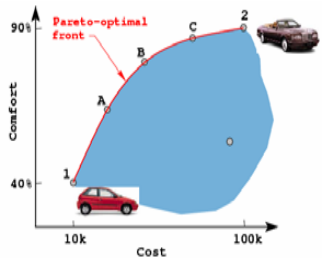
# Lecture 8:
## Advanced Topics in EMO

- Three areas for applications
  - Better decision-making
  - Unveiling common principles (*Innovization*)
  - Solving other optimization problems
- Handling uncertainties
  - Robust and reliability based EMO



## Decision-Making Easier

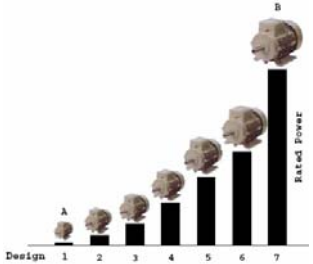
- Existence of multiple trade-off solutions
  - Provide trade-off information
  - A better idea of the nature of Pareto-optimal front
  - An idea of range of solutions
- Weighted scheme with 70-30
  - Always wonder what if 69-31 or 71-29?





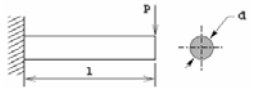

## *Innovization*:
*Discovery of Innovative design principles through optimization*

- Understand important design principles in a routine design scenario
  - Example: Electric motor design with varying ratings, say 1 to 10 kW
    - Each will vary in size and power
    - Armature size, number of turns etc.
  - How do solutions vary?
    - Any common principles!

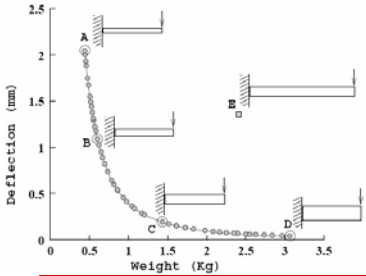





## An Engineering Case Study




- All P-O solutions
  - l=200 mm
  - 18.94 ≤ d ≤ 50.00 mm




$$\text{Minimize} \quad f_1(d,l) = \rho\frac{\pi d^2}{4}l,$$
$$\text{Minimize} \quad f_2(d,l) = \delta = \frac{64Pl^3}{3E\pi d^4},$$
$$\text{subject to} \quad \sigma_{max} \le S_y,$$
$$\delta \le \delta_{max},$$
$$10 \le d \le 50 \text{ mm}, \quad 200 \le l \le 1000 \text{ mm}, \quad \sigma_{max} = \frac{32Pl}{\pi d^3}$$

| Solution | d (mm) | l (mm) | Weight (kg) | Deflection (mm) |
|---|---|---|---|---|
| A | 18.94 | 200.00 | 0.44 | 2.04 |
| B | 21.24 | 200.00 | 0.58 | 1.18 |
| C | 34.19 | 200.00 | 1.43 | 0.19 |
| D | 50.00 | 200.00 | 3.06 | 0.04 |
| E | 33.02 | 362.49 | 2.42 | 1.31 |

## In Support of Common Optimality Properties

**Fritz-John Necessary Condition:**

Solution $x^*$ satisfy

1. $\sum_{m=1}^{M} \lambda_m \nabla f_m(x^*) - \sum_{j=1}^{J} u_j \nabla g_j(x^*) = 0,$ and
2. $u_j g_j(x^*) = 0$ for all $j = 1, 2, 3, \ldots\ldots, J$
3. $u_j \geq 0$, $\lambda_j \geq 0$, for all j and $\lambda_j > 0$ for at least one j

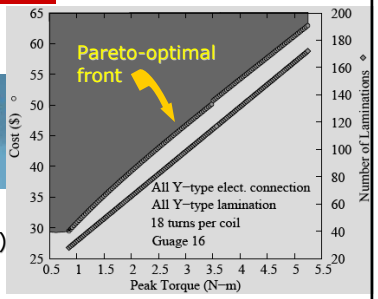
- To use above conditions requires differentiable objectives and constraints
- Yet, it lurks existence of some properties among Pareto-optimal solutions



## Brushless DC Permanent Magnet Motor Design for Cost and Peak Torque

- Six variables (all discrete), three constraints

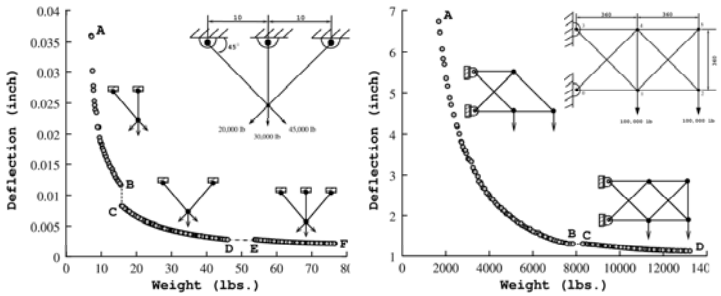





- *Innovizations*:
  - Connection: Y (betn. Y & Δ)
  - Lamination Type: Y (X, Y, Z)
  - 1 out of 16 wire guages
  - 18 turns per coil (10,80)
- *More peak torque by adding linearly more laminations* ⟵ Recipe for design



## Revealing Salient Insights: Truss Structure Design



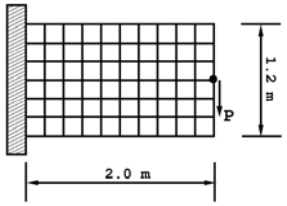

(Deb, Khan and Jindal, 2000)

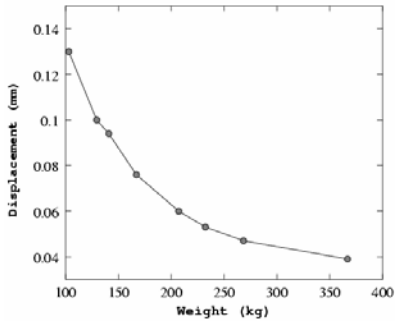


## Revealing Salient Insights: A Cantilever Plate Design

Eight trade-off solutions are chosen

Base Plate

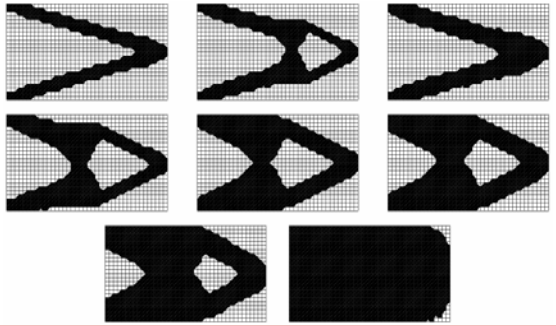



(Deb and Chaudhuri, 2003)

