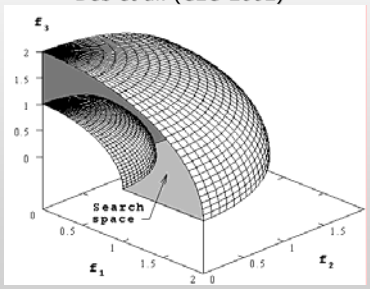
Lecture 12: Recent Issues in EMO

- Test problem design
- Hybrid EMO-Local-Search Methods
- EMO with theoretical convergence property
- Performance metrics in EMO
- Visualization of PO front
- A case study on hydro-thermal power dispatch problem

Scalable Test Problems

- **Step 1** Define Pareto-optimal front mathematically
- **Step 2** Build the objective search space using it
- **Step 3** Map variable space to objective space
- Scalable DTLZ problems suggested

Deb et al. (CEC-2002)

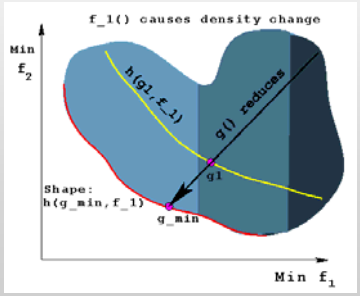


Two-Objective Test Problems

- Pareto-optimal front is controllable and known
- ZDT problems:

$$\text{Min.} \quad f_1(\mathbf{x}) = f_1(\mathbf{x}_I),$$
$$\text{Min.} \quad f_2(\mathbf{x}) = g(\mathbf{x}_{II})h(f_1, g).$$

- Choose $f_1()$, $g()$ and $h()$ to introduce various difficulties



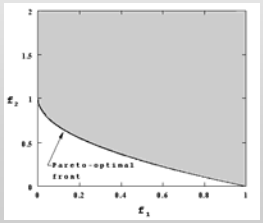
Zitzler-Deb-Thiele's Test Problems

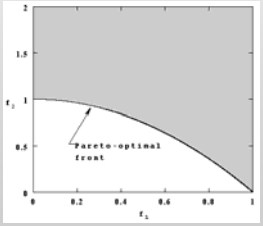
ZDT1

$$f_1(\mathbf{x}) = x_1,$$
$$g(\mathbf{x}) = 1+\frac{9}{n-1}\sum_{i=2}^{n} x_i,$$
$$h(f_1, g) = 1-\sqrt{f_1/g}.$$

ZDT2

$$f_1(\mathbf{x}) = x_1,$$
$$g(\mathbf{x}) = 1+\frac{9}{n-1}\sum_{i=2}^{n} x_i,$$
$$h(f_1, g) = 1-(f_1/g)^2.$$

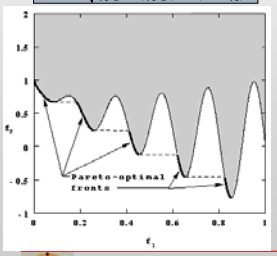




Zitzler-Deb-Thiele's Test Problems

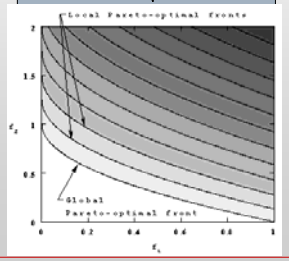
ZDT3

$$f_1 = x_1,$$
$$g = 1 + \frac{9}{n-1}\sum_{i=2}^{n} x_i,$$
$$h = 1 - \sqrt{f_1/g} - (f_1/g)\sin(10\pi f_1).$$



ZDT4

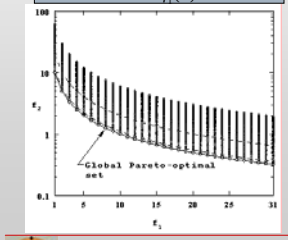
$$f_1 = x_1,$$
$$g = 10n - 9 + \sum_{i=2}^{n}(x_i^2 - 10\cos(4\pi x_i)),$$
$$h = 1 - \sqrt{f_1/g}.$$



Zitzler-Deb-Thiele's Test Problems

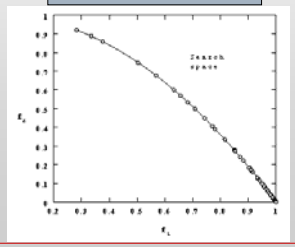
ZDT5

$$f_1 = 1 + u(x_1),\ g = \sum_{i=2}^{11} v(u(x_i))$$
$$v = \begin{cases} 2 + u(x_i) & \text{if } u(x_i) < 5, \\ 1 & \text{if } u(x_i) = 5, \end{cases}$$
$$h = 1/f_1(\mathbf{x})$$



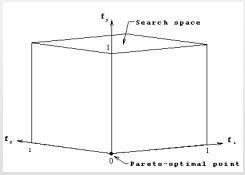
ZDT6

$$f_1 = 1 - \exp(-4x_1)\sin^6(6\pi x_1),$$
$$g = 1 + 9\left[(\sum_{i=2}^{10} x_i)/9\right]^{0.25},$$
$$h = 1 - (f_1/g)^2.$$

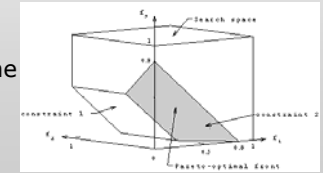


Constraint Surface Approach

- Step 1: Define a rectangular hyper-box
- Step 2: Chop off regions using constraints



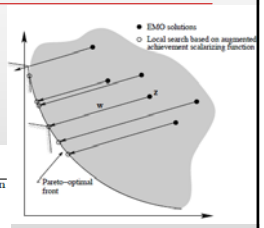
- Adv: Easy to construct
- Disadv: Difficult to define Pareto-optimal front



Hybrid EMO (Karthik, Deb & Miettinen, 2008)

- Improve EMO's convergence properties
- Introduce local search based on achievement scalarization function occasionally

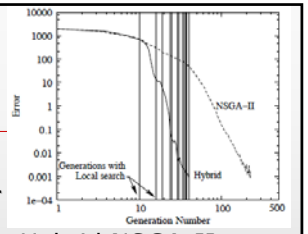
$$\text{Minimize}_{\mathbf{x}} \max_{i=1}^{M} \frac{f_i(\mathbf{X}) - z_i}{f_i^{\max} - f_i^{\min}} + \rho \sum_{j=1}^{M} \frac{f_j(\mathbf{X}) - z_j}{f_j^{\max} - f_j^{\min}}$$
$$\text{subject to } \mathbf{x} \in \mathcal{S},$$



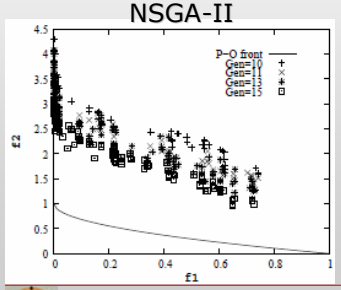
- ASF does not allow converging to weak PO points
- Periodic application of local search
  - Allow good solutions to mix well with population

Hybrid EMO (cont.)

