Transportation Policy Formulation as a Multi-objective Bilevel Optimization Problem

Ankur Sinha[1], Pekka Malo[2], and Kalyanmoy Deb[3]

[1]Productivity and Quantitative Methods
Indian Institute of Management Ahmedabad, Ahmedabad 380015, India
{asinha@iimahd.ernet.in}

[2]Department of Information and Service Economy
Aalto University School of Business, PO Box 21220, 00076 Aalto, Finland
{pekka.malo@aalto.fi}

[3]Department of Electrical and Computer Engineering
Michigan State University, East Lansing, MI, USA
{kdeb@egr.msu.edu}

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***Abstract*—In this paper, we consider multi-objective bilevel optimization problems in the context of transportation policy formulation. In such problems, an authority managing a network of roads is the leader that tries to solve the problem by taking into account the possible actions of the network users who are considered as the followers. In the presence of multiple objectives, the resulting solution set is a Pareto-optimal frontier that consists of optimal decisions of the leader and corresponding optimal responses from the follower. The authority's objectives are to maximize its revenues through tolls and minimize the pollution levels. The network users' objectives are to minimize travel cost and travel time. In addition to accommodating multiple objectives at both levels, the benefits of the proposed formulation is that it allows incorporating various real-world complexities, like admitting complex road network topologies and allowing the modelling of several road user segments with different preferences. A recently proposed algorithm for multi-objective bilevel optimization is used to solve the problem.**

***Index Terms*—Stackelberg game, Bilevel optimization, Multi-objective optimization, Evolutionary algorithms.**

I. Introduction

Bilevel optimization problems are commonly found in practice; for instance in transportation (network design, optimal pricing) [1], [2], economics (Stackelberg games, principal-agent problem, policy decisions) [3], [4], [5], [6], management (network facility location, coordination of multi-divisional firms) [7], [8], engineering (optimal design, optimal chemical equilibria) [9], [10]. Most bilevel problems can be characterized through a leader-follower framework, where the leader tries to optimize her decisions subject to the responses of the follower. The leader is aware that the follower will react optimally to any decision taken by the leader. This induces a two-level hierarchy in the optimization problem. The upper level represents the leader's objectives and constraints, while the (nested) lower level represents the follower's objectives and constraints. The lower level problem is a parameterized optimization task that acts as a constraint to the upper level problem. Only the optimal solutions to the lower level problem are accepted as possible feasible candidates for the upper level problem; thereby making the entire optimization problem very challenging. In this paper, we analyze a multi-objective bilevel optimization problem, where both leader and follower are facing multiple objectives.

The problem considered in this paper belongs to the domain of transportation science, where the upper level decision maker is a government managing a network of highways and the lower level decision makers are the network users. At both levels, the decision makers are faced with multiple objectives. Government wants to maximize its revenues from toll and minimize pollution at the same time, while the highway users want to minimize their travel time and cost. The decision variables at the upper level are different tolls for different highways. For any given set of tolls the highway users can choose different routes to travel from location A to location B. Therefore, the decision variables at the lower level are different choices of the highways. The problem is optimized from the perspective of the government that wants an optimal frontier of different toll settings. The paper contributes to the literature on transportation policy formulation by suggesting different versions of the transportation model that accounts for various real-world complexities, like handling multiple objectives, admitting complex road network topologies and allowing the modelling of several road user segments with different preferences.

To begin with, we provide a survey on multi-objective bilevel optimization. This is followed by a description of multi-objective bilevel optimization where the preferences of the follower are modeled using a value function. We discuss the description in the context of a simple analytical example involving the government and the highway users. This is followed by a brief introduction to the algorithm that we use to solve the problem. Thereafter, we solve different versions of the government-highway users problem and provide the results. Finally, we provide the conclusions and possible extensions to this study.

II. A Survey on Multi-objective Bilevel Optimization

In this section, we highlight the few studies that are available on multi-objective bilevel optimization. One of the first studies in the area of classical optimization that considered multiple objectives at both levels was by [11]. In this study, the author handled the lower level problem using a numerical optimization technique, and the upper level problem using an adaptive exhaustive search method. An exhaustive search method at the upper level makes the solution procedure computationally demanding and non-scalable to large-scale problems. In another study, the ϵ-constraint method was used at both levels by [12]. The ϵ-parameter was elicited from the decision maker, and the problem was solved by replacing the lower level constrained optimization problem with its KKT conditions for different ϵ-parameters, until a satisfactory solution was found.

