Short Course on Applied Optimization

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Course Structure

- 1.5 hours of lecture per session
- 3 sessions per day
- 1 hour of tutorials/reviews at the end of the day
- Ask questions/comments at any time during the lectures

Course Structure (cont.)

- 15 June, 2009:
  - Optimization and its scope in practice
  - Classical optimization methods and practical difficulties
  - Introduction to *Evolutionary Optimization algorithms (EAs)*
    - Binary-coded GA and case studies
- 16 June, 2009:
  - Constraint Handling, theoretical underpinnings
  - Real-parameter EAs
    - DE, PSO, ES, etc.
  - EAs for more complex optimization tasks
    - Multi-modal EAs, meta-model assisted EAs, mixed integer EAs

Course Structure (cont.)

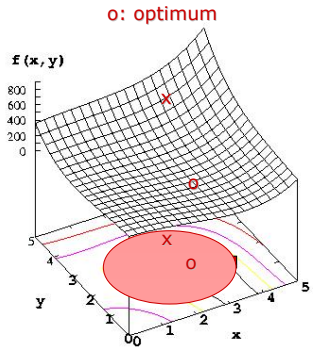
- 17 June, 2009:
  - Evol. Multi-objective optimization (EMO) – Introduction
  - Advanced techniques
    - Constraint handling, uncertainty handling, reliability considerations
  - Knowledge discovery through EMO
- 18 June, 2009:
  - EMO and decision-making
  - Recent practical methods:
    - Bilevel optimization, dynamic optimization
  - Case Studies and Conclusions

Lecture 1: Optimization and Its Need in Practice

- Course Description
- What is optimization?
- Scope of optimization in practice
- Steps in an optimization task
  - Objectives, Constraints and Variables
- Fundamental optimality conditions

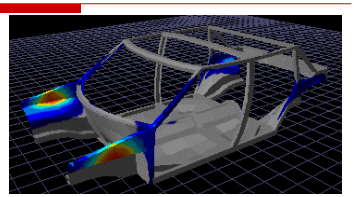
What is Optimization?

- A task of searching for a set of *decision variables* which would minimize or maximize one or more *objective* functions subject to satisfying *constraints*
- Decision variables: *(x,y)*
- Objective function: *f(x,y)*
- Constraint function: Equality or inequality type



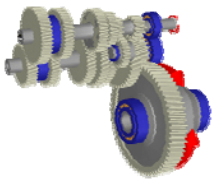
Scope of Optimization in Practice

- *Optimal design & manufacturing* for desired goals
- Major application in engineering



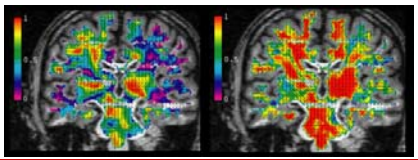






Scope of Optimization (cont.)

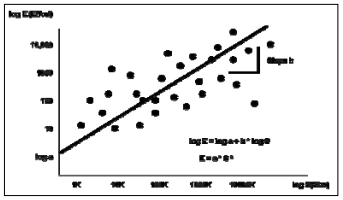
- *Inverse Problems*
  - Output known, find input
  - Often with a goal: minimize distortion, maintain physics, occum's razor (simplest) etc.
  - Tomography, reconstruction, 3D from 2D images
  - Lead to multiple solutions



Reverse current in brain models (Johnson, 2006)

Scope of Optimization (cont.)

- *Parameter optimization* for optimal performance

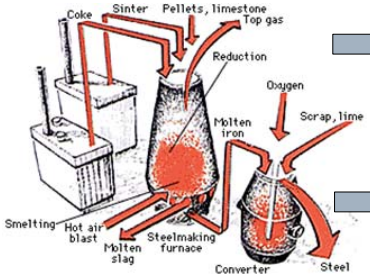


- Scientific experiments, computer experiments

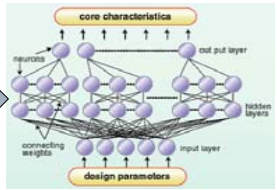


Scope of Optimization (cont.)