## Trade-Off Solutions

- Symmetry in solutions about mid-plane, discovery of stiffener





## Innovized Principles

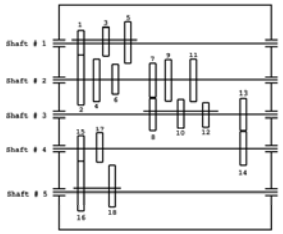
- Mid-line symmetry
- Straight arms to reach load is minimum-weight strategy
- Two ways to increase stiffness
  - Thickening of arms
  - Use of a stiffener
  - Additional stiffening by a combination
- Chamfering of corners helpful

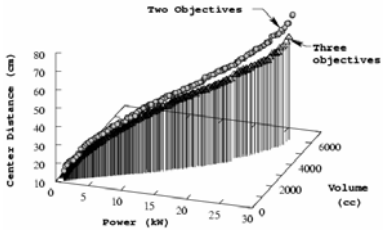


## Gear-box Design

- A multi-spindle gear-box design (Deb and Jain, 2003)
- 28 variables (integer, discrete, real-valued)
- 101 non-linear constraints
- Important insights obtained

(larger module for more power)

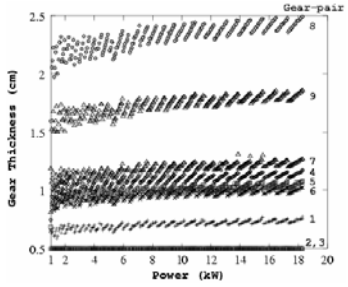


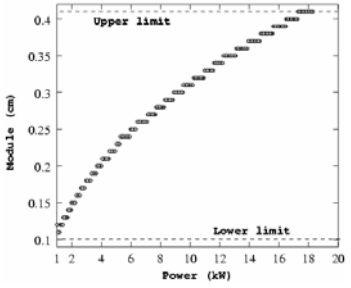





## Innovized Principles

- Module varies proportional to square-root of power
- Keep other 27 variables more or less the same



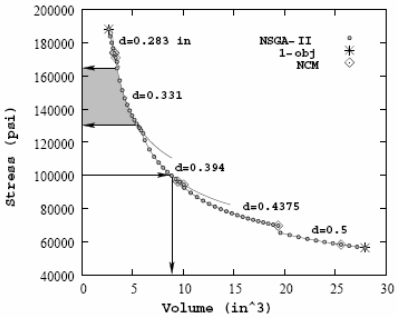
## Mechanical Spring Design

- Minimize material volume
- Minimize developed stress
- Three variables: (d, D, N):
  discrete, real, integer
- Eight non-linear constraints
  - Solid length restriction
  - Maximum allowable deflection (P/k≤6in)
  - Dynamic deflection ($P_m$-P)/k≥1.25in
  - Volume and stress limitations



## Innovized Principles

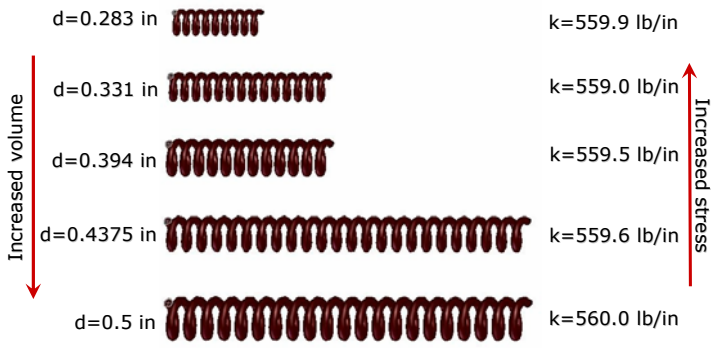
- Pareto-optimal front have niches with d
- Only 5 (out of 42) values of d (large ones) are optimal
- A blue-print for optimal design




(Deb and Srinivasan, 2005)



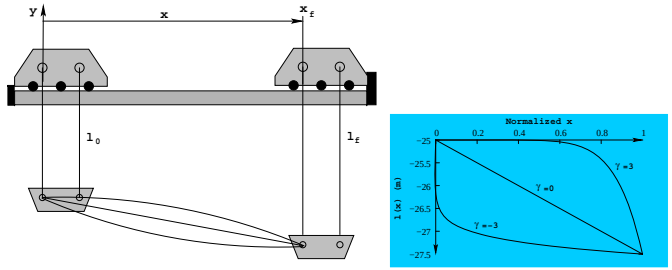
## Optimal Springs, Optimal Recipe






## Overhead Crane Maneuvering

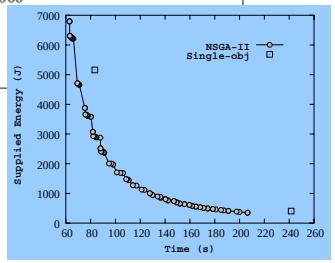
- Minimize time of operation
- Minimize operating energy



## Simulation Results

| $\gamma$ | Force (N) | Pattern |
|---|---|---|
| 2.98 | 149.26 | 1111111100100010000000000000000000000000000000000000000 |
| 3.00 | 187.76 | 11111111001000100000000000000000000000000000 |
| 3.00 | 238.56 | 1111111100100010000000000000000000 |
| 3.00 | 327.86 | 111110110010011000000000000 |
| 2.98 | 427.93 | 11111011001001 1000000 |
| 3.00 | 544.94 | 11111111001000100 |
| 2.87 | 629.62 | 11111111010001 |
| 2.88 | 818.99 | 1101110 10010001 |

- NSGA-II finds trade-off and interesting properties






## Innovative Principles

- For optimum operation:
  - Lower load suddenly at the end
  - Spend energy only at the beginning
- Fast unloading demands more energy
  - For fixed θ, E ∞ l
  - Delay lowering for saving energy

$$E = mg(1 - \cos\theta)l$$



## Epoxy Polymerization

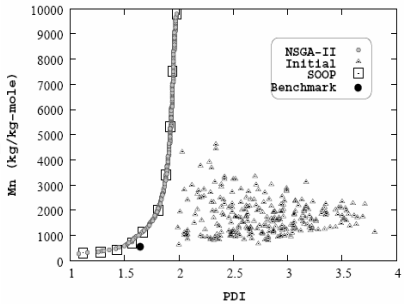
- Three ingredients added hourly
- 54 ODEs solved for a 7-hour simulation
- Maximize Chain length (Mn)
- Minimize Polydispersity index (PDI)
- Total 3x7 or 21 variables

$$\begin{aligned} \text{Maximize} \quad & I_{11} = M_n, \\ \text{Minimize} \quad & I_{12} = \text{PDI}, \\ \text{subject to} \quad & \text{satisfying mass and moment balance equations,} \\ & u_i^{\min} \leq u_i \leq u_i^{\max} \quad i = 1, \ldots, 21. \end{aligned}$$