Evolutionary algorithms have also been used to solve multi-objective bilevel problems. One of the first studies using evolutionary algorithms was by [13]. In this study, the authors solved a transportation planning and management problem with multiple objectives at the upper lever, and a single objective at the lower level using a nested genetic algorithm. Later [14] applied a particle swarm optimization based nested strategy to solve a multi-component chemical system. The lower level problem in their application was linear for which they utilized a specialized linear multi-objective PSO approach. Recently, a hybrid evolutionary algorithm coupled with local search was proposed in [15] [1] to solve bilevel optimization problems with multiple objectives at both levels. In this paper, the authors handled complex bilevel problems with relatively larger number of variables. Other recent work on bilevel multi-objective optimization can be found in [20], [21], [22], [23], [24], [25].

Many of the studies handling multiple objectives at both levels have not considered lower level decision making aspects in the problem. Studies that assume the entire lower level frontier to be the possible feasible set at the upper level end up with an overly optimistic estimate of the upper level frontier. To have a realistic estimate the lower level decisions cannot be ignored. To incorporate lower level decisions, the follower's preferences may be modeled using a value function; thereby converting the lower level optimization problem into a single-objective optimization problem. Such an approach has been used in recent studies by [26], [27].

[1]For earlier versions, refer [16], [17], [18], [19]

III. Multi-objective Bilevel Optimization with Lower Level Decision Making

In this section, we provide a formulation for a bilevel multi-objective optimization problem where both upper and lower levels are faced with multiple objectives. The lower level decision maker has sufficient power to choose the most preferred solution from her own frontier. Therefore, lower level preferences have been modeled using a value function, such that any lower level optimization returns a singleton. This effectively reduces lower level optimization to a single objective problem. If the value function of the lower level decision maker is known, then such an optimization problem can be formulated as follows:

Definition 1: For the upper-level objective function $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ and lower-level objective function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^q$

$$\begin{aligned}
&\underset{x_u \in X_U, x_l \in X_L}{\text{minimize}} \quad F(x_u, x_l) = (F_1(x_u, x_l), \ldots, F_p(x_u, x_l)) \\
&\text{subject to} \\
&\qquad x_l = \underset{x_l}{\text{argmin}} \{ V(f_1(x_u, x_l), \ldots, f_q(x_u, x_l); \omega) : \\
&\qquad\qquad g_j(x_u, x_l) \le 0, j = 1, \ldots, J \} \\
&\qquad G_k(x_u, x_l) \le 0, k = 1, \ldots, K,
\end{aligned}$$

where $x_u \in X_U \subset \mathbb{R}^n$ is the upper level vector, $x_l \in X_L \subset \mathbb{R}^m$ is the lower level vector, V denotes the follower's value function, and ω is the parameter vector of the assumed value function form. For instance, if V is linear, such that $V(f_1(x_u, x_l), \ldots, f_q(x_u, x_l); \omega) = \sum_{i=1}^{q} \omega_i f_i(x_u, x_l)$, then $\omega_i \ \forall \ i \in \{1, \ldots, q\}$ represent the value function parameters.

In the above formulation the follower always returns a singleton for any given upper level vector. We assume that the leader has a complete knowledge of the follower's value function that may be identified through studies or surveys conducted by the leader if the preferences are not readily available. The final solution to the above problem is the upper level Pareto-optimal frontier containing optimal trade-off solutions for the leader. Thereafter, it becomes a multi-criteria decision making problem for the leader who needs to implement the most preferred solution from her frontier.

Figure 1 shows the scenario, where the shaded region ($\Psi(x_u)$) represents the follower's Pareto-optimal solution for any given leader's decision (x_u). These are the possible rational decisions, which the follower may take for a given leader's action. If the leader is aware of the follower's objectives, she will be able to identify the shaded region completely by solving the multi-objective optimization problem for the follower for all x_u. However, the leader can make an appropriate decision only when the follower's preferences on the lower level Pareto-optimal solutions is available. If the preferences of the follower are perfectly known, then the lower level decision for any x_u is given by $\sigma(x_u)$, shown in the figure. Therefore, for a given upper level decision $x_u^{(0)}$, $\Psi(x_u^{(0)})$ is the corresponding lower level Pareto-optimal set and $\sigma(x_u^{(0)})$ is the lower level decision.

