A Review of Hybrid Evolutionary Multiple Criteria Decision Making Methods

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COIN Report Number 2014005

***Abstract*—For real-world problems, the task of decision-makers is to identify a solution that can satisfy a set of performance criteria, which are often in conflict with each other. Multi-objective evolutionary algorithms tend to focus on obtaining a family of solutions that represent the trade-offs between the criteria; however ultimately a single solution must be selected. This need has driven a requirement to incorporate decision-maker preference models into such algorithms – a technique that is very common in the wider field of multiple criteria decision making. This paper reviews techniques which have combined evolutionary multi-objective optimization and multiple criteria decision making. Three classes of hybrid techniques are presented: *a posteriori*, *a priori*, and *interactive*, including methods used to model the decision-makers preferences and example algorithms for each category. To encourage future research directions, a commentary on the remaining issues within this research area is also provided.**

I. Introduction

Real-world decision problems often require the solutions to meet multiple performance criteria (or objectives), simultaneously – they are multi-objective problems (MOPs). These objectives are often conflicting, wherein an improvement in one objective cannot be achieved without detriment of another objective. In this case, there is no single solution to a MOP that can be selected objectively; rather a set of solutions exists, representing different performance trade-offs between criteria. In this setting, a single solution can only be identified (from this set) using the subjective preferences of decision makers (DMs) regarding a favourable resolution of the trade-offs. The MOP itself sits within the wider process of decision making, including governance arrangements, formulation of the criteria, and specification of models for the appraisal of potential solutions against criteria [1]. Without loss of generality, a minimization MOP is defined as follows:

$$\begin{aligned} &\text{minimize } f_m(\mathbf{x}) \quad m = 1,2,...,M \\ &\text{subject to } g_j(\mathbf{x}) \le 0, \quad j = 1,2,...,J \\ &h_k(\mathbf{x}) = 0, \quad k = 1,2,...,K \\ &x_{li} \le x_i \le x_{ui}, \quad i = 1,2,...,n \end{aligned} \tag{1}$$

A solution $\mathbf{x}$ is a vector of n decision variables: $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, $\mathbf{x} \in \mathbb{R}^n$. Each decision variable x_i is subject to a lower bound x_{li}, and an upper bound x_{ui}. f_m represents the m-th objective function. M is the number of objectives (generally, $M \geq 2$). J and K are the number of inequality and equality constraints, respectively.

Pareto-dominance: for two feasible decision vectors $\mathbf{x}$ and $\mathbf{y}$, $\mathbf{x}$ is said to Pareto dominate $\mathbf{y}$ (denoted as $\mathbf{x} \preceq \mathbf{y}$) if and only if $\forall m \in 1, 2, \cdots, M$, $f_m(\mathbf{x}) \leq f_m(\mathbf{y})$ and $\exists m \in 1, 2, \cdots, M$, $f_m(\mathbf{x}) < f_m(\mathbf{y})$.

Pareto optimality: a solution $\mathbf{x} \in \mathbb{R}^n$ is said to be Pareto optimal in $\mathbb{R}^n$ if and only if $\nexists \mathbf{y} \in \mathbb{R}^n$, $\mathbf{y} \preceq \mathbf{x}$.

Pareto optimal set is defined as the set of all Pareto optimal solutions. **Pareto optimal front** is defined as the set of all objective functions values corresponding to all solutions in the **Pareto optimal set** – this set represents the performance trade-offs for the problem.

The field of *multiple criteria decision making* (MCDM) has developed well-established methods over the previous 50 years for helping decision makers address MOPs [2]. Over the last 20 years a largely separate field – known as *evolutionary multi-criterion optimization* (EMO) has arisen that is developing solutions to similar problems [3]. Historically, the EMO research community has tended to emphasise the search for the Pareto optimal set, to inform any subsequent work to select a single solution to a MOP. Within the confines of this perspective, there is no modelling of decision maker preferences (beyond an assumption about a monotonic direction of preference for each objective). Meanwhile, the MCDM community has tended to emphasise the use of preference models either as a precursor to, or during, the search for a single preferred Pareto optimal solution. A further point of discrimination is the nature of the search process: even in cases where MCDM methods have focused on identifying the Pareto optimal set, the search for each solution in that set is regarded as a distinct search in its own right. However, EMO methods are traditionally characterised as employing a parallel search, with information sharing, such that the Pareto optimal set emerges via a single 'run' of the optimizer. Recently, there have been initiatives to integrate and blend the EMO and MCDM fields together, including MCDM special tracks at the two most recent EMO conferences [4], [5].