## Pareto-Optimal Front

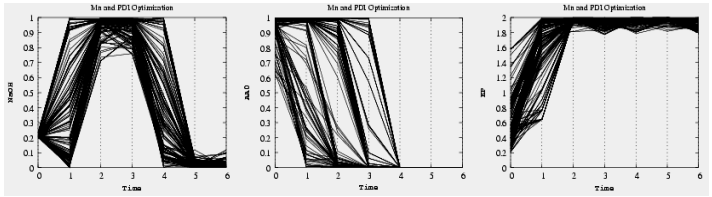
- NSGA-II results
- Verified with NCM
- Batzer and Zahir (1992)'s solution as benchmark




A non-convex frontier

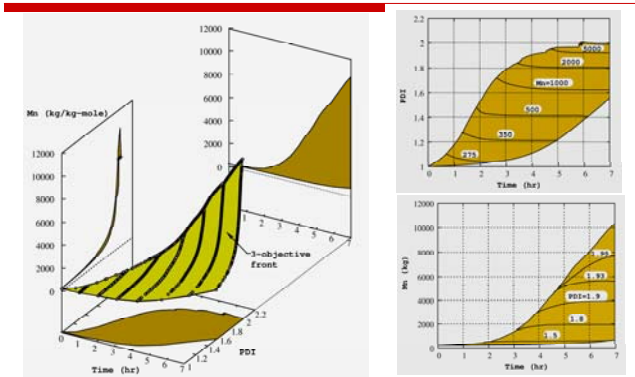
## Epoxy Polymerization (cont.)

- Some patterns emerge among obtained solutions
- Chemical significance unveiled





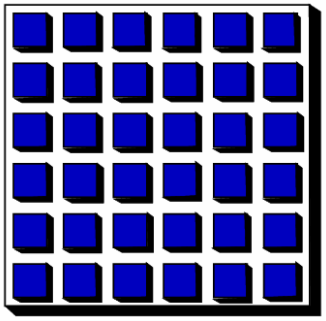

## Higher-Level Innovization:
## *More Objectives and Operating Chart*






## VLSI Layout Design

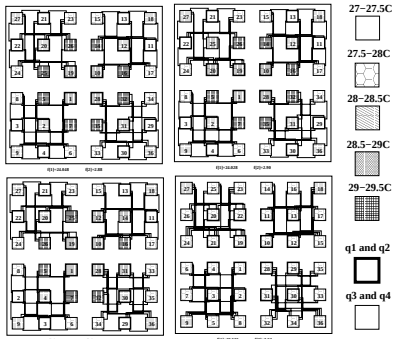
- Connectivity of components known
- Thermal properties known
- Minimize wire length
- Minimize failures due to heat localization





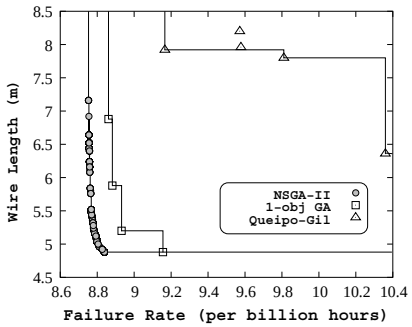
## A Case Study

- 6X6 components
- Structured connectivity
- More heat producing elements at the boundary
- Minimum connectivity



## Comparison with Other Methods

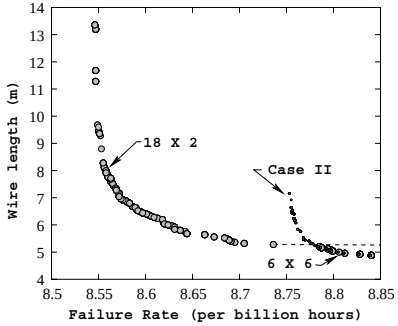
- Classical and another GA
- NSGA-II results are better






## Board Shape and Connectivity

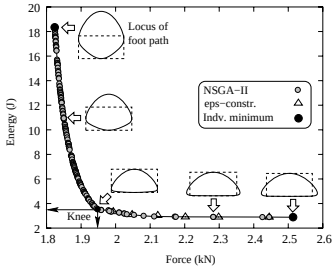
- 6X6, 9X4, 12X3, 18X2 shapes
- Combined optimization
- Difficult for any other methods

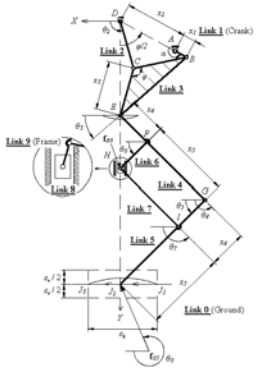





## Robotic Leg Mechanism Design

- Minimize energy
- Minimize force

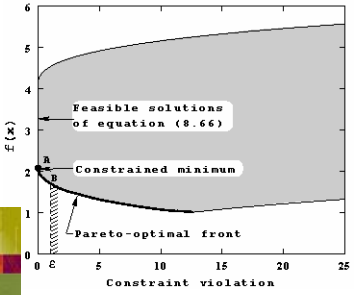




## *Multiobjectivization* for Other Optimization Tasks

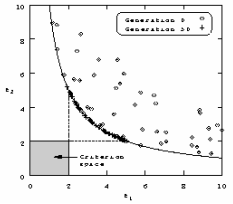
- Constrained handling
  - Constraint violations as additional objectives
- Find partial front near zero-CV
- May provide a flexible search
- A new book ->






## Goal Programming and Others

- Goal programming to find multiple solutions
- Avoids fixing a weight vector (Deb, 2001)

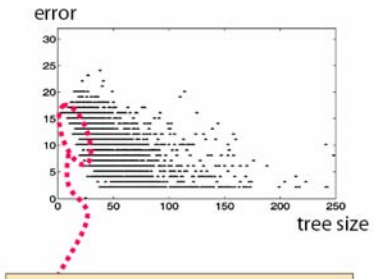


- Reducing the chance of getting trapped in local optima (Knowles et al., 2001)
- Use secondary objectives for maintaining diversity (Abbass and Deb, 2003, Jensen, 2003)



## Reducing Bloating in GP

- Bleuler et al., (2001)
- Find small-sized programs with small error
- Minimization of Size of Program as second objective




Keep and optimize small trees (potential building blocks)



## Optimization Problems with Uncertainty

- Decision variables: $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$
- Constraints restrict some solutions to be feasible

$$\begin{aligned} &\text{Min. } f(\boldsymbol{x},\boldsymbol{a}) \\ &\text{s.t. } g_j(\boldsymbol{x},\boldsymbol{b}) \geq 0 \quad j = 1,2,\ldots,J \\ &\quad\; h_k(\boldsymbol{x},\boldsymbol{c}) = 0 \quad k = 1,2,\ldots,K \\ &\quad\; x_i^L \leq x_i \leq x_i^U \; i = 1,2,\ldots,n \end{aligned}$$

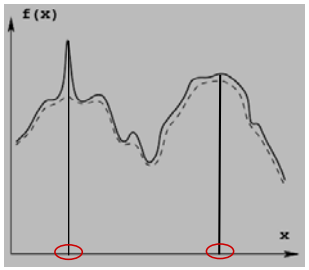
- Multiple conflicting objectives can exist
- Other problems: permutations, mixed types
- What sources of uncertainties?