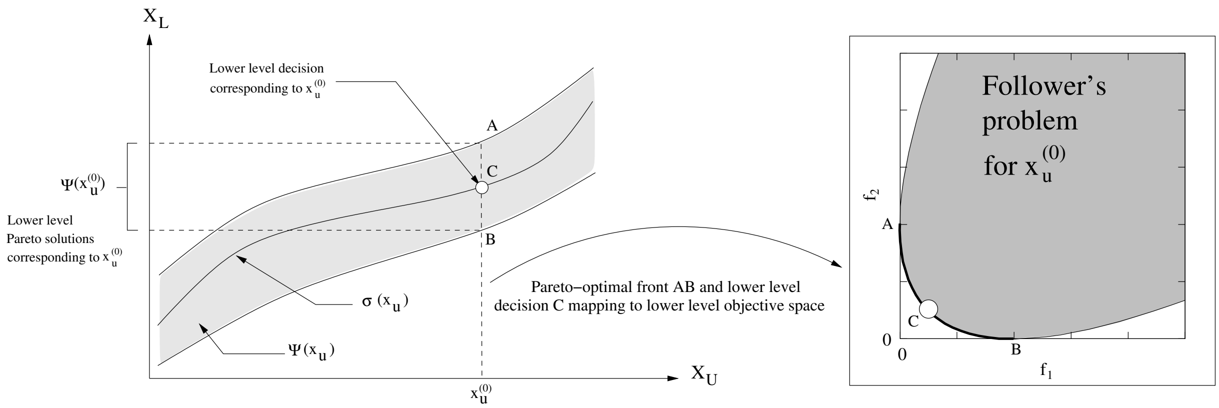

Fig. 1. Lower level Pareto-optimal solutions ($\psi(x_u)$) and corresponding decisions $\sigma(x_u)$.

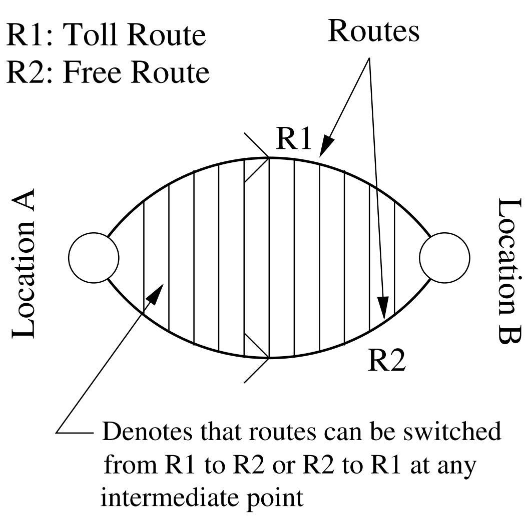

Fig. 2. Roads connecting location A and location B.

IV. Transportation Policy Formulation Problem

In this section, we describe a simple version of the transportation problem involving the government and the highway users. Consider that there are two roads, R1 and R2 shown in Figure 2 that can be chosen to travel from location A to location B. Road R1 is a highway on which the government charges a toll for every unit distance traveled, while there is no toll on road R2. One can switch from R1 to R2 or R2 to R1 at any point one feels like. Road R1 is faster than road R2, therefore the users have to make a trade-off between travel time and travel cost. We consider an average user in our study for whom we are aware of the preferences through a value function. The upper level variable is toll τ charged on road R1 for every unit distance traveled, and lower level variables are y_1 and y_2 representing the distance traveled on roads R1 and R2 respectively. Road R2 being a slow road leads to higher pollution. The multi-objective bilevel optimization problem to determine the optimal tolls has been formulated in Table I where all objectives are supposed to be minimized. The government's objectives are to minimize the negative of revenues ($F_1(\tau, y_1)$)) and minimize pollution ($F_2(y_1, y_2)$). The user's objectives are to minimize travel cost ($f_1(\tau, y_1, y_2)$) and minimize travel time ($f_2(y_1, y_2)$). The lower level equality constraint requires that the weighted distances traveled on roads R1 and R2 should sum to 1 in order for the user to travel from location A to B.

TABLE I
Multi-objective bilevel transportation policy problem with known lower level preferences

Example	Level	Formulation
Variables	Upper level	$x_u = (\tau)$
	Lower level	$x_l = (y_1, y_2)$
Objectives	Upper level	$F_1(\tau, y_1) = -\tau y_1$ $F_2(y_1, y_2) = a_1 y_1 + a_2 y_2$
	Lower level	$f_1(\tau, y_1, y_2) = (b_1 + \tau) y_1 + b_2 y_2$ $f_2(y_1, y_2) = c_1 y_1 + c_2 y_2$
Constraints	Upper level	$\tau' \leq \tau$
	Lower level	$d_1 y_1 + d_2 y_2 = 1$ $0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1$
Preference Structure	Lower level	$V(f_1, f_2) = (f_1 - e_1)^2 + (f_2 - e_2)^2$
Constants	Upper level	$a_1 = 1, a_2 = 2, \tau' = 0.5$
	Lower level	$b_1 = 0.5, b_2 = 1, c_1 = 1, c_2 = 2,$ $d_1 = 1, d_2 = 1, e_1 = 1, e_2 = 1$

It is possible to solve this problem analytically. We skip the analytical steps for the problem and directly provide the results through Figures 3 and 4. Figure 3 shows the lower level Pareto-optimal frontiers for different values of τ, where x-axis represents travel cost and the y-axis represents travel time. The lower level decision maker decides to operate at a particular point on the lower level frontier for any given value of τ. These operating points can be identified using the lower level value function and are referred as lower