According to when decision maker preferences are incorporated, i.e., before, during or after the search, MCDM and EMO approaches can be divided into three classes – *a priori*, *interactive* and *a posteriori*, respectively.

In an ***a priori* decision making approach**, the DM preferences are incorporated prior to the search process. The weighted sum approach is one of the most commonly employed *a priori* methods, where the DM preferences are formulated by a weight vector that indicates the relative importance of the objectives. When the DM preferences can be faithfully captured in a mathematical model, an *a priori* method would be effective and efficient. However, this is rarely the case.

In an ***interactive*** **decision making approach**, the DM preferences are incorporated progressively during the optimization process. This enables a DM to learn about the problem and fine-tune his/her preferences if needed, effectively guiding the search towards regions of interest and away from exploring non-interesting solutions. The main limitation of this scheme is that DM may need to be involved intensively during the search process.

In an ***a posteriori*** **decision making approach**, the DM preferences are incorporated after the search; an approximation of the Pareto optimal front is found first followed by selection of a preferred solution by the DM from the set of trade-off solutions presented. An *a posteriori* approach can be effective for MOPs with 2 or 3 objectives – a good approximation of the Pareto optimal front can be obtained and easily be presented to the DM, enabling him/her to confidently select a preferred solution. However, *a posteriori* schemes become less effective on MOPs with higher number of objectives, sometimes termed *many-objective problems* (MaOPs [6]). Not only does the computational burden for solving these problems become very expensive, the approaches become more inefficient since the DMs often are only interested in particular regions of the Pareto front. Furthermore, the number of Pareto optimal solutions required for describing the entire Pareto optimal front of a MaOP is usually very large. Selecting one preferred solution from all these solutions is cognitively difficult.

To date, considerable effort has been spent on developing efficient EMO approaches for finding a well-converged and well-distributed set of Pareto optimal solutions, supporting *a posteriori* decision making, including, for example, MOGA [7], [8], NSGA-II [9], SPEA2 [10], and HypE [11]. See [12] for a review. However, as mentioned above, this scheme faces difficulties when applied onto MaOPs, and such problems arise regularly in the real-world [13]. Thus, to facilitate the process of decision making, the alternative is to consider incorporating DM preferences *a priori* or *interactively* into the EMO approaches. Such hybrid approaches might take advantages of both EMO and MCDM methods.

Early work on hybrid EMO-MCDM approaches was reviewed by Coello in 2000 [14], with an update by Rachmawati and Srinivasan in 2006 [15]. Since this time, the tempo of development for hybrid EMO-MCDM schemes has increased considerably. Branke reviewed *a priori* methods in 2008 [16], with a further review of interactive methods by Jaszkiewicz and Branke in the same year [17]. Our paper presents an updated review of both these classes, whilst also considering *a posteriori* approaches. The paper also sets out a prospectus for future hybrid developments. The remainder of the paper is structured as follows: in Section II representative hybrid EMO-MCDM approaches are reviewed. In Section III challenging research issues are identified. Section IV concludes.

II. Hybrid EMO and MCDM approaches

A. *Decision maker preferences*

Priori to describing the various approaches, we briefly summarize the methods developed for modelling DM preferences. According to [18], these methods can be roughly divided into the following categories: (i) reference point (aspirations), (ii) weights related methods, e.g, lexicographical ordering, relative importance order, reference direction and light beam search [19], (iii) trade-off information, (iv) utility function. Amongst these methods, some elicit a direct model of preferences, e.g. reference point, reference direction, and trade-off information. Other methods construct a preference model indirectly based on elicitation of some examples of holistic judgements, such as utility function. As discussed in [20], eliciting direct preference information from the DM requires a high cognitive effort, and so can be counterproductive in real-world decision making situations. Eliciting indirect preferences tends to be less demanding in terms of cognitive effort.

B. A priori *schemes*

This section review some of representative *a priori* approaches. These approaches are classified as reference point based, weight information based, trade-off information based and other forms.