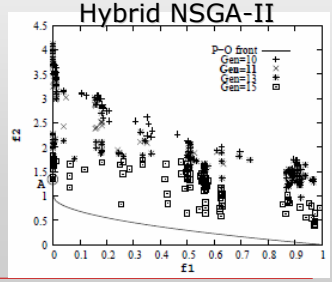
- Difference is clear
- Local search makes EMO faster



NSGA-II



Hybrid NSGA-II



Hybrid EMO

Function Evaluations

Test Problem	Original NSGA-II			Hybrid NSGA-II		
	Best	Median	Worst	Best	Median	Worst
ZDT1	19,400	20,900	24,600	**4,665**	7,554	8,580
ZDT2	20,600	21,700	23,200	**4,826**	6,351	7,198
ZDT3	21,900	23,500	26,300	**10,736**	16,731	22,137
ZDT4	27,100	34,500	60,300	**6,003**	7,658	14,479
3-DTLZ1	59,800	76,200	97,200	**48,352**	60,323	76,890
3-DTLZ2	38,200	54,000	93,800	**32,611**	61,508	69,650
4-DTLZ2	52,200	68,200	120,800	**45,005**	61,879	114,899

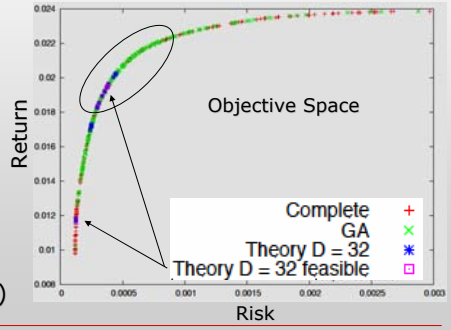
Diversity Measure (smaller, the better)

Test Problem	Original NSGA-II			Hybrid NSGA-II		
	Best	Median	Worst	Best	Median	Worst
ZDT1	0.0043	0.0047	0.0054	**0.0034**	0.0042	0.1630
ZDT2	0.0044	0.0053	0.0064	**0.0037**	0.0070	0.0499
ZDT3	0.0012	0.0016	0.0023	**0.0007**	0.0009	0.0010
ZDT4	0.0042	0.0047	0.0055	0.0042	0.1135	0.2851
3-DTLZ1	0.0196	0.0341	0.0403	**0.0187**	0.0224	0.0296
3-DTLZ2	0.0000	0.0000	0.0000	0.0000	0.0001	0.0030
4-DTLZ2	0.0049	0.0066	0.0086	**0.0048**	0.0067	0.0085

Portfolio Optimization

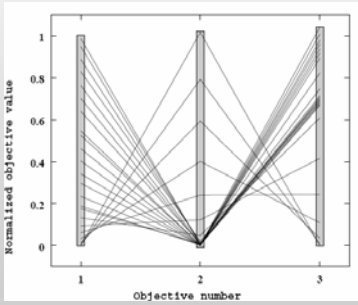
$$\min \{x^T \Sigma x\}$$
$$\max \{\mu^T x\}$$
$$s.t. \quad 1^T x = 1$$
$$x_i = 0 \text{ or } x_i \in [\alpha, \omega] \quad \alpha \geq 0$$

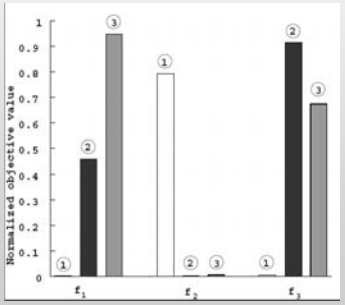
- Minimize risk, maximize return and Card(x)=D
- Complete: Assuming any x
- GA: x=0 or [α,w] and fixed D=32
- Theory D=32: Complete with D=32
- Theory D=32 feasible: D=32 & feasible (9 solns.)



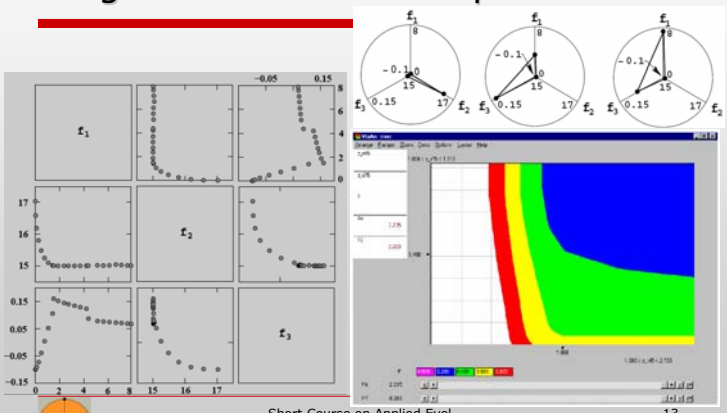
Visualization of Solutions

Value Path and Bar Diagrams



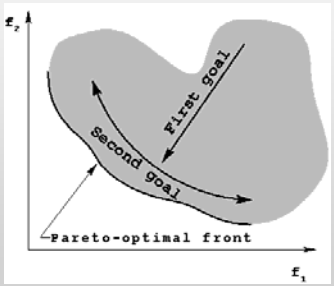


Scatter Plots, Star Coordinate Diagrams & Decision Maps



Performance Metrics

- A recent study by Zitzler et al. (2003) suggests at least *M* metrics
- Two essential metrics (functionally)
  - Convergence measure
  - Diversity measure



Metrics for Convergence

- Error ratio:

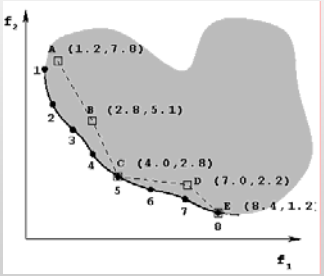
$$ER = \frac{\sum_{i=1}^{|Q|} e_i}{|Q|}$$

- Set Coverage:

$$C(A,B) = \frac{|\{b \in B \mid \exists a \in A : a \circ b\}|}{|B|}$$

- Generational distance:

$$GD = \frac{(\sum_{i=1}^{|Q|} d_i^p)^{1/p}}{|Q|}$$



Metrics for Diversity

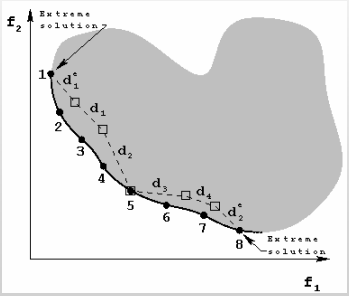
- Spacing:

$$S = \sqrt{\frac{1}{|Q|}\sum_{i=1}^{|Q|}(d_i - \bar{d})^2}$$

- Spread:

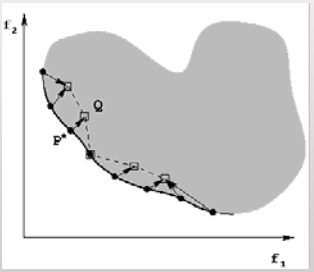
$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{|Q|} |d_i - \bar{d}|}{\sum_{m=1}^{M} d_m^e + |Q|\bar{d}}$$

- Chi-square like deviation measure

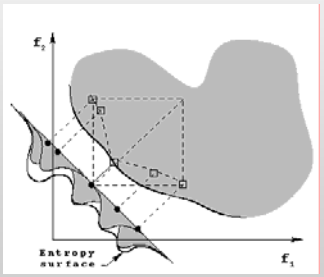


Metrics for Diversity (Cont.)

Distance from P*

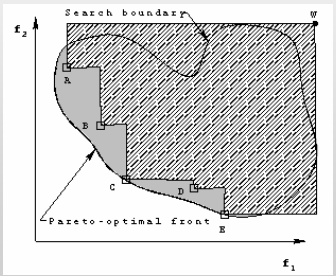


Entropy Measure

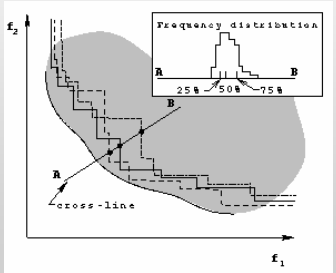


Metrics for Convergence and Diversity

Hypervolume



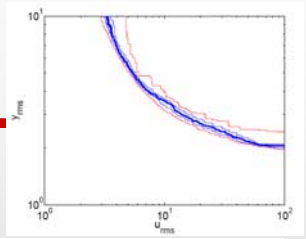
Attainment Surface Method

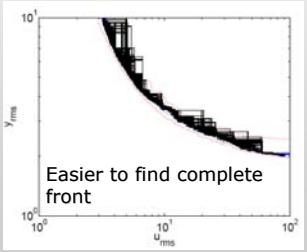


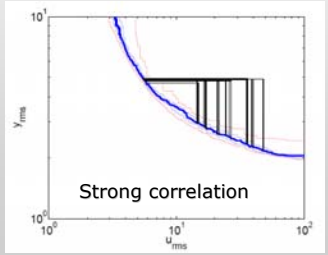
Attainment Surface Methods

- Positive and negative relationships

Fonseca (2005; 2007)



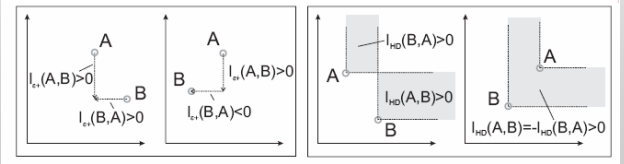




Indicator-Based EMO

- Zitzler et al. (2004)
- Move point A so as to make weakly-dominated with B
- Positive if reduction in hypervolume
- Negative, otherwise

$$I_{\epsilon}+(A,B) = \min_{\epsilon}\{\forall \boldsymbol{x}^2 \in B\ \exists \boldsymbol{x}^1 \in A\ :\ f_i(\boldsymbol{x}^1) - \epsilon \leq f_i(\boldsymbol{x}^2) \text{ for } i \in \{1,\ldots,n\}\}$$



IBEA (cont.)

- Develop a single-objective EA with indicator functions

$$F(\boldsymbol{x}^1) = \sum_{\boldsymbol{x}^2 \in P \setminus \{\boldsymbol{x}^1\}} -e^{-I(\{\boldsymbol{x}^2\},\{\boldsymbol{x}^1\})/\kappa}$$

Theorem 1. *Let I be a binary quality indicator. If I is dominance preserving, then it holds that $\boldsymbol{x}^1 \succ \boldsymbol{x}^2 \Rightarrow F(\boldsymbol{x}^1) > F(\boldsymbol{x}^2)$.*

- Smaller the fitness F, worse is the solution
- Some niche preservation is needed
- Better solutions reported



Evolutionary Optimization and Theory

- Discussions restricted to real-parameter problems
- Two pertinent questions:
  - Are EA solutions theoretically optimal?
  - Does EA gets its solutions closer to theoretically optimum?
- EAs since 1980s, but above is not addressed adequately
- Answers may attract our theoretical counterparts to EAs



KKT Necessary Conditions

A solution $\boldsymbol{x}$ is optimum,
if following conditions are true:

- At $\boldsymbol{x}$, grad of g (for active constraints) and grad of h are linearly independent
- $\boldsymbol{x}$ satisfies following two conditions:

$$\begin{array}{ll} 1) & \sum_{i=1}^{M} \lambda_i \nabla f_i(\mathbf{x}) + \sum_{j=1}^{J} u_j \nabla g_j(\mathbf{x}) + \sum_{k=1}^{K} v_k \nabla h_k(\mathbf{x}) = 0, \\ 2) & u_j g_j(\mathbf{x}) = 0 \text{ for } j = 1, \ldots, J. \end{array}$$

- $\boldsymbol{\lambda}$ non-negative, non-zero vector
- $\boldsymbol{u}$ non-negative vector