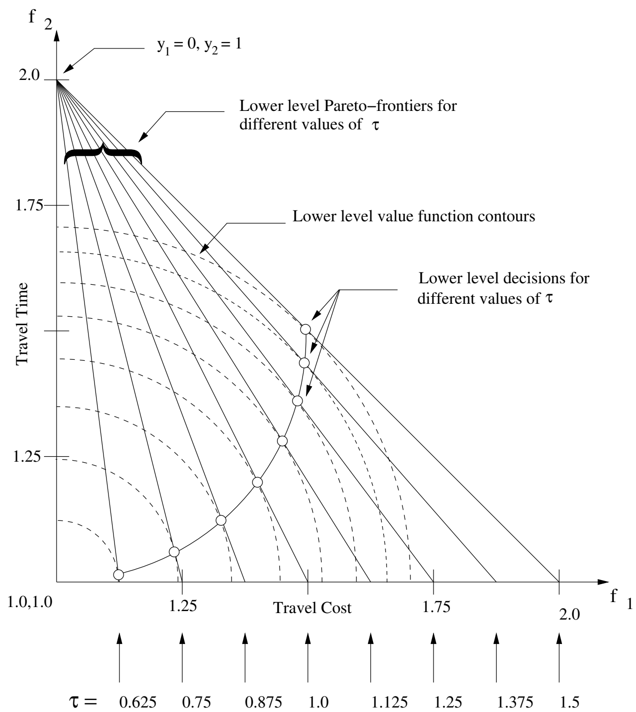

Fig. 3. Lower level Pareto-optimal frontiers for different tolls and corresponding lower level decisions.

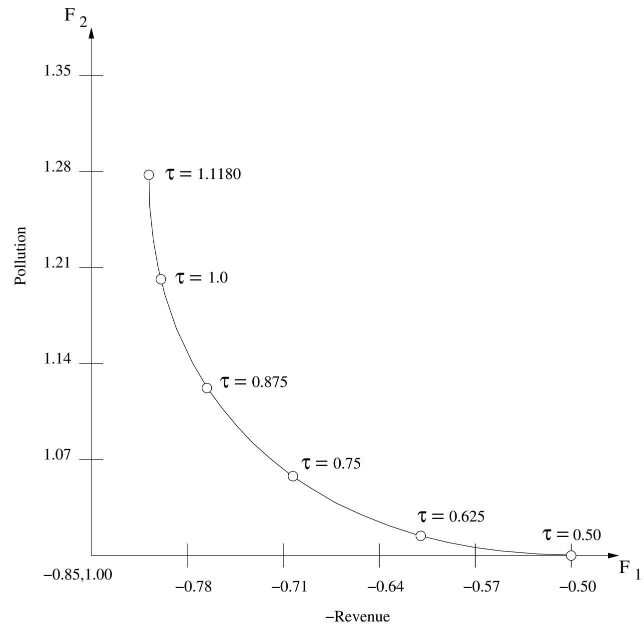

Fig. 4. Upper level Pareto-optimal frontier.

level decisions. They have been marked with small circles on each frontier. Figure 4 shows the upper level Pareto-optimal frontier. There is a clear trade-off between revenues generated (x-axis) and extent of pollution (y-axis). Different tolls τ have been marked on the frontier that shows that $\tau \in [0.5, \frac{\sqrt{5}}{2}]$. Some interesting conclusions that can be drawn from this analysis are as follows:

1) Having a τ larger than $\frac{\sqrt{5}}{2}$ does not correspond to the Pareto-optimal frontier. Therefore, a very high toll leads to lower revenues as well as higher pollution.
2) Trade-off between the objectives at upper level is not very obvious. If the lower level optimization problem is ignored, one might not observe any trade-off between the two upper level objectives as a higher use of highway would lead to a higher revenue as well as lower pollution. However, when the behavior of the lower level users is taken into account one obtains a set of trade-off solutions.

The problem has only a few variables that allows us to arrive at an analytical solution. However, there are at least two other complexities that need to be taken into account in realistic cases. First, the network usually contains a number of highways and alternative routes which makes the lower level problem much more complex. Second, the above analysis has been done for a single user representing a large number of people that is far from realism. If one accounts for these complexities, it is not possible to handle such problems analytically. Therefore, in the next section we describe an algorithm that can handle more complex versions of the transportation problem. Thereafter, we handle two versions of the transportation problem that contain these difficulties.