1) Reference point information: There is a large body of *a priori* approaches based on the reference point. Perhaps MOGA [7] developed by Fonseca and Fleming is the earliest such approach. The DM preference is specified as aspirations and the non-dominated ranking mechanism is extended to accommodate aspiration levels, enabling the search to be gradually guided towards the DM region of interest (ROI). MOGA is further extended by introducing a preferability operator, with which both goals and priorities can be accommodated in the ranking scheme [8]. This new ranking scheme provides a unification of Pareto optimality, the lexicographic method, goal programming, constraint satisfaction and constrained optimization. MOGA has been successfully used in optimising a low-pressure spool-speed governor of a Pegasus gas turbine engine and many other applications [21], [13]. The main weakness of this approach is that it requires a DM to know the ranges of objective values so as to initialize coherent aspiration levels. In addition, MOGA uses parallel coordinates to visualize solutions. Although this method can aid DM cognition of high-dimensional trade-offs, it does not provide any indication of the location of chosen solutions on the Pareto optimal front and lacks quantitative statistical analysis of the solutions. Tan et al. [22] also extended MOGA by introducing a new goal-sequence domination scheme to allow advanced specifications such as priorities and hard/soft constraints to be incorporated.

Another representative approach that uses aspirations was proposed by Molina et al. [23]. A dominance relation called g*-dominance* (g refers to goals) is defined; solutions satisfying all the aspirations and solutions fulfilling none of the aspirations are preferred over solutions satisfying some aspirations. An approach called g-NSGA-II that combines g-dominance and NSGA-II is proposed to search for solutions satisfying the specified aspirations. This algorithm works regardless of whether the specified goal vector is feasible or infeasible. However, it is demonstrated in [24] that g-NSGA-II faces difficulties when the provided goal vector is close to the true Pareto front (as the approach does not preserve a Pareto based ordering). Handling of multiple ROIs by g-NSGA-II is not considered. Intuitively, the *g-dominance* relation is not easy to extend to handle multiple ROIs as an individual can *g-dominate* one goal vector, and simultaneously, be *g-dominated* by another goal vector.

Deb and Sundar [25] proposed a reference point based NSGA-II (R-NSGA-II) for searching for solutions close to a DM specified reference point. The reference point is not applied in a classical way, i.e. together with an achievement scalarizing function [26], but rather by establishing a biased crowding scheme. Solutions near reference points are emphasized by the selection mechanisms. The extent and the distribution of the solutions is maintained by a user defined parameter ε. The efficiency of R-NSGA-II is demonstrated on MOPs with up to ten objectives. R-NSGA-II can also handle multiple ROIs simply by using multiple reference points.

Thiele et al. [27] hybridized the reference point with indicator based evolutionary algorithm (PBEA). The reference point is applied to an achievement scalarizing function, and this is then incorporated into the binary indicator function, the ε-indicator [28] (which is Pareto-dominance preserving). The spread range of the obtained solutions is controlled by an additional parameter which might be not easy to configure.

Ben Said et al proposed another reference point based approach, the r-NSGA-II [24]. In their study, the reference point is employed to modify the usual dominance principle, resulting in a new dominance relation, named *r-dominance*, which can be used to create a strict partial order over non-dominated solutions. The *r-dominance* relation prefers solutions that are closer to the specified reference point, and simultaneously preserves the order induced by Pareto-dominance relation. The approach r-NSGA-II is derived from NSGA-II by replacing the Pareto-dominance relation with the *r-dominance* relation. The algorithm has other two additional parameters δ and $\mathbf{w}$. $\delta \in [0, 1]$ is used to control the range of the ROIs, and $\mathbf{w}$ expresses the bias of the DM. The performance of r-NSGA-II is assessed on a set of benchmarks ranging from 2 to 10-objective problems and is shown to be good in guiding the search towards both single and multiple ROIs. However, as pointed out by the authors, r-NSGA-II faces difficulties on multi-modal problems, such as ZDT4.

The reference point method has also been used in multi-objective particle swarm optimization (MSPSO) algorithms [29], [30]. The idea of these approaches is to incorporate the DM preferences (reference points) into the selection of leaders.

2) Weight information: Deb and Kumar [31] combined the reference direction with NSGA-II. The reference direction is incorporated into an achievement scalarizing function which is used to guide the search towards a preferred region. Multiple ROIs are obtained by specifying multiple reference directions. The efficiency of this approach is demonstrated on MOPs with up to ten objectives. Again, the spread range of the ROI is controlled by a user defined parameter.

