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SC2: Evolutionary Methods for Single and Multi-Objective Optimization

Assignment III: Evolutionary Multi-Objective Optimization

Marks: 20

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1. Determine if the first solution dominates the second solution? (3)

   (a) (min, min): $\mathbf{f}^{(1)} = (1.2, 3.5)^T$, $\mathbf{f}^{(2)} = (1.5, 3.0)^T$.

   (b) (min, max, min): $\mathbf{f}^{(1)} = (10.5, 1.5, -10.0)^T$, $\mathbf{f}^{(2)} = (5.0, 0.5, -12)^T$.

   (c) (max. max, min): $\mathbf{f}^{(1)} = (5, 5, 3)^T$, $\mathbf{f}^{(2)} = (2, 5, 4)^T$.

2. Consider the following three objective functions:

$$\begin{array}{ll} \text{Minimize} & f_1(\mathbf{x}) = x_1^2 + 2\sin\pi x_2, \\ \text{Maximize} & f_2(\mathbf{x}) = x_1 x_2 - 10, \\ \text{Minimize} & f_3(\mathbf{x}) = 5x_1^3 - x_1 x_2^2. \end{array}$$

   and the following points:

$$\text{(i) } (1,0)^T, \quad \text{(ii) } (0,1)^T, \quad \text{(iii) } (1,1)^T, \quad \text{(iv) } (2,1)^T.$$

   Which solutions are non-dominated solutions? (5)

3. Consider the following parent and offspring populations for a problem of minimizing the first objective and maximizing the second objective:

Soln.	P_t	
	f_1	f_2
1	5.0	2.5
2	1.0	1.0
3	1.5	0.0
4	4.5	1.0
5	3.5	2.0

Soln.	Q_t	
	f_1	f_2
a	3.5	0.0
b	2.2	0.5
c	5.0	2.0
d	3.0	3.0
e	3.0	1.5

   Create P_{t+1} using NSGA-II. (6)

4. Consider the problem:

$$\begin{array}{ll} \text{Minimize} & f_1(\mathbf{x}) = (x_1 - 2)^2 + x_2^2, \\ \text{Maximize} & f_2(\mathbf{x}) = 9x_1 - x_2^2, \\ \text{subject to} & x_1^2 + x_2^2 \leq 225, \\ & x_1 - 3x_2 + 10 \leq 0, \\ & -20 \leq x_1, x_2 \leq 20. \end{array}$$

   Normalize the constraints. Sort the following points in increasing level of constraint non-domination: (6)

$$\begin{array}{lll} \mathbf{x}^{(1)} = (0,0)^T, & \mathbf{x}^{(2)} = (5,10)^T, & \mathbf{x}^{(3)} = (-10,-15)^T, \\ \mathbf{x}^{(4)} = (-11,0)^T, & \mathbf{x}^{(5)} = (10,10)^T, & \mathbf{x}^{(6)} = (0,15)^T. \end{array}$$