Practicalities of Using KKT conditions

- Gradients may not exist
  - Numerical gradient computation
  - Subdifferential approach can be used
- Computing $\boldsymbol{x}^*$ from KKT conditions is difficult
  - Non-linear root-finding problem
  - Unknowns: ($\mathbf{x}$, $\boldsymbol{\lambda}$, $\mathbf{u}$), usually avoided
- But KKT conditions can be checked for a $\boldsymbol{x}$
  - Linear system of equations to be solved
  - Unknowns: ($\boldsymbol{\lambda}$, $\mathbf{u}$)



Checking KKT Conditions

- Given an EA solution ***x*** in R^n
- # of active *g(**x**)* and *h(**x**)* ≤ *n*
  - Constraint qualification is usually met
- # of active *g(**x**)* and *h(**x**)* > *n*
  - Check grad *g* and *h* for linear independance
- Consider only active constraints *g*
  - *Takes care of condition 2*

Remaining Tasks

- Find (**λ**, **u**)≥**0** and **λ**≠**0** for which

$$\sum_{i=1}^{M} \lambda_i \nabla f_i(\mathbf{z}^{(i)}) + \sum_{j=1}^{J} u_j \nabla g_j(\mathbf{z}^{(i)}) + \sum_{k=1}^{K} v_k \nabla h_k(\mathbf{z}^{(i)}) = 0.$$

- Assume, $\Sigma\lambda_i=1$ and substitute $\lambda_M = 1 - \sum_{i=1}^{M-1} \lambda_i$
- Above equations: *Ay=b*

$$\begin{aligned} A &= ((\nabla f_1 - \nabla f_M), \ldots, (\nabla f_{M-1} - \nabla f_M), \\ &\quad \nabla g_1, \ldots, \nabla g_J, \nabla h_1, \ldots, \nabla h_k), \\ b &= -\nabla f_M, \\ y &= (\lambda_1, \ldots, \lambda_{M-1}, u_1, \ldots, u_J, v_1, \ldots, v_K)^T. \end{aligned}$$

(n x p), p=M+J+K-1

(n x 1)

(p x 1)

Three Scenarios

- n>p: More equations than unknowns
  - Least square solution: $\bar{y} = (A^T A)^{-1}(A^T b)$
  - Define an error measure: $\bar{e} = \|b - A\bar{y}\|/\|b\|$
  - If error is close to zero, ***x*** is a KKT point
- n<p: Less equations than unknowns
  - Pose as an optimization problem:

$$\begin{aligned} \text{Minimize} \quad & e(\mathbf{y}) = \|b - A\mathbf{y}\|/\|b\|, \\ \text{Subject to} \quad & \mathbf{y} \geq 0, \end{aligned}$$

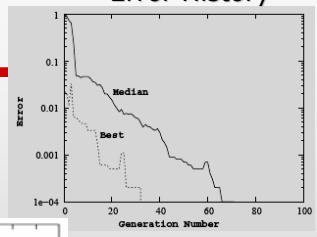
  - If *e** is close to zero, ***x*** is a KKT point
- n=p: # of equations and unknowns are same
  - Compute: $\bar{y} = A^{-1}b$ $e(\bar{y}) = \|b - A\bar{y}\|/\|b\|$
  - If *e* is close to zero, ***x*** is a KKT point

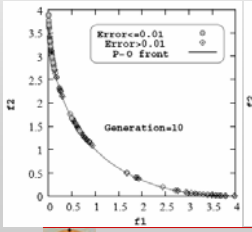
Simulation Results

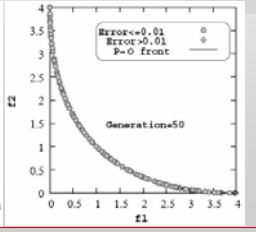
$$\begin{aligned} \text{Minimize} \quad & f_1(x, y) = x^2 + y^2, \\ \text{Minimize} \quad & f_2(x, y) = (x-2)^2 + y^2. \end{aligned}$$

- More and more solutions become KKT points with generation

Error History



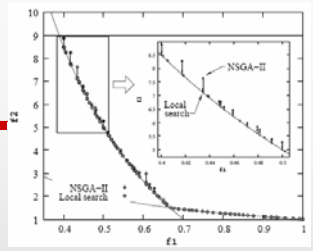




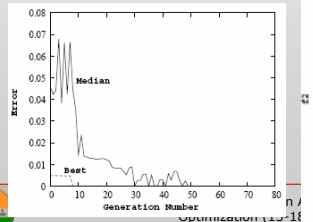
- Can potentially indicate which regions are difficult to converge
- Termination crietrion

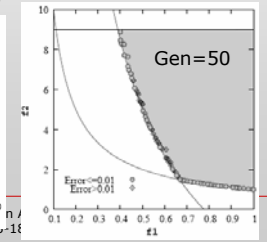
A Constrained Problem

$$\begin{aligned}\text{Minimize} \quad & f_1(\mathbf{x}) = x_1, \\ \text{Minimize} \quad & f_2(\mathbf{x}) = \frac{1+x_2}{x_1}, \\ \text{subject to} \quad & g_1(\mathbf{x}) \equiv x_2 + 9x_1 \geq 6, \\ & g_2(\mathbf{x}) \equiv -x_2 + 9x_1 \geq 1, \\ & 0.1 \leq x_1 \leq 1, \quad 0 \leq x_2 \leq 5.\end{aligned}$$



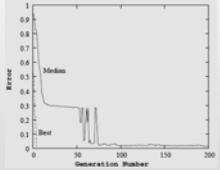
- Constrained part is difficult to converge
- Local search after 50th generation makes zero error

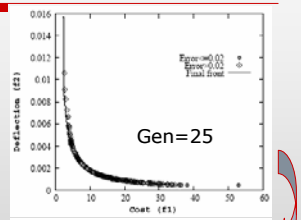


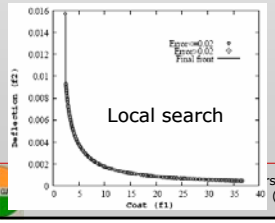


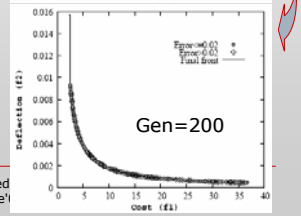
Welded Beam Design

Improvement with generation









Welded Beam Design (cont.)