Deb and Kumar [32] also hybridized the light beam search method with NSGA-II. The hybridized approach is able to search for part(s) of Pareto optimal fronts illuminated by the light beam emanating from a starting point to the reference point with a span controlled by a threshold. This approach also performs well on MOPs with up to ten objectives. The light beam search is also hybridized with MSPSO algorithm in [33]. Again, the issue is how to appropriately control the spread range of the obtained solutions.

3) Trade-off information: Branke [34] proposed a guided evolutionary multi-objective optimization approach, denoted as G-MOEA. In G-MOEA the DM preferences are manifested through a modification of the dominance relation, specifying an maximally acceptable trade-off rate between objectives, i.e., one unit improvement in objective f_i is worth at most a_{ij} units in objective f_j. G-MOEA works well for two objectives. However, providing all pair-wise information for a problem with many objectives is cognitively intensive and needs $\frac{M^2-M}{2}$ comparisons.

4) Other forms: Branke and Deb [35] suggested a modified and controllable biased crowding approach. Their approach aims to search for a set of solutions that are parallel to an *iso-utility* function defined by a specified reference direction. Specifically, a parameter is applied to control the range of ROI along the Pareto optimal front. This parameter is defined as the ratio of the real distances between neighboring solutions on the Pareto optimal front and the projected distance of the same solutions on a plane defined by a linear utility function.

In [36], the authors integrate weight preferences in the calculation of hypervolume indicator. The weighted hypervolume indicator serves as a means of integrating the DM preferences. Auger et al. [37] applied this idea to HypE and proposed the weighted hypervolume based HypE. W-HypE is demonstrated to perform well on searching for preferred solutions for both bi and many-objective problems. The only issue is that the spread range of the ROI is controlled by a deviation parameter in the weight distribution function. Defining a proper value for this parameter is not easy for a decision maker.

Karahan and Köksalan [38] proposed a steady-state elitist evolutionary algorithm, named the territory defining evolutionary algorithm (TDEA). Similar to ε-MOEA [39], TDEA defines a territory around each individual so as to prevent crowding. A smaller territory corresponds to a denser coverage of solutions (i.e., more neighboring solutions), and a larger territory corresponds to a sparser coverage of solutions. Based on TDEA, the authors developed an *a priori* approach, named prTDEA, in which the DM specifies his/her preferred region by a weight set. Solutions in the preferred region and non-interesting region are then assigned different territories such that more solutions are obtained in the preferred region(s).

Although in this review the above approaches are classified as *a priori* approaches, most of these methods (e.g., [7], [34], [25], [27], [24], [38]) can be turned into *interactive* approaches simply by allowing the DM to adjust preferences and continue the optimization interactively. For example, Köksalan and Karahan proposed iTDEA [40] as an interactive extension of the TDEA. In the iTDEA, the DM is asked to choose his/her preferred solutions from a set of representative solutions at each interaction. A territory is then defined around those preferred solutions so as to obtain more solutions around them, obtaining denser coverage of these interesting regions. This procedure continues till the algorithm finds a satisfactory solution. The iTDEA is tested on three problems using three different utility function types to simulate the DM responses. Experimental results show that iTDEA converges the DM simulated preferred regions well.

C. Interactive schemes

In *interactive* approaches, the preference information requested from the DM is usually much simpler than the preference information required by *a priori* methods. Also, in comparison to *a posteriori* methods, they have moderate computational requirements. More importantly, as the DM controls the search process, he/she gets more involved in the process, learns about potential solutions, and can be more confident about the final choice[1].

Greenwood et al. [43] suggested a procedure which asks the DM to order a provided set of solutions, and use this preference information to derive constraints for linear weighting of the objectives consistent with the given ordering. These constraints are used in EMO approaches to check whether there exists a feasible linear weighting, such that solution **x** is preferred over solution **y**. Although in [43] this procedure is implemented as an *a priori* approach, it can easily be applied interactively.

Phelps and Köksalan [44] proposed a conceptually similar interactive optimizer in which the DM preference is elicited by pairwise comparisons of solutions. This preference information is further used to obtain a "most compatible" weight vector via linear programming methods, resulting a linearly weighted sum of objectives. This aggregate objective is optimized in the subsequent generations using an evolutionary algorithm till new comparisons of solutions are provided. It needs to be mentioned that as multiple objectives are combined into one single objective, the power of EMO approaches in searching for multiple solutions with different trade-offs is not exploited.