- *Modeling* (system, process)



$$(1)\ \frac{\partial P}{\partial t} = -\gamma P + \beta \lambda I_A - \kappa P S_A$$
$$(2)\ \frac{\partial I_A}{\partial t} = r I_A - g_A I_A + \kappa P S_A - \lambda I_A + m b_{B \to A} I_B - m b_{A \to B} I_A$$
$$(3)\ \frac{\partial S_A}{\partial t} = r S_A - g_A S_A - \kappa P S_A + m b_{B \to A} S_B - m b_{A \to B} S_A$$
$$(4)\ \frac{\partial I_B}{\partial t} = -g_B I_B - m b_{B \to A} I_B + m b_{A \to B} I_A$$
$$(5)\ \frac{\partial S_B}{\partial t} = -g_B S_B - m b_{B \to A} S_B + m b_{A \to B} S_A$$



Scope of Optimization (cont.)

- *Scheduling and planning* (combinatorial optimization)

Timetabling



Routing & Scheduling



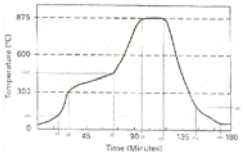
Traveling Salesperson problem

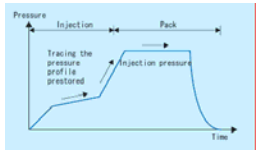


Scope of Optimization (cont.)

- *Optimal control*
- Time-variant profiles are to be found
  - How to lower load?
  - How to control temp, pressure?



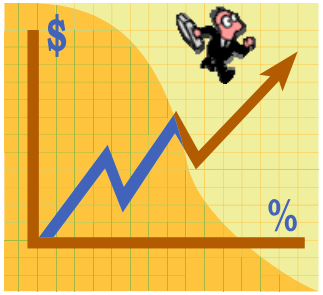


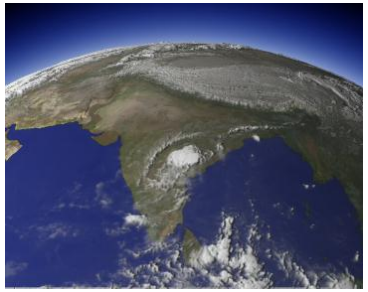




Scope of Optimization (cont.)

- *Forecasting and prediction*





Scope of Optimization (cont.)

- *Data mining* (classification, clustering, pattern recognition)



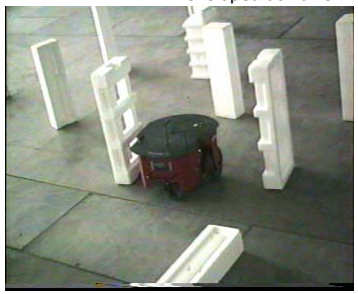


Scope of Optimization (cont.)

- *Machine learning*
  - Designing intelligent systems

Developed at KanGAL

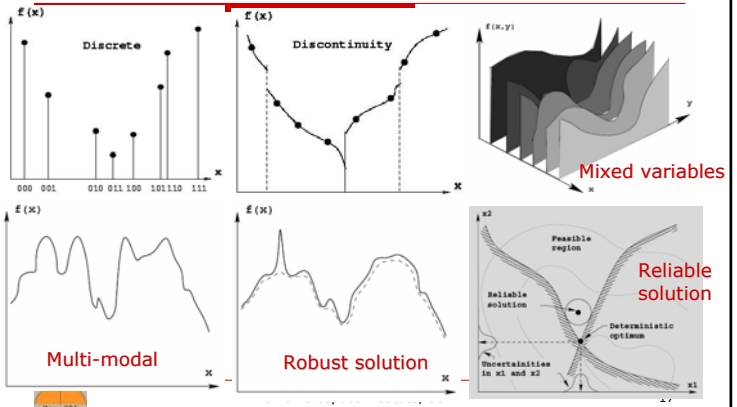




Properties of Practical Optimization Problems

- Non-differentiable functions and constraints
- Discontinuous search space
- Discrete search space
- Mixed variables (discrete, continuous, permutation)
- Large dimension (variables, constraints, objectives)
- Non-linear constraints
- Multi-modalities
- Multi-objectivity
- Uncertainties in variables
- Computationally expensive problems
- Multi-disciplinary optimization