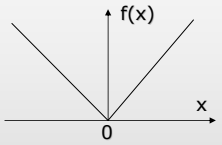
- All solutions have zero or small error
- Largrange multipliers are non-negative
- Minimum cost solution is relatively difficult to find

h	ℓ	t	b	f_1	f_2	Error	λ_1	u_1	u_3	u_4	u_t	u_b
0.244	6.214	8.298	0.244	2.381	0.016	3.780e-04	1.054e-01	0.0627	0.0437	0.000	0.145	
0.235	5.178	10.000	0.235	2.486	0.009	0.000e+00	5.891e-03	0.0071	0.0035	1.0e-06	0.022	
0.310	3.606	10.000	0.310	3.003	0.007	0.000e+00	3.193e-03	0.0036	0.0013		0.016	
0.438	2.350	10.000	0.438	3.942	0.005	0.000e+00	1.541e-03	0.0018	0.0003		0.011	
0.653	1.470	10.000	0.714	6.009	0.003	0.000e+00	5.780e-04	0.0008			0.007	
0.857	1.075	10.000	1.263	10.033	0.002	0.000e+00	1.900e-04	0.0003			0.004	
1.139	0.777	10.000	2.235	17.006	0.001	0.000e+00	6.200e-05	0.0001			0.002	
1.341	0.644	10.000	3.070	22.911	0.001	0.000e+00	3.300e-05	7.7e-05			0.001	
1.558	0.542	10.000	4.088	30.055	0.001	0.000e+00	1.900e-05	4.9e-05			0.001	
1.223	0.751	10.000	5.000	36.723	0.000	0.000e+00	0.000e+00				0.001	0.0004

Handling Non-differentiable Functions

- Clarke's subdifferential:
  - $\delta^c f(x=0) = [-1,1]$
- First KKT condition:

$$0 \in \sum_{i=1}^{M} \lambda_i \partial^c f_i(\mathbf{x}) + \sum_{j=1}^{J} u_j \partial^c g_j(\mathbf{x})$$



- Our approach: For every **λ** and **u**
  - Define: $s_i \in \partial^c f_i(\mathbf{x})$ and $t_j \in \partial^c g_j(\mathbf{x})$ are additional variables
  - Solve:

$$\begin{aligned}\text{Minimize} \quad & |e(\mathbf{s},\mathbf{t})|, \\ \text{Subject to} \quad & s_i \in \partial^c f_i(\mathbf{x}), \\ & t_j \in \partial^c g_j(\mathbf{x}).\end{aligned}$$

where $e(\mathbf{s},\mathbf{t}) = A(\mathbf{s},\mathbf{t})y - b(\mathbf{s})$

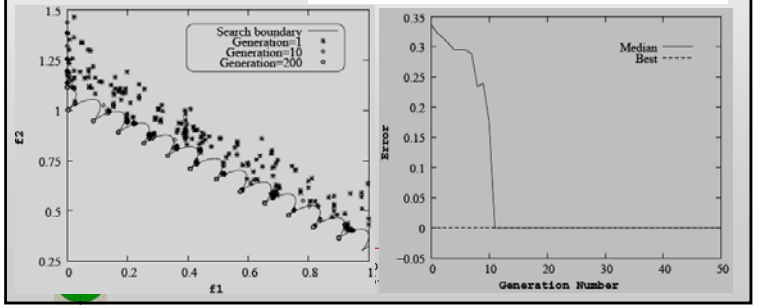
  - If e^* is close to zero, ***x*** is a KKT point

Simulation Results

- CTP3 problem: (Kinks at optima)

$$
\begin{aligned}
\text{Min.} \quad & f_1(\mathbf{x}) = x_1, \\
\text{Min.} \quad & f_2(\mathbf{x}) = g(\mathbf{x})\left(1 - \frac{f_1(\mathbf{x})}{g(\mathbf{x})}\right), \\
\text{s.t.} \quad & C(\mathbf{x}) \equiv \cos(\theta)\left[f_2(\mathbf{x}) - e\right] - \sin(\theta) f_1(\mathbf{x}) \geq \\
& \quad a\left|\sin\left\{b\pi\left[\sin(\theta)(f_2(\mathbf{x}) - e) + \cos(\theta) f_1(\mathbf{x})\right]^c\right\}\right|^d, \\
\text{where} \quad & g(\mathbf{x}) = 1 + \sum_{i=2}^{n}(x_i - 0.5)^2, \\
& 0 \leq x_i \leq 1, \quad i = 1, 2, \ldots, n.
\end{aligned}
$$

$\theta = -0.2\pi,\ a = 0.1,\ b = 10,\ c = 1,\ d = 0.5,\ e = 1.$



CTP3 Problem (cont.)

- Population at 200 generation
- s within [-1,1], $\boldsymbol{\lambda}$ and $\boldsymbol{u}$ non-negative
- All $\boldsymbol{x}$ are KKT points

f_1	f_2	Error	s	λ_1	u	u_{x_1}
0.000	1.114	0.000	0.350	0.692	0.321	0.496
0.003	1.004	0.000	−0.600	1.000	1.000	0.000
0.087	0.949	0.000	0.000	0.385	0.769	
0.170	0.892	0.000	0.000	0.385	0.769	
0.254	0.838	0.000	0.050	0.612	0.716	
0.333	0.776	0.000	0.000	0.385	0.769	
0.408	0.710	0.000	0.000	0.385	0.769	
0.494	0.658	0.000	−0.050	0.679	0.700	
0.573	0.597	0.000	0.050	0.767	0.679	
0.655	0.539	0.000	−0.050	0.711	0.693	
0.734	0.479	0.000	0.000	0.385	0.769	
0.815	0.420	0.000	−0.050	0.775	0.677	
0.901	0.367	0.000	0.050	0.867	0.656	