Similar to [44], Fowler et al. [45] used a more general quasi-concave utility function to form the DM preference as a preference cone consisting of inferior solutions. Combined with Pareto dominance, the preference cone is applied to drive the search towards preferred regions. This approach is tested on multi-objective knapsack problems and is found to perform well. As argued by the authors, this approach guarantees correct partial orders of solutions provided that the DM preferences are consistent with the assumed utility function forms.

Jaszkiewicz [46] proposed another interactive approach based on the Pareto memetic algorithm (PMA[2]) [47] which also uses linear value functions to model the DM preference. The DM preference is again elicited from pairwise comparison of solutions. However, this strategy does not aim to identify a single most likely utility function but, rather, simultaneously maintains a range of utility functions compatible with the elicited preferences. In other words, the preference information is not applied to create a single compatible weight vector but it is to reduce the set of possible weight vectors.

Greco et al. [48] proposed a method for interactive multi-objective optimization, which is based on application of a logical preference model built using the Dominance-based Rough Set Approach (DRSA). DRSA [49] is a methodology of multiple criteria decision analysis which is used for structuring the DM's preferences in terms of the most general and understandable "if ..., then ..." decision rules [50]. In [48], once an approximation of solutions is obtained, the DM is then asked to indicate those relatively good solutions. Having this information, a preference model structured in terms of "if ..., then ..." decision rules is induced using DRSA. This preference model is then applied to refine the obtained solutions, cutting off non-interesting solutions. The procedure continues until a satisfactory solution is found. This main advantage of this approach is that the preference model used during the search is composed of a set of user-friendly decision rules.

Follow the study of [48], Greco et al. [51] proposed two interactive schemes, called DRSA-EMO and DRSA-EMO-PCI, where the preference information from the DM is elicited by sorting some solutions in the current population into "relatively good" and "others", or by pairwise comparison of solutions, respectively. The resulting two interactive schemes also have the potential to take into account robustness concerns simply because DRSA can handle a plurality of scenarios in case of decision making under uncertainty and dynamism [52].

Branke et al. [20], [53] incorporated the Generalized Regression with Intensities of Preference (GRIP) methodology [54] into a modified NSGA-II (where the dominance-based ranking is replaced by the *necessary ranking* and the crowding distance is calculated in utility space rather than objective-space), and proposed another interactive multi-objective optimizer, the Necessary-preference-enhanced Evolutionary Multi-objective Optimizer (NEMO). In NEMO, the DM is asked to compare some pairs of solutions and specify which is preferred, or compare intensities of preferences between pairs of solutions. These results are then used to construct all possible additive value functions (based on the robust ordinal regression method

[1]There are other interactive approaches to those presented here, such as the Interactive Surrogate Worth Trade-off method [41] and the NIMBUS approach [42]. However they have yet to be combined with EMO approaches and are not reviewed here

[2]The PMA employs a scalarizing function with a randomly generated weight in each iteration for local search and recombination. The use of random weights corresponds to searching for solutions in different regions of the Pareto front.

[55]) as a preference model. These compatible value functions are then applied to guide the search towards regions of interest to the DM. NEMO is tested on bi-objective problems and performs well. However, its performance scalability has not been examined on many-objective problems.

Some hybrid EMO/MCDM approaches model the DM preferences by a value function (VF), such as the framework of PI-EMO-VF [56]. In [56], the DM is asked to order a given set of alternatives from best to worst. This preference information is then used to model a strictly increasing polynomial value function. The construction procedure involves solving a single-objective optimization problem to determine the optimal parameters of the value function. This constructed value function is then utilized to redefine the dominance principle, and drive the EMO approach (NSGA-II is applied in [56]) to search for preferred solutions for the subsequent iterations until the next "DM call". In addition, this value function is also used to build a preference based termination criterion. The effectiveness of PI-NSGA-II-VF is demonstrated on MOPs with up to five objectives. However, as identified by the authors for future studies, this approach has not be extended to handle constrained problems. Furthermore, this study has also suggested some interesting directions, such as modelling preferences with other value functions, building robust value functions, using value function based variation operators, and being more restrictive in the use of DM calls.