## Robust Optimization:

*Handling uncertainties in variables/parameters*

- Not noise in obj/constraints
- DV/Parameters uncertain and sensitive to implementation
  - Tolerances in manufacturing
  - Material properties are uncertain
  - Loading is uncertain
- Who wants a sensitive optimum solution?




- Real Issue: How to perform EAs sub-optimally?

## Robustness in Evolutionary Optimization

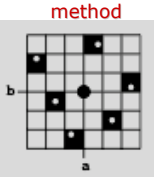
- Relate to solution quality or performance
  - Every $\boldsymbol{x}$ represents a set of neighboring solutions $\boldsymbol{y}^j \in B(\boldsymbol{x})$, $j=1,\ldots,H$
  - Good expected performance
    - Minimize $\mu_f$ instead of $f$
  - Good worst-case performance
    - Minimize (max $\boldsymbol{y}^j$)
  - Low variability in performance
    - Minimize $\sigma_f$ <- *not a good idea alone*
    - Minimize $\mu_f + k\,\sigma_f$ is a better idea



## Expected Performance, $\mu_f$

- Most popular approach (Ex., Jin and Branke, 2005)
- Assume a distribution of variation
- Compute objective variation
- Reduce to a single performance measure
- Cannot be computed analytically
- Estimation procedures
  - EA studies vary on different estimation schemes
  - Archieving, samples, meta-models

Latin Hypercube method





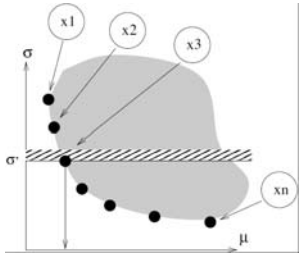

## Worst Case Performance, max $\boldsymbol{y}^j$

- Complete risk aversion
- Many research groups (Ong, Haftka, Tan)
- Difficult to find worst neighbor analytically
- Mostly simulation based
  - Nested optimization -> Minimize (maximize $\boldsymbol{y}^j$)
- Meta-modeling for a quick computation
- *Co-evolutionary method may be a good idea*
  - One population handles $\boldsymbol{x}$, other population relative neighboring points ($\boldsymbol{y}^j - x$)



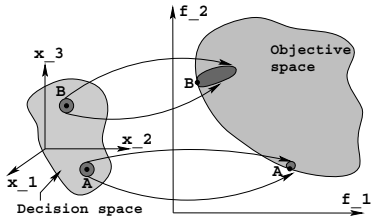
## Other Approaches

- Bi-objective approach (Avigad, Jin+Sendhoff)
  - Optimize performance and minimize its variation
    - Provide valuable information through a Pareto-optimal frontier
  - Involves a decision maker
- Good survey (Beyer and Sendhoff, 2007)



## Robust EAs for Multi-Objective Optimization

- Not all Pareto-optimal points may be robust
- A is robust, but B is not
- Decision-makers will be interested in knowing robust part of the front
- Generic decision-making idea






## Multi-Objective Robust Solutions of Type I and II (Deb and Gupta, ECJ 2004)

- Similar to single-objective robust solution of type I

$$\begin{aligned} &\textit{Minimize} && (f_1^{\text{eff}}(\mathbf{x}), f_2^{\text{eff}}(\mathbf{x}), \ldots, f_M^{\text{eff}}(\mathbf{x})), \\ &\textit{subject to} && \mathbf{x} \in \mathcal{S}, \end{aligned}$$

- Type II

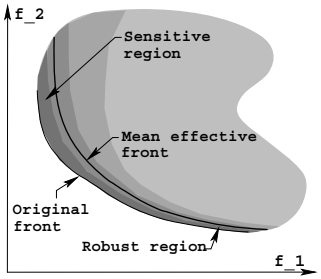
$$\begin{aligned} &\textit{Minimize} && \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})), \\ &\textit{subject to} && \frac{\|\mathbf{f}^p(\mathbf{x}) - \mathbf{f}(\mathbf{x})\|}{\|\mathbf{f}(\mathbf{x})\|} \leq \eta, \\ & && \mathbf{x} \in \mathcal{S}. \end{aligned}$$

- Other ideas can be used as well

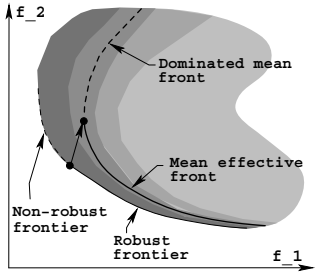


## Different Scenarios: Cases 1 and 2 (No Local Front)

Case 1: Complete Pareto-optimal front is robust



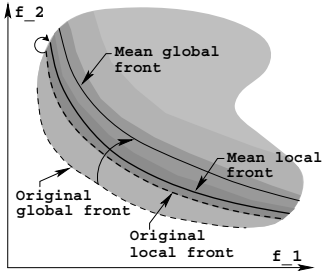

Case 2: Part of original PO front is robust



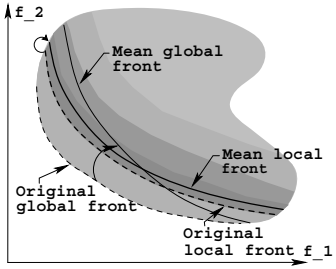



## Cases 3 and 4 (Multimodal Fronts)

Case 3: Original local PO front is robust

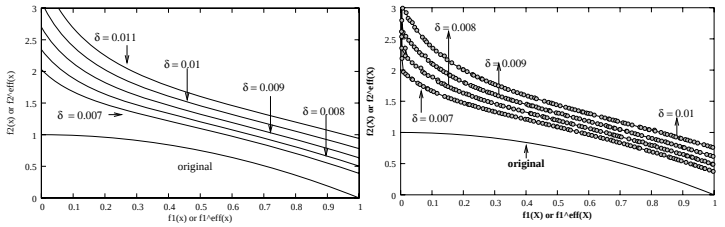



Case 4: Parts of original global and local PO fronts make robust frontier



## Type I Robustness: Effect of δ-neighborhood Size

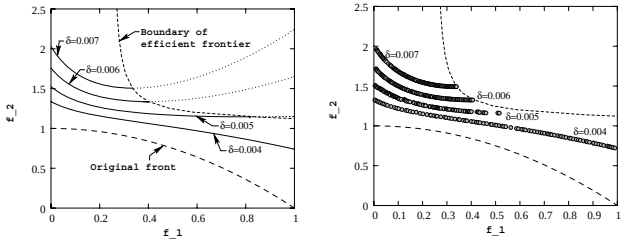
- Theory and NSGA-II simulation on Case 1
- Larger $\delta$, more shift from original front
- Some part is more sensitive than others






