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Applied Evolutionary Optimization
Ohio Aerospace Institute, USA

Tutorial 3: Evolutionary Multi-Objective Optimization

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1. Consider the two-objective optimization problem:

$$\begin{array}{rl} \text{Minimize} & f_1(\mathbf{x}) = x_1^2 + x_2^2, \\ \text{Minimize} & f_2(\mathbf{x}) = (x_1 - 2)^2 + x_2^2. \end{array}$$

   (a) Using the weight vector $\mathbf{w} = (w_1, 1 - w_1)^T$ $(0 \leq w_1 \leq 1)$, find the Pareto-optimal solutions in terms of w_1.

   (b) Does a uniform set of $\mathbf{w}$ vectors produce a uniformly distributed set of Pareto-optimal solutions? Explain.

   (c) What is the relationship between f_1 and f_2 for the Pareto-optimal solutions?

2. Consider the following parent and offspring populations for a problem of minimizing the first objective and maximizing the second objective:

Soln.	P_t	
	f_1	f_2
1	5.0	2.5
2	1.0	1.0
3	1.5	0.0
4	4.5	1.0
5	3.5	2.0

Soln.	Q_t	
	f_1	f_2
a	3.5	0.0
b	2.2	0.5
c	5.0	2.0
d	3.0	3.0
e	3.0	1.5

   Create P_{t+1} using NSGA-II.

3. Consider the problem:

$$\begin{array}{rl} \text{Minimize} & f_1(\mathbf{x}) = (x_1 - 2)^2 + x_2^2, \\ \text{Maximize} & f_2(\mathbf{x}) = 9x_1 - x_2^2, \\ \text{subject to} & x_1^2 + x_2^2 \leq 225, \\ & x_1 - 3x_2 + 10 \leq 0, \\ & -20 \leq x_1, x_2 \leq 20. \end{array}$$

   Normalize the constraints. Sort the following points in increasing level of constrain-non-domination:

$$\begin{array}{lll} \mathbf{x}^{(1)} = (0, 0)^T, & \mathbf{x}^{(2)} = (5, 10)^T, & \mathbf{x}^{(3)} = (-10, -15)^T, \\ \mathbf{x}^{(4)} = (-11, 0)^T, & \mathbf{x}^{(5)} = (10, 10)^T, & \mathbf{x}^{(6)} = (0, 15)^T. \end{array}$$