So far effort has been made along some of these directions. Sinha et al. [57] augmented the polynomial value function into a generalized polynomial value function that fits a wider variety of quasi-concave preference information. The value function takes into account the indifference of the decision maker towards a pair of alternatives. The efficacy of PI-NSGA-II-VF is evaluated on three and five-objective test problems with constraints. Moreover, in [57], the value function fitting procedure is tested on other commonly used value functions like the Cobb-Douglas value function and the CES value function in the literature [58], and the generality of the PI-EMO-VF is demonstrated. Sinha et al. [59] proposed another progressively interactive EMO algorithm (PI-EMO-PC), where a polyhedral cone is used to construct the DM preference. The constructed polyhedral cone is then applied to modify the domination principle of an EMO and drives the search towards a preferred region. Instead of providing an order of solutions in PI-EMO-VF, in PI-EMO-PC the DM is asked to choose the best solution from a provided set of alternatives. The best solution is selected using an advanced selection technique known as VIMDA [60]. This is a visual interactive method that uses the reference point technique to allow the DM to select the best point from a set of non-dominated points. Using the best point, a polyhedral cone is constructed based on the end points (that have the best value in one objective). An instantiation of PI-EMO-PC, PI-NSGA-II-PC, is evaluated on two to five-objective unconstrained test problems and is shown to be effective. In [61], Sinha et al. studied how the PI-EMO-PC and PI-EMO-VF framework is used under a limited budget of DM calls. The preference information from the DM is elicited only after pre-defined progress has been made, such progress being measured by the distance between the best solution and the ideal point [62]. The efficiency of this approach is demonstrated on two to five-objective constrained and unconstrained test problems. Moreover, in [61], it has also been demonstrated that the more DM calls are made, the better the accuracy of obtained solutions (this is also identified in [40], [57], [59], respectively).

A further interactive multi-objective decision support system named I-MODE has been proposed [63], [64]. This system is a GUI-based, user-friendly software, and is built over a number of existing multi-objective evolutionary algorithms (MOEAs) and different decision making approaches. I-MODE allows the DM to interactively focus on interesting region(s) of the Pareto front using tools such as weighted sum approach, Chebyshev function approach, utility function based approach and trade-off information. These preferences are then incorporated into a MOEA to search for new solutions in a ROI or multiple ROI(s). So far the main limitation of the I-MODE is that it can only consider a maximum of 3 objectives due to the use of a Cartesian coordinate system. However, this can be addressed by using parallel coordinates [65]. In addition, other widely used decision making tools, e.g., the light beam search [19], could be included.

D. A posteriori *schemes*

Most of the evolutionary multi-objective algorithms that focus on finding a full and satisfactory approximation of the Pareto optimal front are classic examples of *a posteriori* approaches. In these methods, the decision making aspect is not considered until the entire Pareto optimal front is generated. However, as previously mentioned, *a posteriori* approaches often face difficulties in obtaining a full approximation of the entire Pareto optimal front. It has been demonstrated that the search ability of Pareto-dominance based methods degrades significantly as the number of objectives increases [66]. As a result, the obtained solutions are usually not close to the true Pareto front [67], [68], [69].

In order to obtain a satisfactory approximation of the entire Pareto optimal front, considerable effort has been invested in other types of MOEA, many of which draw on preference schemes originally developed by the MCDM community. However these preferences are not used to steer the search toward a specific subset of preferred solutions – rather they are *synthetic preferences* that act only to provide discrimination between solutions in high-dimensional objective spaces. A number of representatives are:

(i) modified Pareto-dominance relation based MOEAs, e.g. ε-MOEA [39], [70];
(ii) decomposition based MOEAs, e.g. CMOGA [71], MSOPS [72] and MOEA/D [73];
(iii) preference-inspired co-evolutionary algorithms, e.g. PICEA-g [74], [75], PICEA-w [76].
(iv) use of a predefined multiple target approach, e.g. NSGA-III [77].

There remains some debate about the usefulness of attempting to obtain trade-off surfaces for MOPs with greater than three objectives. The number of solutions required to represent such a surface at a given resolution grows exponentially with the number of objectives. Also it might be cognitively challenging for the DM to choose the most preferred solution from such large sets of candidate solutions.