A Hydro-Thermal Power Dispatch Problem

- Thermal power generation P^s_{sm} and hydroelectric power generation P^h_{hm} are variables

$$
\begin{aligned}
\text{Minimize} \quad & f_1(\mathbf{x}) = \sum_{m=1}^{M}\sum_{s=1}^{N_s} t_m[a_s + b_s P^s_{sm} + c_s(P^s_{sm})^2 + |d_s \sin(e_s(P^s_{s,\min} - P^s_{sm}))|], \\
\text{Minimize} \quad & f_2(\mathbf{x}) = \sum_{m=1}^{M}\sum_{s=1}^{N_s} t_m[\alpha_s + \beta_s P^s_{sm} + \gamma_s(P^s_{sm})^2 + \eta_s \exp(\delta_s P^s_{sm})], \\
\text{subject to} \quad & \sum_{s=1}^{N_s} P^s_{sm} + \sum_{h=1}^{N_h} P^h_{hm} - P_{Dm} - P_{Lm} = 0, \quad m = 1,2\ldots,M, \\
& \sum_{m=1}^{M} t_m(a_{0h} + a_{1h}P^h_{hm} + a_{2h}(P^h_{hm})^2) - W_h = 0, \quad h = 1,2,\ldots,N_h. \\
& P^h_{h,\min} \leq P_{hm} \leq P^h_{h,\max}, \quad h = 1,2,\ldots,N_h, m = 1,2,\ldots,M, \\
& P^s_{s,\min} \leq P_{sm} \leq P^s_{s,\max}, \quad s = 1,2,\ldots,N_s, m = 1,2,\ldots,M.
\end{aligned}
$$

- Minimize Cost (non-differentiable) and NOx emission
- Power balance and water head limits
- Known power demand P_{Dm} with time m

GA Coding and Constraint Handling

- Four thermal, two hydroelectric, M time steps

$$((\underbrace{P^h_{11}\,P^h_{21}}_{\mathcal{H}_1}\ \underbrace{P^s_{11}P^s_{21}P^s_{31}P^s_{41}}_{\mathcal{T}_1})(\underbrace{P^h_{12}\,P^h_{22}}_{\mathcal{H}_2}\ \underbrace{P^s_{12}P^s_{22}P^s_{32}P^s_{42}}_{\mathcal{T}_2})\cdots$$

- Two sets of constraints: $(\underbrace{P^h_{1M}\,P^h_{2M}}_{\mathcal{H}_M}\ \underbrace{P^s_{1M}P^s_{2M}P^s_{3M}P^s_{4M}}_{\mathcal{T}_M}).$
  - Handle water availability constraint first
  - Quadratic constraint, repair to find $P^h_{h\mu}$

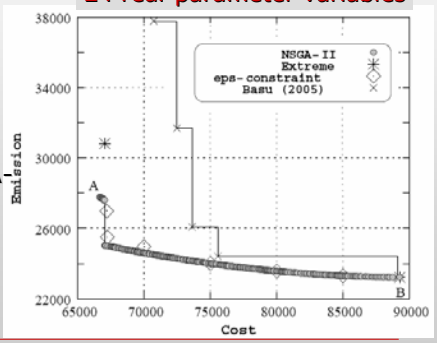
$$(P^h_{h\mu})^2 + \frac{a_{1h}}{a_{2h}}P^h_{h\mu} + \frac{1}{t_\mu a_{2h}}\left(-W_h + a_{0h}T + \sum_{\substack{m=1\\ m\neq\mu}}^{M} t_m a_{1h} P^h_{hm} + \sum_{\substack{m=1\\ m\neq\mu}}^{M} t_m a_{2h}(P^h_{hm})^2\right) = 0.$$

  - If not within bounds, penalize
  - Similarly, handle other constraint set

NSGA-II Simulation and Verifications

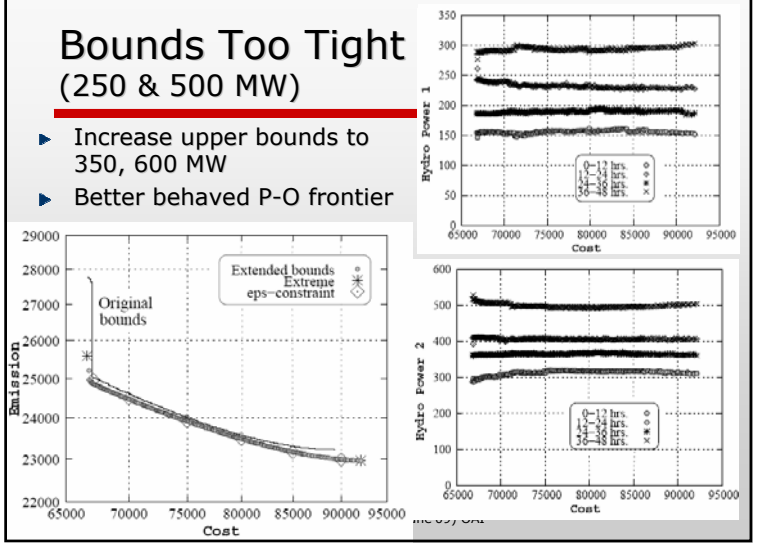
- Single-objective opt. for extreme solns.
- Intermediate ε-constraint optimizations
- Better than a SA-approach with naive penalty
- Min-cost soln. difficult to optimize

4 time steps of 12 hr. each, 24 real-parameter variables



Bounds Too Tight (250 & 500 MW)

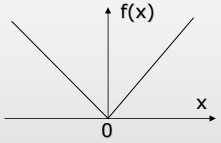
- Increase upper bounds to 350, 600 MW
- Better behaved P-O frontier



Handling Non-differentiable Functions

- Clarke's subdifferential:
  - $\delta^c f(x=0) = [-1,1]$
- First KKT condition:

$$0 \in \sum_{i=1}^{M} \lambda_i \partial^c f_i(\mathbf{x}) + \sum_{j=1}^{J} u_j \partial^c g_j(\mathbf{x})$$



- Our approach: For every **λ** and **u**
  - Define: $s_i \in \partial^c f_i(\mathbf{x})$ and $t_j \in \partial^c g_j(\mathbf{x})$ are additional variables
  - Solve:

$$\begin{array}{ll} \text{Minimize} & |e(\mathbf{s},\mathbf{t})|, \\ \text{Subject to} & s_i \in \partial^c f_i(\mathbf{x}), \\ & t_j \in \partial^c g_j(\mathbf{x}). \end{array}$$

where $e(\mathbf{s},\mathbf{t}) = A(\mathbf{s},\mathbf{t})y - b(\mathbf{s})$

  - If e^* is close to zero, $\boldsymbol{x}$ is a KKT point

Verify Obtained Solutions Through KKT Conditions

$$\begin{array}{rl} \lambda_1 \nabla f_1(\mathbf{x}) + \lambda_2 \nabla f_2(\mathbf{x}) + \sum_{k=1}^{M} v_k^{(1)} \nabla h_k^{(1)}(\mathbf{x}) + \sum_{l=1}^{N_h} v_l^{(2)} \nabla h_l^{(2)}(\mathbf{x}) + & \\ \sum_{j=1}^{M(N_h+N_s)} u_j^{(L)} \nabla g_j^{(L)}(\mathbf{x}) + \sum_{j=1}^{M(N_h+N_s)} u_j^{(U)} \nabla g_j^{(U)}(\mathbf{x}) & = 0, \\ h_k(\mathbf{x}) & = 0, \\ g_j(\mathbf{x}) & \leq 0, \\ u_j g_j(\mathbf{x}) & = 0, \\ u_j & \geq 0. \end{array}$$