V. Algorithm Description

In this section, we provide a brief description to an evolutionary algorithm for solving bilevel problems with multiple upper level objectives and known preference structure at the lower level. The approach has been proposed in [26], [27] and is referred as multi-objective bilevel evolutionary algorithm based on quadratic approximations (m-BLEAQ). It is an extension of an earlier approach [28], [29] that was proposed for solving single objective bilevel optimization problems. The algorithm is based on estimation of unknown lower level decisions using quadratic approximations, when lower level decisions corresponding to a few upper level vectors are known. A number of such approximations are performed during the course of the algorithm that help in reducing the number of lower level optimization calls. The working of the algorithm has been shown through a flowchart in Figure 5. For the problems solved in this paper, lower level optimization task can be handled using a sequential quadratic programming approach (SQP). Therefore, in m-BLEAQ we do not invoke genetic algorithm at the lower level, but use SQP whenever lower level optimization is to be performed.

VI. Transportation Policy Formulation Problem with Additional Complexities

We describe two transportation models in this section. The first model contains multiple roads connecting location A

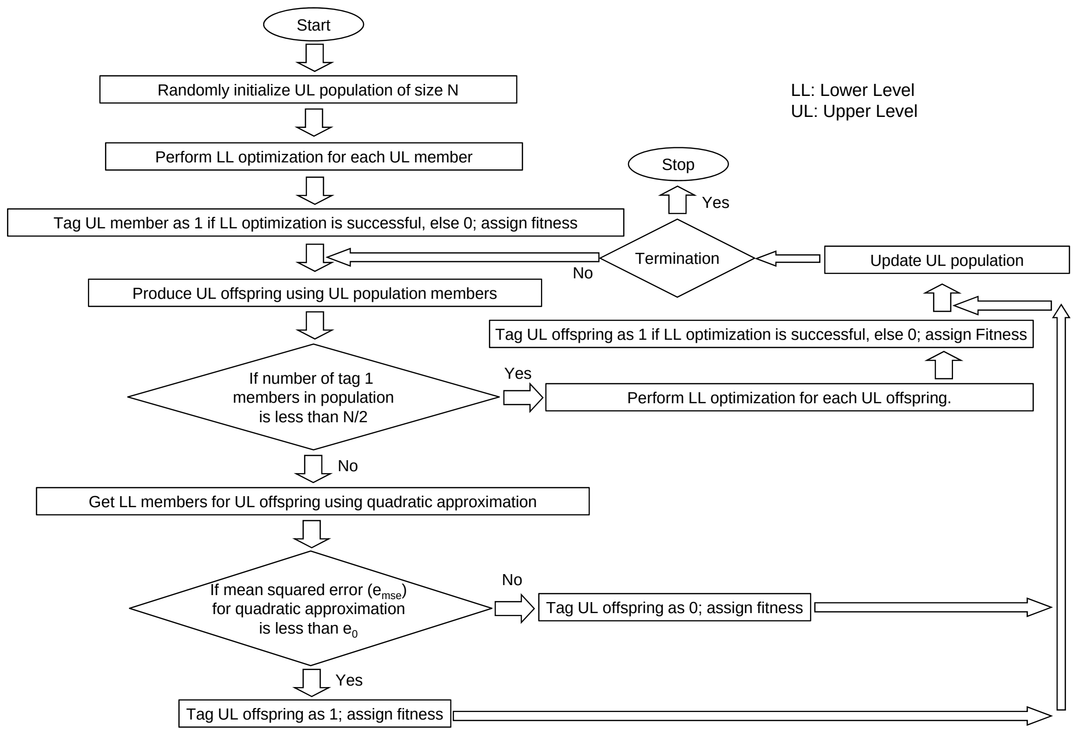

Fig. 5. Flowchart for m-BLEAQ.

to location B, while the second model has an additional complexity of users with different preference structures to be accounted simultaneously. For both models we consider the same network of roads that is shown in Figure 6.

Model 1

The first model is shown in Table II that contains many possible routes from location A to location B. We solve the problem using m-BLEAQ with a network of 9 roads, where the government charges tolls on 5 roads (R1-R5) while 4 roads (R6-R9) are free. Therefore, the number of upper level variables in this model is 5, with each variable representing toll charged on each road for every unit distance traveled. The number of lower level variables in the model is 9 that corresponds to the distance traveled on each of the roads R1 to R9.

Figure 7 provides lower level Pareto-optimal frontiers for a few toll vectors. The corresponding decisions of the highway users are marked on each frontier. Whenever a lower level call is made by the m-BLEAQ algorithm, the lower level decision corresponding to the given toll vector is returned after solving the lower level optimization problem. Lower level decisions that correspond to the non-dominated set at the upper level are preferred by the algorithm causing the population members to converge towards the upper level Pareto-optimal set. The optimal trade-off set produced by the m-BLEAQ algorithm is shown in Figure 8. The upper and lower level function evaluations required by m-BLEAQ to converge towards the upper level frontier are given in Table V. In order to verify the results of m-BLEAQ algorithm we used a nested procedure with NSGA-II [30] algorithm at the upper level and SQP at the lower level. The nested and the m-BLEAQ procedure were executed for a fixed number of upper level evaluations and the lower level function evaluations were recorded. While there was negligible difference in hypervolumes of the frontier obtained by the two approaches, the quadratic approximation scheme in m-BLEAQ was helpful in saving a large number of lower level evaluations.