## Case 2: Partial Frontier is Robust

- Theory and NSGA-II simulation on Case 2
- Larger $\delta$, more shift from original front
- Some part is no more robust






## Type I Robustness may not be Practical

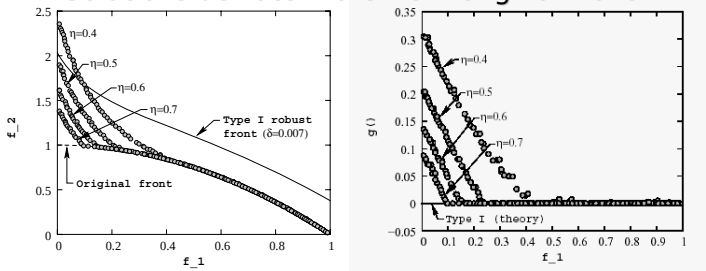
- Entirely depend on averages
- Decision-maker may like to have control on extent of robustness
- Type II is more pragmatic
- Define extent of relative perturbation ($\eta$) allowed

$$\begin{aligned} \textit{Minimize} \quad & \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})), \\ \textit{subject to} \quad & \frac{\|\mathbf{f}^p(\mathbf{x}) - \mathbf{f}(\mathbf{x})\|}{\|\mathbf{f}(\mathbf{x})\|} \leq \eta, \\ & \mathbf{x} \in \mathcal{S}. \end{aligned}$$



## Type II Robustness on Case 1

- As $\eta$ reduces, more stringent requirement for robustness
- Solutions deviate more from original front



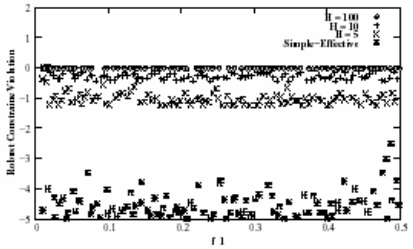
## Robustness in Presence of Constraints

- Constraints will make boundary solutions infeasible
- Naive Approach: Simply declare a solution infeasible, if any of *H* neighboring points is infeasible
  - If any $\boldsymbol{y}^j$ is infeasible, $\boldsymbol{x}$ is infeasible
- Use constraint tournament selection in NSGA-II



## Test Problem
(Deb and Gupta, ECJ 2004)

- Results sensitive to *H* (number of points used for averaging)
- Larger *H*, better satisfaction of constraints

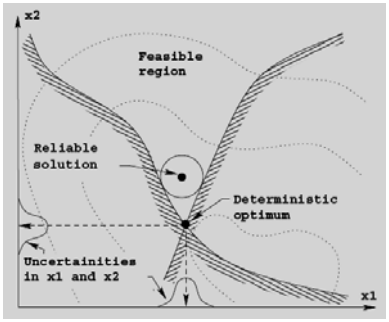



- … but more computations
- Use a reliability based approach



## Reliability-Based Optimization:
*Making designs safe against failures*

- Deterministic optimum is not usually reliable
- Reliable solution is an interior point
- *Chance* constraints with a given reliability



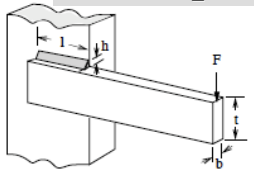

Minimize $\mu_f + k\sigma_f$
Subject to $Pr(g_j(x) \geq 0) \geq \beta_j$
$\beta_j$ is user-supplied



## An Illustrative Example

Minimize Cost
Subject to:
stress ≤ strength
deflection ≤ d_allow

- Decision variables:
  - Dimensions: *h, l, t, b*
- Parameters:
  - Material strength
  - Load *F*, weld quality, etc.




- Each (or some) vary with known distribution
- Causes objective (cost) and constraints (failure) to vary
- Optimal solution is not desired
- Specify an *overall* reliability (*R*) against failure and identify which solution has best cost w.r.t. *R*

## RBDO: Basics

- $x$: uncertain variables
- $d$: fixed variables
- $p$: uncertain parameters

Original Problem

$$\begin{aligned} \text{Minimize}_{(\mathbf{x},\mathbf{d})} \quad & f(\mathbf{x},\mathbf{d},\mathbf{p}), \\ \text{Subject to} \quad & g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0, \quad j=1,2,\ldots,J, \\ & h_k(\mathbf{d}) \geq 0, \quad k=1,2,\ldots,K, \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}, \\ & \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$

- $x$ and $p$ are assumed to vary with ($\mu$, $\sigma$)
- Functions $f$ and $g_j$ are uncertain and have distributions of variation
- Makes it difficult to use an optimization method
  - Dealing with function distributions



## RBDO Basics (cont.)

- Reliability $R$
- Revised problem
- Joint prob. $P$ difficult to compute

$$\begin{aligned} \text{Minimize}_{(\mu_{\mathbf{x}},\mathbf{d})} \quad & f(\mu_{\mathbf{x}},\mathbf{d},\mu_{\mathbf{p}}), \\ \text{Subject to} \quad & P(\textstyle\bigwedge_{j=1}^{J}(g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0)) \geq R \\ & h_k(\mathbf{d}) \geq 0, \quad k=1,2,\ldots,K, \\ & \mathbf{x}^{(L)} \leq \mu_{\mathbf{x}} \leq \mathbf{x}^{(U)}, \\ & \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$

- Most studies use $P(g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0) \geq R_j, \quad j=1,2,\ldots,J$ $R_j$=R
- System reliability versus component reliability
- Thus, the worst constraint govern the reliability constraint
- First, we discuss single-constraint strategies



## Determining a Solution's Reliability

- Probability of success against constraint satisfaction

$$P_j(\mu_{\mathbf{x}},\mathbf{d},\mu_{\mathbf{p}}) = \int_{g_j(\mathbf{x},\mathbf{d},\mathbf{p})<0} \varphi_{\mu_{\mathbf{x}},\mu_{\mathbf{p}}}(\mathbf{x},\mathbf{p}) d\mathbf{x} d\mathbf{p},$$