- f_1 broken into two parts, s_i in [-1,1]

$$\nabla (f_1)_i^1(\mathbf{x}) = \begin{cases} t_m (b_s + 2c_s x_i), & \text{for } i \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases}$$

$$\nabla (f_1)_i^2(\mathbf{x}) = \begin{cases} -s_i \left(t_m e_s d_s \cos(e_s (x^{\min} - x_i)) \right), & \text{for } i \in \mathcal{T}, \\ 0, & \text{otherwise.} \end{cases}$$

- KKT conditions reduce to following:

$$0 \in \lambda_1 \left(\nabla f_1^1(\mathbf{x}) + s_i D_i \right) + \lambda_2 \nabla f_2(\mathbf{x}) + \sum_{k=1}^{M} v_k^{(1)} \nabla h_k^{(1)}(\mathbf{x}) + \sum_{k=1}^{N_h} v_k^{(2)} \nabla h_k^{(2)}(\mathbf{x})$$

Error Minimization and KKT Results

- Solutions are close to KKT points, if (i) error is close to zero, (ii) λ≥0, (iii) s_i in [-1,1]

$$\begin{aligned} \text{Minimize}_{\mathbf{s}} \quad & \tilde{e}(\mathbf{s}) = \|b - A(\mathbf{s})\ \bar{y}(\mathbf{s})\|/\|b\|, \\ \text{subject to} \quad & s_i \in [-1,1], \quad \forall i. \end{aligned}$$

Soln.	(f_1, f_2)	$\|s\|$	i near kink	s	$\tilde{e}(s)$	λ_1/λ_2
Feasible Solutions at Final Generation						
1	(92120.45, 22963.09)	0	–	–	0.0024	0.000
2	(89929.82, 22991.40)	0	–	–	0.0169	0.019
3	(84997.68, 23169.00)	2	(5,21)	(0.99, -0.60)	0.0141	0.051
4	(79966.72, 23488.10)	4	(5,9,17,21)	(0.69, -0.92, 0.87, -0.38)	0.0126	0.077
5	(75023.50, 23920.95)	7	(4,5,9,11, 17,21,24)	(0.88, 0.45, -0.86, 0.88, 0.56, -0.63, -0.90)	0.0207	0.099
6	(70018.99, 24467.52)	11	(3,4,5,9, 11,12,13,15, 17,21,24)	(-0.85, 0.92, 0.45, -0.52 0.89, -0.92, -0.86, 0.90 0.57, -0.20, -0.55)	0.0241	0.125

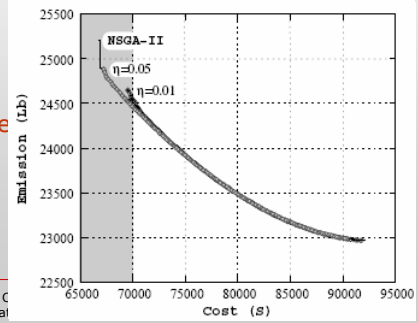
Solutions in Earlier Generations are Not Close to KKT Points

- λ<0 and/or high error

Soln.	(f_1, f_2)	$\|s\|$	i near kink	s	$\tilde{e}(s)$	λ_1/λ_2
Feasible Solutions at Generation 10						
1	(83900.92, 24760.04)	0	–	–	0.2915	0.066
2	(82273.16, 25924.39)	1	(12)	(-1)	0.3469	0.051
3	(81281.44, 26298.55)	4	(5,6,9,22)	(-1,1,1,1)	0.3512	−0.078
Feasible Solutions at Generation 30						
1	(83145.39, 23610.54)	1	(6)	(-1)	0.1265	0.056
2	(73140.35, 24720.39)	4	(3,6,12,18)	(-0.57,1.00,1.00,1.00)	0.1931	0.097
3	(71869.35, 25234.02)	7	(5,6,11,12 16,18,23)	(1.00 -0.64 0.31 -1.00 1.00,-1.00,1.00)	0.2338	0.105
Feasible Solutions at Generation 70						
1	(81952.12, 23425.20)	2	(3,23)	(-0.42, 1.00)	0.0515	0.065
2	(74994.34, 24045.11)	4	(6,11,15,17)	(-1.00,0.42,-0.81,0.88)	0.0713	0.096
3	(70424.45, 24922.91)	8	(5,6,11,16 17,18,23,24)	(-0.93,-0.46,0.52,1.00 1.00,-1.00,1.00,-1.00)	0.1459	0.107

Investigating Minimum-Cost Region: Robust Frontier

- Generator values are uncertain
  - Power generation ±5 MW
  - Type-II robustness
  - Max_i $\Delta f_i/f_i \le \eta$
  - Less η means more strict requirement
  - Min-cost solutions are sensitive
    - Prefer near minimum-emission solutions

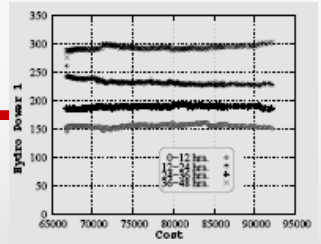


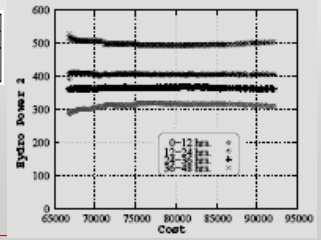
Innovizations

- Hydroelectric power generation almost constant
- Higher power demand, more generation of hydroelectric power

	0-12 Hrs.	12-24 Hrs.	24-36 Hrs.	36-48 Hrs.
Demand P_{dm} (MW)	900.00	1100.00	1000.00	1300.00
P^h_{1m} (MW)	155.45	232.70	189.29	293.39
P^h_{2m} (MW)	311.68	406.30	364.37	498.69