Model 2

The second model is given in Table III and corresponds to the same network of roads as in model 1. However, in this model we consider that the network users belong to 4 different categories. Their objective functions are given as (f_{1j}, f_{2j}), where $j \in \{1, \ldots, 4\}$ denotes each type of user. Their value functions are given by $V_j(f_{1j}, f_{2j})$. The proportion of users belonging to each category is given by p_j. Note that the objectives at the upper level are represented as a weighted sum, where the weights correspond to the proportion of each user type.

In this model, whenever the lower level optimization is called, we have to solve the optimization problem for each

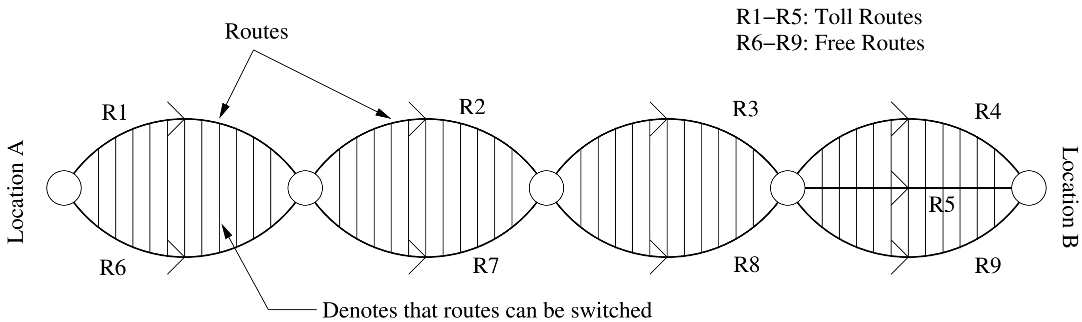

Fig. 6. Network of roads connecting location A and location B.

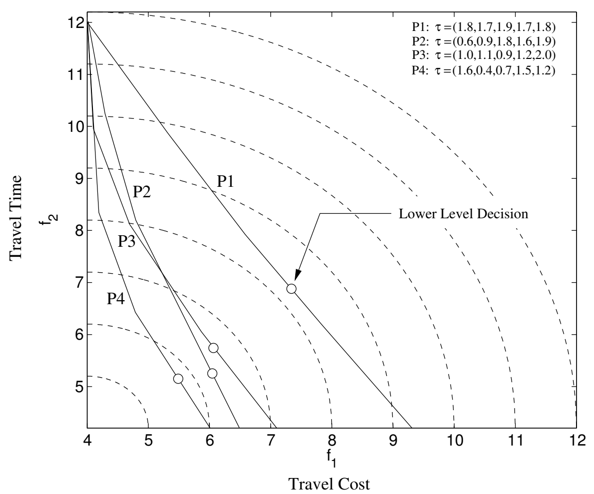

Fig. 7. Model 1: Lower level Pareto-optimal frontiers for different tolls and corresponding lower level decisions.

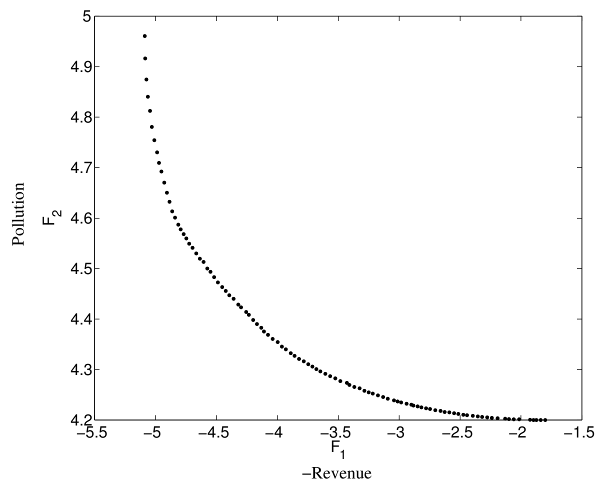

Fig. 8. Model 1: Upper level Pareto-optimal frontier obtained using m-BLEAQ algorithm.

type of user to determine how they use the network of roads to travel from A to B. Therefore, the model contains 4 independent optimization problems at the lower level. This makes the number of variables at the lower level as 36 with 9 variables corresponding to each user type. The number of upper level variables is 5 that represents the toll for roads R1 to R5.