- Difficult to compute exactly
- Monte-Carlo simulation: $r_j$ success of $N$ scenarios:

$$1 - \frac{r_j}{N} \geq R_j$$ (Loughlin and Ranjithan, 1999)

- If $R$=0.9999, $N$>10,000 (Expensive!)
- Importance sampling: May still be large

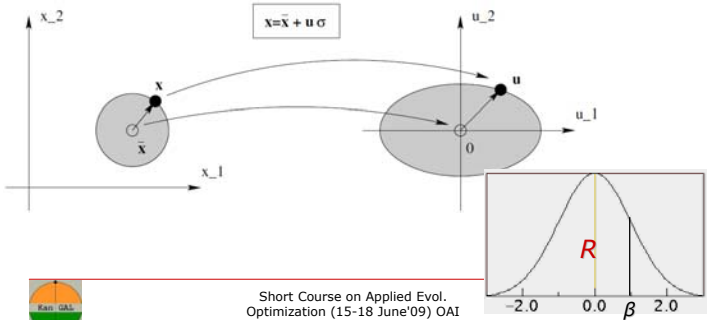


## Optimization Based Procedure: *Classical Approaches*

- Given a solution, determine the closest point on the constraint boundary
  - Most probable point (MPP)
- To generalize, computation is performed on std. normal coordinate system $U$ for every $x$
- $g_j(x,d,p)=0$ => $G_j(U)=0$
- Two optimization approaches:
  - Performance Measure Approach (PMA)
  - Reliability Index Approach (RIA)

## Standard Normal Coordinate System

- Variance is supplied
- Any vector $x$ is mapped to $u$ as follows:

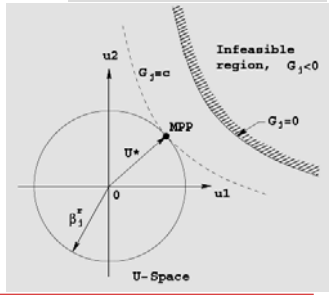





## PMA Approach (for every $x,d,p$)

- $\beta_j$ comes from $R_j$
- Find $U^*$ using an optimization method
- MPP on reliability circle
- Check if MPP is feasible
- Replace $P(g_j(x,d,p) \geq 0) \geq R_j$ with $G_j(U^*) \geq 0$

$$\begin{aligned} &\text{Minimize} && G_j(\mathbf{U}), \\ &\text{Subject to} && \|U\| = \beta_j^r, \end{aligned}$$

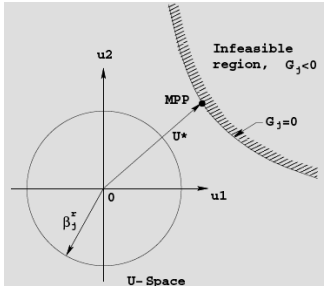





## RIA Approach (for every $x,d,p$)

- Find $U^*$ using an optimization method
- MPP on constraint
- Check if it equals or exceeds given $R$
- Replace $P(g_j(x,d,p) \geq 0) \geq R_j$ with $\|U^*\| \geq \beta_j$

$$\begin{aligned} &\text{Minimize} && \|\mathbf{U}\|, \\ &\text{Subject to} && G_j(\mathbf{U}) = 0. \end{aligned}$$



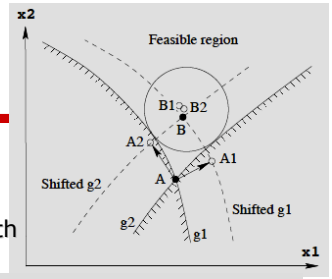



## Reliability Based Optimization Procedures

- Double-loop methods (Agarwal, 2004)
  - Nested optimization with PMA or RIA
    Computationally expensive, but accurate
- Single-loop methods (Liang et al., 2004)
  - Approximate methods of PMA/RIA
  - Computationally fast, but not accurate
- Decoupled methods (Du and Chen, 2004)
  - Sequential, repetitive application of both optimization tasks
  - A good compromise

## A Decoupled Method: SORA (Du and Chen, 2004)

- Start with deterministic optimum
- Find MPPs ($J$ lower level opt.)
- Next, original opt. problem with revised constraints



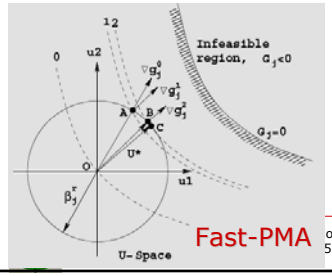

$$\begin{aligned} \underset{(\mu_{\mathbf{X}},\mathbf{d})}{\text{Minimize}} \quad & f(\mu_{\mathbf{X}},\mathbf{d},\mu_{\mathbf{p}}), \\ \text{Subject to} \quad & g_j(\mu_{\mathbf{X}}-\mathbf{s}_j^t,\mathbf{d},\mathbf{p}_{MPP,j}^t)\geq 0, \quad j=1,2,\ldots,J, \\ & h_k(\mathbf{d})\geq 0, \qquad\qquad\quad k=1,2,\ldots,K, \\ & \mathbf{x}^{(L)}\leq \mu_{\mathbf{X}}\leq \mathbf{x}^{(U)}, \\ & \mathbf{d}^{(L)}\leq \mathbf{d}\leq \mathbf{d}^{(U)}. \end{aligned}$$

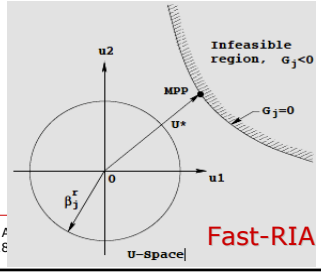
$(\mu_{\mathbf{X}}^t,\mathbf{d}^t)\downarrow$ $\qquad\uparrow\ t=t+1,\ \mathbf{s}_j^t=\mu_{\mathbf{X}}^t-\mathbf{x}_{MPP,j}^t, \quad \forall j$

For $j=1,2,\ldots,J$
PMA Approach to find MPP for $j$-th constraint $\rightarrow (\mathbf{x}_{MPP,j}^t,\mathbf{p}_{MPP,j}^t)$

## Reliability Based EAs

- For every EA solution, reliability constraints are checked using faster methods of PMA/RIA
  - Fast-PMA: Gradient of $G$ is computed at rel. circle but projected from origin
  - Fast-RIA: First Fast-PMA, then Newton's search