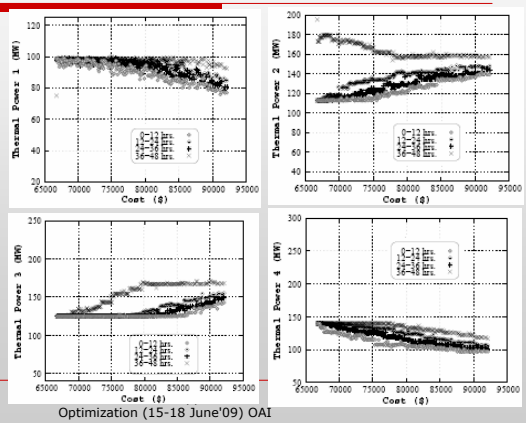
- Thermal power causes trade-off
- Other details in original study





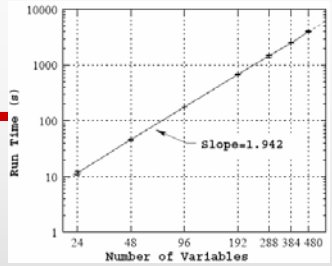
Further Innovizations

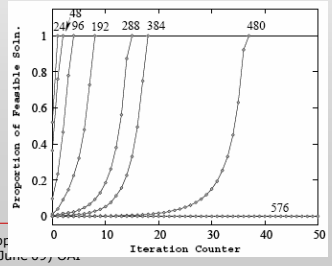
- Smaller cost with higher Ts1 and Ts4, and smaller Ts2 and Ts3
- Smaller range of optimal values



Scale-up Study

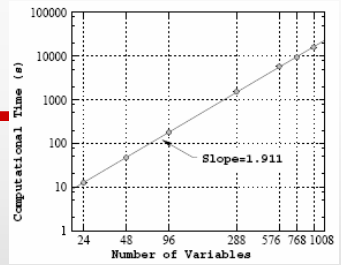
- More time steps
- *N=8n*
- Polynomial scaling up to 480 real variables
- Feasible solutions are difficult to find for higher number variables
- Need problem information through customization

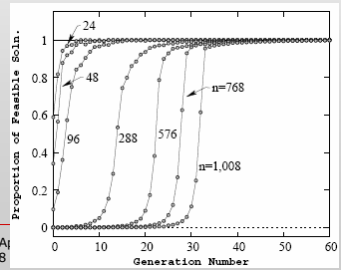




Scale-up Study (cont.)

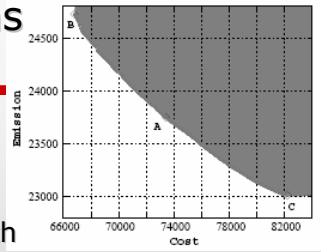
- Problem information in generating initial population
- One solution with equal Ts and Th for each time period
- Feasible solutions are easy to find
- Performance is better (1,008 real variables, *17.2 minutes vs. 12 hourly change in demand*)

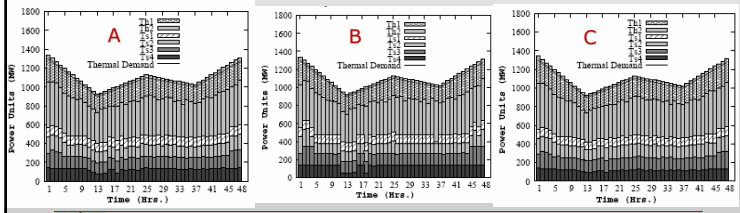




Trade-off Solutions (4 Time Steps)

- Demand is met
- Min-cost solution is abrupt and isolated
- Min-emission sol. smooth





Conclusions

- Hybrid EMOs are fast and reliable
  - Using local search methods
  - Using theoretical optimality conditions
- Performance metrics are important for termination
- Credible applications using various optimization methods
  - Way forward for practical applications



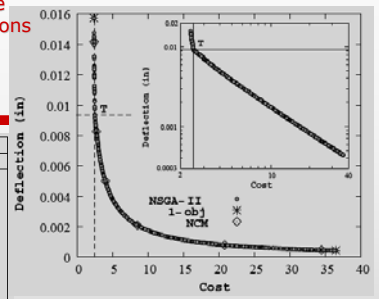
Credible EA Applications

- Why would an EA be necessary for the problem?
  - Failure of usual methods, unsatisfactory solutions, demonstrate that an EA work as well
- What features of an EA make the application worth and unique?
  - Power of recombination, flexibility, scalability, parallel use, hybrid, interactive methodologies, etc.
- How can near-optimality be established?
  - Restricted use, but optimization theory may help
- Does the application provide information beyond finding one or more optimal solutions?



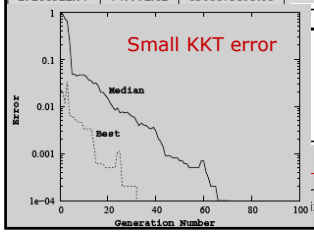
Build Up Confidence

Multiple optimizations



Multiple runs

$P(x^*)$ (€)	A_C (m²)	E (kJ)	P_{AUX} (kW)
29201019.61	749999.99	6346926947.98	92768.27
29201018.75	749999.94	6346929408.75	92768.26
29201018.62	749999.94	6346929669.93	92768.26
29201018.62	749999.94	6346929478.79	92768.27
29200967.67	749997.61	6347055515.19	92768.46
29200967.42	749997.62	6347055515.19	92768.46
29200957.23	749997.01	6347087739.59	92768.27
29200953.07	749997.00	6347092303.42	92768.31
29200922.51	749999.96	6346929711.66	92777.49
29200522.97	749992.62	6318673893.16	92768.70



Case	Best	Median	Worst
1	4.939027	7.183082	12.084255
2	3.820984	8.899963	14.298933
3	2.442714	3.834121	7.444246
4	2.381191	2.392892	2.645833
5	2.381187	2.392025	2.645833
6	2.381447	2.382634	2.383554

Consistent performance

Summary

- Evolutionary algorithms allow flexibility and practical considerations
- Customization using problem information
  - Requires domain-specific expertise, optimization expertise and computing expertise
- Optimization, for not only finding optimal solutions
  - Better understanding of problems through optimization and analysis
- EAs hold promise, but demonstrating it needs careful and credible research