It is interesting to note that this problem can also be handled by solving a lower level multi-objective optimization problem with 9 variables, and then choosing the decisions corresponding to each user type from the frontier. However, we have not used this approach as it would have involved modifications in the m-BLEAQ procedure. Figure 9 provides the lower level decisions corresponding to each user type for a few toll vectors. The upper level Pareto-optimal frontier obtained using m-BLEAQ is shown in Figure 10. Function evaluations required by m-BLEAQ to converge towards the upper level frontier are given in Table V. Results of the comparative study against the nested procedure are also given in the table. It is quite clear from the results that the m-BLEAQ procedure requires fewer lower level function evaluations to solve the problem as compared to the nested approach.

VII. Conclusions

We have discussed a real-world multi-objective bilevel problem from the area of transportation science. In the considered problem, both upper and lower level decision makers face multiple objectives at their respective levels. The leader manages a network of roads and wishes to maximize revenues and minimize pollution at the same time. The follower is the user of the highways whose objectives are to minimize travel costs and time. Single objective bilevel formulations of such a problem have been studied in the past, but to the best knowledge of the authors multi-objective bilevel formulations have not been studied. A multi-objective formulation allows to take environmental issues into account along with revenues during policy formulation.

The transportation problem has been modeled by adding different layers of real-world complexities. The resulting bilevel multiobjective programs have been solved using a recently proposed m-BLEAQ algorithm and the results have been confirmed using a nested procedure. The m-BLEAQ algorithm is able to handle the models successfully but

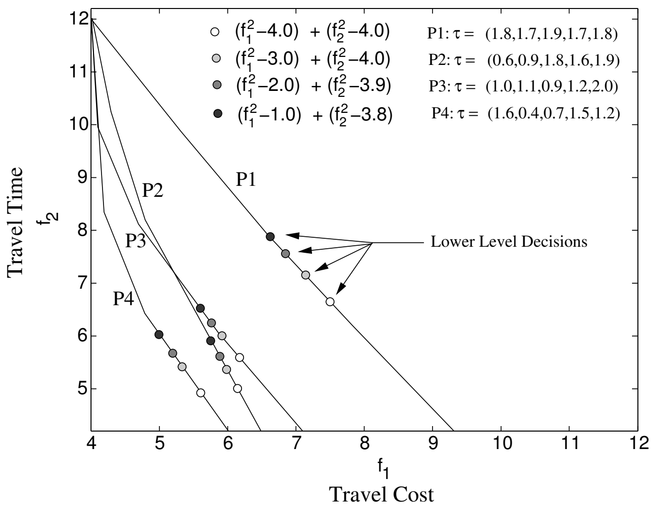

Fig. 9. Model 2: Lower level Pareto-optimal frontiers for different tolls and corresponding lower level decisions.

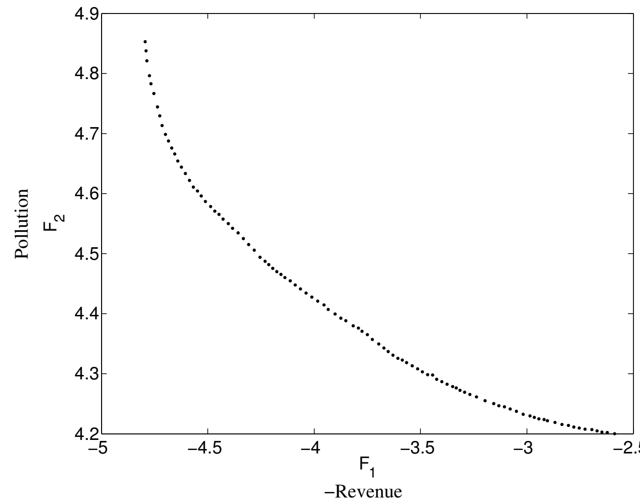

Fig. 10. Model 2: Upper level Pareto-optimal frontier obtained using m-BLEAQ algorithm.

TABLE II
MULTI-OBJECTIVE BILEVEL TRANSPORTATION POLICY PROBLEM WITH KNOWN LOWER LEVEL PREFERENCES

Model 1	Level	Formulation
Variables	Upper level	$x_u = (\tau_i) : i \in \{1, \dots, 5\}$
	Lower level	$x_l = (y_i) : i \in \{1, \dots, 9\}$
Objectives	Upper level	$F_1(\tau_i, y_i) = \sum_{i=1}^{5} \tau_i y_i$
		$F_2(y_i) = \sum_{i=1}^{9} a_i y_i$
	Lower level	$f_1(\tau_i, y_i) = \sum_{i=1}^{5} \tau_i y_i + \sum_{i=1}^{9} b_i y_i$
		$f_2(y_i) = \sum_{i=1}^{9} c_i y_i$
Constraints	Upper level	$\tau_i' \leq \tau_i \ \forall \ i \in \{1, \dots, 5\}$
	Lower level	$d_1 y_1 + d_6 y_6 = 1$
		$d_2 y_2 + d_7 y_7 = 1$
		$d_3 y_3 + d_8 y_8 = 1$
		$d_4 y_4 + d_5 y_5 + d_9 y_9 = 1$
		$0 \leq y_i \leq 1 \ \forall \ i \in \{1, \dots, 9\}$
Preference Structure	Lower level	$V(f_1, f_2) = (f_1 - e_1)^2 + (f_2 - e_2)^2$
Constants (Refer Table IV)	Upper level	$a_i : i \in \{1, \dots, 9\}, \tau_i' : i \in \{1, \dots, 5\}$
	Lower level	$b_i, c_i, d_i : i \in \{1, \dots, 9\}$
		e_1, e_2