III. Challenging issues in hybridized EMO and MCDM approaches

We have briefly discussed the approaches used to model the preferences of decision makers for use within MOEAs and raised particular attributes or shortcomings of each approach. We now place these findings in a more general context concerning the performance and applicability of hybrid EMO/MCDM approaches. Specifically, we will discuss issues raised concerning: (i) preservation of Pareto-dominance; (ii) transitivity of preferences; (iii) scalability of the approaches based on the number of objectives; (iv) presence of more than one DM; (v) performance measures for assessment of these hybridized approaches; (vi) unification of preferences; (vii) limiting the burden on the DM; and (viii) fuzzy preferences.

Most MOEAs rely on the Pareto dominance concept to effectively drive the search toward solutions that are Pareto optimal. However, the introduction of preferences within MOEAs alters the standard dominance relation between solutions as shown in the g-NSGA-II [23]. This alteration of the Pareto-dominance relationship may introduce difficulty in obtaining a robust ordering between the solutions.

Often, DM preference information is attained by requesting the decision maker to select his preferred solution from a subset of solutions (e.g. solution *a* or *b*). However, this approach to elicitation affects the transitivity of the preferences. If solution *a* is better than solution *b*, and solution *b* is better than solution *c*, it does not necessarily mean that solution *b* is better than solution *c*. This is issue is particularly pertinent in the outranking method of modelling preferences.

Another common problem with most of these hybrid approaches is their applicability to problems with many objectives, which is a common trait for real-world problems. Modelling the preferences for MaOPs becomes more taxing and may lead to other issues involving computational complexity and accuracy of the preferences models.

The presence of more than one DM introduces a set of preferences which may be overlapping or fundamentally divided. For instance, we may extract information from a group of DMs to integrate into the hybrid approaches to form a set of preferred solutions. For overlapping preferences, the group of decision makers may not select the same solutions: DMs A and B may express their preference of solutions *a* and *b*, while DM C prefers solutions *a*, *b*, and *c*, where solutions *a*, *b*, and *c* are similar. For cases where the set of preferences are divided, the DMs are in disagreement: DMs A and B may express their preference of solution *a* over solution *b*, while DM C prefers solution *b* over solution *a*, where solutions *a* and *b* are different. This raises an issue during the implementation of the hybrid approaches – which set of solutions should be presented to the DMs? The overlapping preferences may be dealt with incorporating uncertainty within the hybridized framework. However, this is not often incorporated within the EMO-based approaches and for approaches that do incorporate uncertainty, appropriate measures should be taken to ensure its sufficiency. Appropriate measures should also be developed for handling divided preferences.

Despite a number of reports on hybridizing MCDM approaches with MOEAs, the literature currently lacks techniques to provide a performance measure of how well these approaches deliver the needs of the DMs. The main drive of this research area is to provide assistance to DMs in MCDM with use of MOEAs. In order to encourage a fair comparison among these techniques, or at least a guide for potential studies, a measure of performance of these approaches should be developed. There are difficulties such as imprecision of DM preferences as expressed in [13]. A technique enabling the performance of these approaches to be measured would encourage the growth of this research area and help identify weaknesses that may be resolved from the insight gained. To date, there are few studies addressing this issue – one notable exception is [78].

Obtaining a preferred solution under a limited budget of DM calls is another challenging problem. A first attempt has been made in [61]. However, the method is rather limited and more effective strategies are required. Other related approaches include that of Todd and Sen [79], who used preference information provided by the DM to train an artificial neural network, which was then used to automatically evaluate solutions for the subsequent iterations of an evolutionary algorithm. Similar studies are also found in [80], [81]. This literature is limited, and further studies that focus on the reduction of DM calls are urgently needed.

It is interesting to note that all of the works to date constrain the DM to a particular formulation of preferences. However different DMs may be more comfortable expressing their preferences in different ways. Search methodologies that can unify different preference models, or retain flexibility with regard to expressions of preference, would be highly beneficial. Some early steps have been made in this direction [82]. With human DMs It is usually natural for preferences to be expressed linguistically. Fuzzy logic offers an appropriate methodology, with some existing works in this direction (e.g., [83], [84], [85]), although construction of a suitable fuzzy inference system requires substantial further research.

IV. Conclusion

EMO methods can used together with MCDM techniques to assist DMs in finding the best solutions satisfying multiple objectives. In this paper we reviewed hybrid EMO/MCDM approaches based on the interaction between the DM preference model and the optimization process. We have identified eight key challenges for hybrid approaches and argue that these challenges should be priority research themes for new work blending EMO and MCDM methods.

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