	5%
	15	5000	100%	0%
	3	1000	100%	100%
	5	1500	90%	0%
WFG7	8	2000	50%	0%
	10	3000	45%	0%
	15	5000	75%	0%



(a) WFG6 (10 Objectives)



(b) WFG6 (10 Objectives)

Fig. 7. Comparison of the number of reference points associated with at least one population member after each generation and value path plot for the obtained solutions by NSGA-III and EliteNSGA-III for WFG6 test problems with 10 objectives. Figure (a) show the average value of 20 independent runs.

V. CONCLUSION

In this paper, we proposed a novel elite archive-based NSGA-III, called EliteNSGA-III. EliteNSGA-III uses an elite population to store previously generated individuals that can probably be eliminated by NSGA-III's pruning mechanism. EliteNSGA-III also modifies the parent selection mechanism employed by NSGA-III to diversify the parent selection pool by giving equal chance to the elite population archive and the current population. The performance of EliteNSGA-III was compared with NSGA-III on 11 widely used many-objective test problems with dimension ranging from 3 to 15 objectives. Experimental results on these test problems showed that the performance of the traditional NSGA-III algorithm can significantly be improved through the introduction of an elite population archive.

Almost in all test problems, the proposed algorithm outperformed the parent algorithm in terms of convergence and accuracy of the obtained solutions. As the number of objectives increased, the performance of the corresponding EliteNSGA-III algorithm significantly outperformed NSGA-III in almost all instances. Moreover, our study on the influence of elite population archive proved the capability of the proposed algorithm in preserving diverse sets of elite population and hence improving the diversity of an offspring population.

In the future, first, we would like to extend this study to investigate the impact of an elite population archive when alternate recombination operators are used (e.g. differential evolution (DE) instead of SBX and polynomial mutation operators). Second, we would like to investigate the impact of an elite population archive when only neighbouring elite solutions are used to create offspring population. And last, we would like a penalty mechanism to avoid the generated solutions being attracted to few reference points.

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