Different Problem Complexities

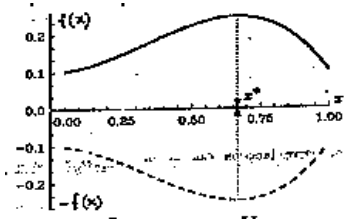


Design Variables, Constraints, and Objectives

- List any and every parameter related to the problem
- Specify the type of each parameter (binary, discrete, continuous)
- Choose a few of them as design variables
- First thumb rule: Use as few variables as possible
  - Usually from the experience of the user
  - From minimum variability consideration
  - From sensitivity analysis
- Constraints are of two types:
- - Inequality: $g(x) \geq 0$ or $g(x) \leq 0$
- - Equality: $h(x) = 0$

Further Comments

- Second thumb rule: Avoid complex equality constraints, use simple equality constraints to eliminate a variable
- $\delta(x) = 5$ can, be relaxed as $\delta \geq 4$ and $\delta \leq 6$
- Objective function: Minimize or maximize
- Duality principle helps unification
- Variable bounds: $x^L \leq x \leq x^U$



Steps in An Optimization Task

- Need for optimization
- Problem formulation
  - Identify problem parameters
  - Choose design variables from parameters
  - Formulate Constraints
  - Formulate objective function
  - Set up variable bounds
  - Requires 50-60% of the effort
- Choose an optimization algorithm
- Obtain solution
- Reformulation and rerun, if desired

Optimization Problem

- Decision variables: $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$
- Constraints restrict some solutions to be feasible

Min. $f(\boldsymbol{x})$
s.t. $g_j(\boldsymbol{x}) \geq 0 \quad j = 1,2,\ldots,J$
$h_k(\boldsymbol{x})=0 \quad k = 1,2,\ldots,K$
$x_i^L \leq x_i \leq x_i^U \quad i = 1,2,\ldots,n$

- Equality and inequality constraints
- Minimum of $f(\boldsymbol{x})$ need not be constrained minimum
- Constraints can be non-linear

Fundamental Optimality Conditions for Unconstrained Minimum Points

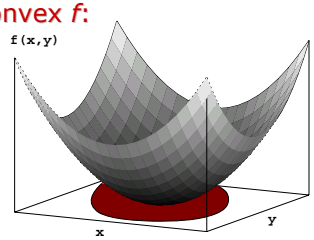
- Single variable: $df/dx = 0$, $d^2f/dx^2>0$
  - General rule: identify first non-zero derivative at point x, say its order is n. If n is odd, the sign of that derivative (+ve or –ve) determines minimum or maximum, respectively. If n is even, it is a saddle point.
- Multiple variables: grad f = 0, Hessian matrix H(f) is +ve definite
  - A matrix is +ve definite, if all eigenvalues are positive

Constrained Optimization

- Kuhn-Tucker (K-T) conditions for optimality
  - First-order necessary conditions
  - Convex search space, convex f: K-T point is minimum

$$\nabla f(x)-\sum_{j=1}^{J} u_j \nabla g_j(x)-\sum_{k=1}^{K} v_k \nabla h_k(x)=0$$

$g_j(\boldsymbol{x}) \geq 0 \quad j = 1,2,\ldots,J$
$h_k(\boldsymbol{x})=0 \quad k = 1,2,\ldots,K$
$u_j g_j(\boldsymbol{x})=0 \quad j = 1,2,\ldots,J$
$u_j \geq 0 \quad j = 1,2,\ldots,J$



- Involve derivatives and solve for roots
- Difficult for classical methods

No Free Lunch (NFL) Theorem

- In the context of *optimization*
  - Wolpert and McCardy (1997)
  - Algorithms A1 and A2
  - All possible problems **F**
  - Performances P1 and P2 using A1 and A2 for a fixed number of evaluations
  - P1 = P2
- NFL breaks down for a class of problems or algorithms
- Research effort: Find the best algorithm for a class of problems
  - Unimodal, multi-modal, quadratic etc.

Summary of Lecture 1

- Optimization is widespread in practice
  - Better solution (design)
  - Better control
  - Search for innovative solutions
  - Understand systems better
- Theoretically sound, but often theory not easy to use in practice
- NFL theorem requires one to alternative methods
  - Strong need for a course like this one