## Three Problem Niches for EAs

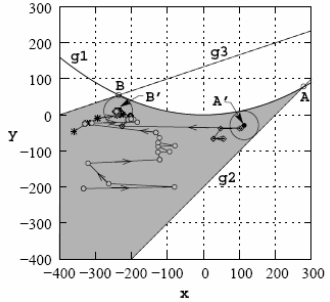
1. Finding reliable optimum in multi-modal problems
2. Enhancing knowledge of reliable solutions with increased reliability
3. Locating reliable frontier in multi-objective optimization problems



## 1. Finding Global Reliable Solution: *Single-objective reliable solution*

- Multiple deterministic optima
- Reliable solution for local is better
- Classical methods starts from deterministic global optimum
- EAs solve the overall problem and are better

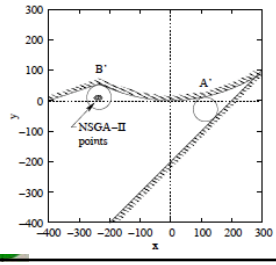
PMA approach is used



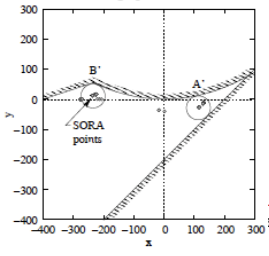
## Comparison with SORA

| Method | # success | Proportion success | Solution Evaluations | | |
|---|---|---|---|---|---|
| | | | Best | Average | Worst |
| GA ($\epsilon_{PMA} = 0.001$) | 2,500 | 100% | 9,483 | 9,756.2 | 9,768 |
| GA ($\eta_{PMA} = 2$) | 2,500 | 100% | 3,663 | 3,663.0 | 3,663 |
| SORA | 1,630 | ~65% | 491 | 1,394.0 | 15,679 |
| Double-loop | 1,219 | ~49% | 2,910 | 147,994.7 | 413,040 |

GA



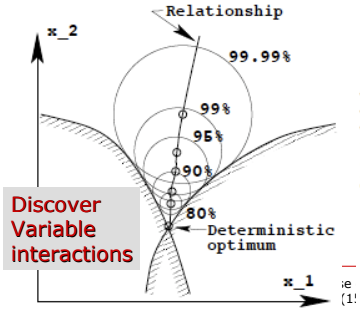

SORA

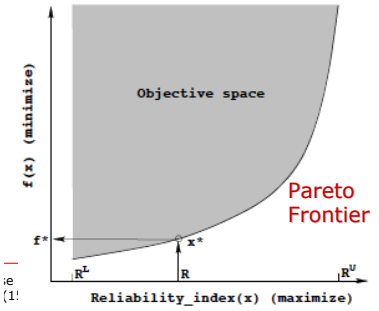



## 2. Solutions with Increased Reliability

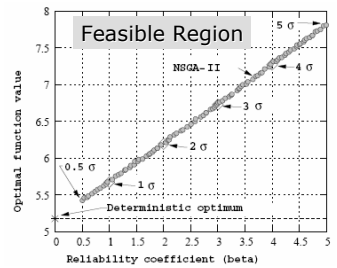
- Bi-objective problem
- Use an EMO

$$\begin{aligned} &\text{Minimize}_{(\mu_{\mathbf{X}},\mathbf{d})} && f(\mu_{\mathbf{X}},\mathbf{d},\mu_{\mathbf{p}}),\\ &\text{Maximize}_{(\mu_{\mathbf{X}},\mathbf{d})} && R(\mu_{\mathbf{X}},\mathbf{d},\mu_{\mathbf{p}})\\ &\text{Subject to} && h_k(\mathbf{d}) \geq 0,\\ & && \mathbf{x}^{(L)} \leq \mu_{\mathbf{X}} \leq \mathbf{x}^{(U)},\\ & && \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$

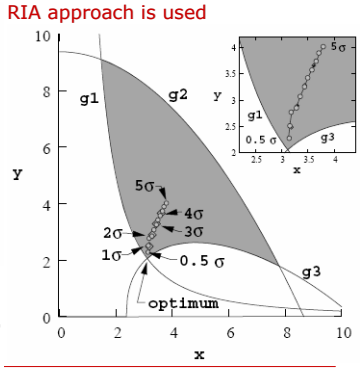



## An Example Problem: *Get a better insight*




RIA approach is used

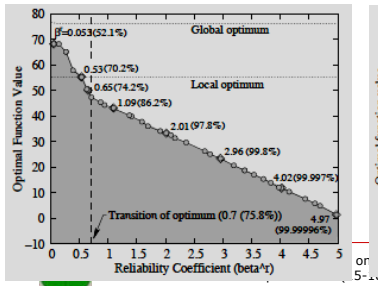


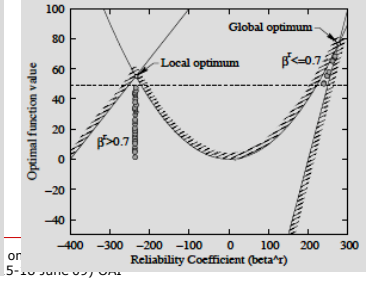

$$\begin{aligned} \text{Maximize} \quad & x+y,\\ \text{Subject to} \quad & g_1(x,y) \equiv \tfrac{1}{30}x^2y - 1 \geq 0,\\ & g_2(x,y) \equiv \tfrac{1}{30}(x+y-5)^2 + \tfrac{1}{120}(x-y-12)^2 - 1 \geq 0,\\ & g_3(x,y) \equiv \tfrac{80}{x^2+8y+5} - 1 \geq 0,\\ & 0 \leq x,y \leq 10. \end{aligned}$$



## Bi-Modal Problem Revisited

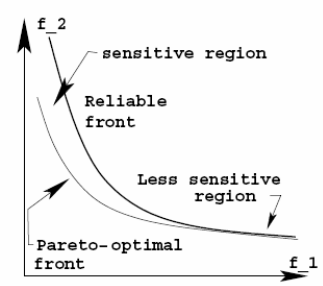
- Trade-off is clear (maximize *(f,R)*)
- After 76% reliability, local optimum is robust



## 3. Multi-Objective Reliable Frontier

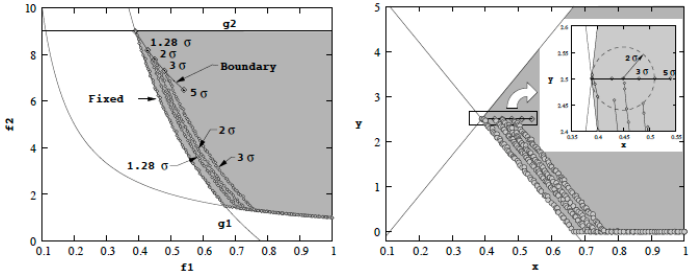
- Instead of finding deterministic Pareto-optimal front, find the reliable front
- Chance constraints
- Objectives as they are
- PMA approach is used