TABLE III
MULTI-OBJECTIVE BILEVEL TRANSPORTATION POLICY PROBLEM WITH KNOWN LOWER LEVEL PREFERENCES

Model 2	Level	Formulation
Variables	Upper level	$x_u = (\tau_i) : i \in \{1, \dots, 5\}$
	Lower level	$x_l = (y_{ij}) : i \in \{1, \dots, 9\}, j \in \{1, \dots, 4\}$
Objectives	Upper level	$F_1(\tau_i, y_{ij}) = \sum_{j=1}^{4} p_j \sum_{i=1}^{5} \tau_i y_{ij}$
		$F_2(y_{ij}) = \sum_{j=1}^{4} p_j \sum_{i=1}^{9} a_i y_{ij}$
	Lower level	$f_{1j}(\tau_i, y_{ij}) = \sum_{i=1}^{5} \tau_i y_{ij} + \sum_{i=1}^{9} b_i y_{ij}$
		$f_{2j}(y_{ij}) = \sum_{i=1}^{9} c_i y_{ij}$
	Note:	$\forall \ j \in \{1, \dots, 4\}$ a lower level optimization problem is required to be solved
Constraints	Upper level	$\tau_i' \leq \tau_i \ \forall \ i \in \{1, \dots, 5\}$
	Lower Level	$d_1 y_{1j} + d_6 y_{6j} = 1$
		$d_2 y_{2j} + d_7 y_{7j} = 1$
		$d_3 y_{3j} + d_8 y_{8j} = 1$
		$d_4 y_{4j} + d_5 y_{5j} + d_9 y_{9j} = 1$
		$0 \leq y_{ij} \leq 1 \ \forall \ i \in \{1, \dots, 9\}, j \in \{1, \dots, 4\}$
Preference Structure	Lower level	$V_j(f_{1j}, f_{2j}) = (f_{1j} - e_{1j})^2 + (f_{2j} - e_{2j})^2$ $\forall \ j \in \{1, \dots, 4\}$
Constants	Upper level	$a_i : i \in \{1, \dots, 9\}, \tau_i' : i \in \{1, \dots, 5\}$, $p_j : j \in \{1, \dots, 4\}$
Refer Table IV	Lower level	$b_i, c_i, d_i : i \in \{1, \dots, 9\}$ $e_{1j}, e_{2j} : j \in \{1, \dots, 4\}$

requires a large number of function evaluations at the lower level. Our future studies will be directed towards improving the computational requirements of the solution procedure.

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TABLE IV
VALUES FOR CONSTANTS USED IN MODEL 1 AND MODEL 2.

i	a_i	b_i	c_i	d_i	τ_i'
1	1.0	0.5	1.0	1.0	0.5
2	1.1	0.7	1.1	1.0	0.3
3	1.2	0.4	1.2	1.0	0.6
4	0.9	0.6	0.9	1.0	0.4
5	1.0	0.4	1.1	1.0	0.6
6	1.5	1.0	3.0	1.0	
7	1.5	1.0	3.0	1.0	
8	1.5	1.0	3.0	1.0	
9	1.5	1.0	3.0	1.0	

$e_1 = 4.0$	$e_{11} = 4.0$
$e_2 = 4.2$	$e_{21} = 4.0$
$p_1 = .2$	$e_{12} = 3.8$
$p_2 = .3$	$e_{22} = 3.9$
$p_3 = .4$	$e_{13} = 3.6$
$p_4 = .1$	$e_{23} = 3.9$
	$e_{14} = 3.5$
	$e_{24} = 3.8$

TABLE V
UPPER LEVEL (UL) AND LOWER LEVEL (LL) FUNCTION EVALUATIONS (FE) REQUIRED BY M-BLEAQ AND NESTED PROCEDURE FOR MODEL 1 AND MODEL 2.

Model 1			
Algorithm	UL FE (Fixed)	LL FE (mean)	LL FE (std)
m-BLEAQ	20,000	212,124	23,414
nested	20,000	1,104,253	62,536
Model 2			
Algorithm	UL FE (Fixed)	LL FE (mean)	LL FE (std)
m-BLEAQ	20,000	1,835,435	63,654
nested	20,000	11,457,534	189,615

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