## A Test Problem

- PO front moves inside with more *R*
- Provides information in decision space
- Can be used a decision-making tool



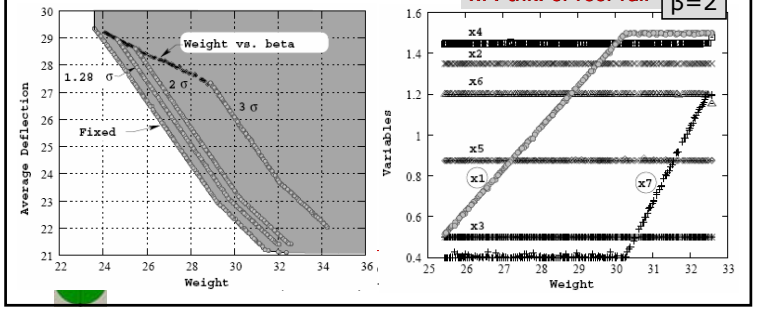



*Innovization*: Innovative design principles through optimization

## A Car Side Impact Design

- Reliable fronts show rate of movement
- What remains unchanged and what gets changed!
  - Better resource management

x1: thk. of B-Pillar inner
x7: thk. of roof rail
β=2




## Handling Joint Probability Better with Ditlevsen's Bound

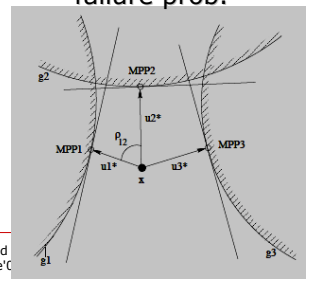
- First, find all failure prob. $P_j$
- Order from highest to lowest
- Replace chance constraint using

$$1 - \left( \sum_{i=1}^{J} P_i - \sum_{i=2}^{J} \max_{j|j<i} P_{ji} \right) \geq R$$

$$P_{ji} = \Phi(-\beta_j, -\beta_i, \rho_{ji})$$

$$\rho_{ji} = \frac{\langle \mathbf{u}_j^*, \mathbf{u}_i^* \rangle}{\|\mathbf{u}_j^*\| \, \|\mathbf{u}_i^*\|}$$

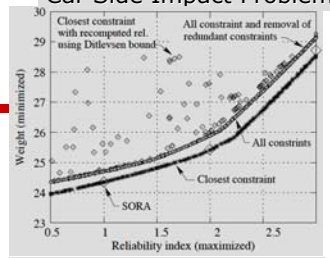
- Exact till two constraint interactions, else overestimates failure prob.

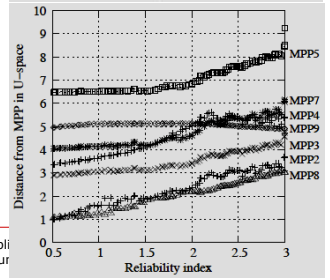


## Weight Vs. Reliability

- Contribution from all constraints make reliability tight
  - Closest constraint strategy overestimates feasibility
- No difference with redundant constraint strategy
  - MPP shows strong two constraint interaction
  - 32% saving in constraint calls

Car Side Impact Problem

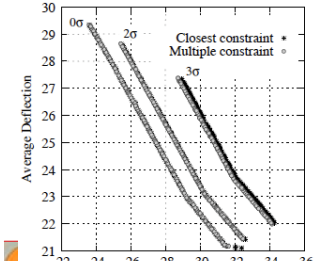


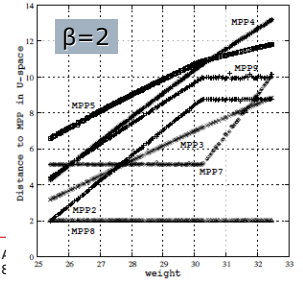






## Weight Vs. Deflection:
*Car Side Impact Problem*

- Up to β=2, identical performance
  - Almost single critical constraint
- At β=3, difference shows up

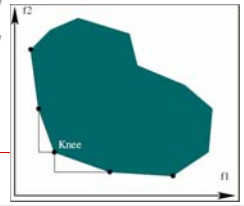





## Robustness/Reliability in Decision Making

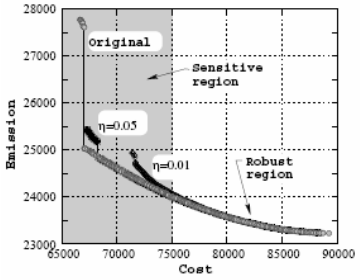
- EMO and decision making
  - An important remaining task
- Decision making usually subjective
- Collaborations with MCDM in progress
- Objective decision making
  - Select robust/reliable front, instead of PO front
  - Select knee-like regions



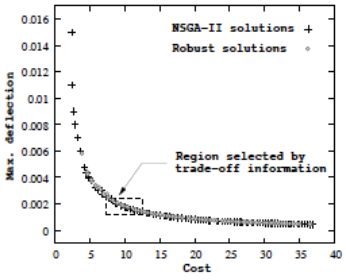



## Practical Design Problems

Hydro-electric power dispatch




Welded Beam Design



## Summary

- Practice demands handling constraints and uncertainties
  - Optimal solution is not reliable
- Monte Carlo simulation not sufficient
- Use RBDO using EAs:
  - Get parameter/variable distributions
  - Find reliable solution(s) for one or more objectives
  - Learn how reliability affects solution quality
- A pragmatic application and a definite niche for EAs





## Conclusions

- EAs have an edge in applied optimization
  - Flexibility, population approach, modularity
- Uncertainty in variables and parameters
  - Highly pragmatic and desired
- Robust and reliability are different
- Classical methods are also recent in reliability based optimization
- EAs show promise in at least three areas
- More studies are needed





## Acknowledgments & References

- Coauthors:
  - Dhanesh Padmanabhan, Sulabh Gupta, Abhishek Mall, Juergen Branke, David Daum
- General Motors R&D, Bangalore, Detroit
- Related Literature:
  - Deb, K., Padmanabhan, D., Gupta, S. and Mall. A. K. (in press). Reliability-Based Optimization Using Evolutionary Algorithms. IEEE Trans. on Evolutionary Computation
  - Deb, K., Padmanabhan, D., Gupta, S. and Mall, A. K. (2007). Reliability-Based Multi-Objective Optimization Using Evolutionary Algorithms. EMO-2007
  - Daum, D., Deb, K., and Branke, J. (2007). Reliability-based optimisation for multiple constraints with evolutionary algorithms. CEC-2007

