Uncertainty Handling Using Evolutionary Multi-Objective Optimization

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Outline of Lecture

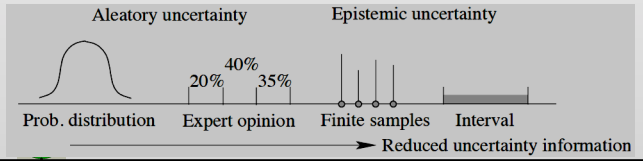
- Uncertainties in practice
- Ha... pa...
- Keywords:
  - • Optimization
  - • Multi-objective optimization
  - • Evolutionary optimization
  - • Uncertainty handling
- Evolutionary optimization and its niches
- Evolutionary multi-objective optimization (EMO)
- Conclusions

Engineering Design and Presence of Uncertainties

- Conceptual or detail design stages
- Sources of uncertainties: model (objective, constraints), design variables, parameters, etc.
- Often handled using a large factor of safety
- Technology advanced to consider certain uncertainties in the past decade
- Uncertainties:
  - Design variables (dimensions, shapes etc.)
  - Problem parameters (material properties, constants)
- Deterministic (usual) optimization is not meaningful in such cases

Two Uncertainty Handling Methods

- Alleatory: Random and irreducible, problem-specific, uncertainty information available, occurs in detail design stage
- Epistemic: Subjective and reducible, exact information not available, occurs in conceptual design stage
- Here, consider two methods of each type



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Optimization Problems

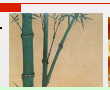
- Decision variables: $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$
- Constraints restrict some solutions to be feasible

$$\begin{aligned} &\text{Min. } f(\boldsymbol{x},\boldsymbol{a}) \\ &\text{s.t. } g_j(\boldsymbol{x},\boldsymbol{b}) \geq 0 \quad j = 1,2,\ldots,J \\ &\quad h_k(\boldsymbol{x},\boldsymbol{c}) = 0 \quad k = 1,2,\ldots,K \\ &\quad x_i^L \leq x_i \leq x_i^U \quad i = 1,2,\ldots,n \end{aligned}$$

- Multiple conflicting objectives can exist
- Other problems: permutations, mixed types
- What sources of uncertainties?

Nature as a Problem Solver
Natural Evolutionary Optimization

- Nature as structural optimizer
  - Stem, Bamboo, insect trachea, bee-hive
- Natur
  - Bi
- Natur
  - Pe



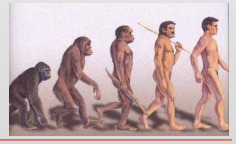

Good reasons to mimick natural evolutionary process




Genesis of Evolutionary Computation

- Natural evolution + genetics
  - Population of solutions
  - Good solutions are emphasized at the expense of weak solutions
  - New offspring are created by duplication, mutation, recombination etc.
  - Evolution of better species with iterations


+

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Evolution of Evolutionary Algorithms

- Ann Arbor, Michigan in 1962
  - John Holland working with Cellular Automata
  - ANAS (MIT Press) in 1975
  - Holland's PhD students: Genetic Algorithms
- San Diego, California in 1965
  - Larry Fogel working with finite state machines
  - Evolutionary Programming
- Berlin in 1965
  - Ingo Rechenberg, Hans-Paul Schwefel working on experimental optimization
  - Evolution Strategy
- Merger in 1991: Evolutionary Algorithms
- Genetic programming, Particle swarm opt., Diff. evolution, etc.






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Evolutionary Algorithms:
Major Early Events

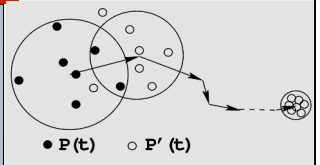



- Kenneth DeJong's PhD thesis in 1975
- David E. Goldberg's GA book in 1989
- First ICGA Conference in CMU in 1985
- First Foundation of GA conference in 1990
- First issue of Evolutionary Computation Journal by MIT Press in 1993
- First issue of IEEE Trans. on EC in 1997
  - *High Impact Factors*
- GECCO and CEC conferences since 1999
- *ACM SIGEVO in 2005*
- *IEEE Society on Computational Intelligence*

Evolutionary Optimization:
A Computerized Algorithm

```
begin
Solution Representation
t := 0;  // generation counter
Initialization P(t);
Evaluation P(t);
while not Termination
do
  P'(t)  := Selection (P(t));
  P''(t) := Variation (P'(t));
  Evaluation P''(t);
  P(t+1):= Survivor (P(t),P''(t));
  t := t+1;
od
end
```

Recombination
Mutation



- Mean approaches optimum
- Variance reduces

Theoretical Underpinnings

- EAs are stochastic, multi-faceted, iterative procedures
- Not easy to do a perfect math. modeling and analysis
- Two main approaches to theory:
  - Functional decomposition approach: Engineering approach (N=kn, high p_c, low $p_m=1/\ell$, etc.)
  - Dynamical systems approach: Markov-chain approach
- Asymptotic convergence proof with elite-preservation and a finite mutation exists (Rudolph, 1996)
- However, time complexity proof to specific problems exists

Niches of EAs

- Applicable in problems where no (good) method is available
  - Discontinuities, non-linear constraints, multi-modalities
  - Discrete variable space
  - Mixed variables
  - implicitly defined models (*if-then-else* constructs)
  - Noisy problems
- Most suitable in problems where multiple solutions are sought
  - Multi-modal optimization problems
  - Multi-objective optimization problems
- Use of parallel computing is desired

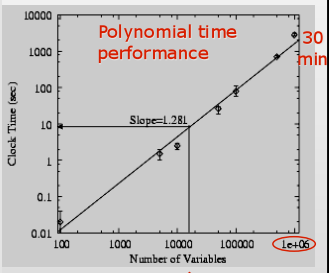
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Advanatges and Disadvantages

- Population approach
- Ability to recombine
- Operator flexibility
- Algorithmic flexibility
- However, onus is on developer to design EAs right for a problem
- Need know-how about what algorithm feature good for what problem class!
- *Never try as the first algorithm!*

Large-Scale Optimization

- Customized GA has sub-quadratic complexity and up to **one million integer** variables
- Solutions may not be optimal, but pragmatic
  - At least 99.97% utilization
  - Solutions approximate to 200 gm of molten metal in a 650 kg vessel



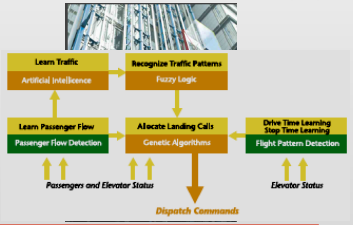
30 min

Casting Scheduling

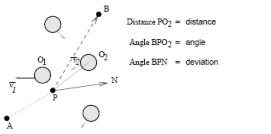
Adopted by Industries

- Nose of Shinkansen (Bullet Train in Japan) N700 Series http://english.jr-central.co.jp/news/n20040616/index.html
- KONE (Finland) elevator's control system (http://www.kone.com/countries/en_MP/Documents/Brochures/KONE%20Alta.pdf)





EAs with Other Soft Computing Approaches

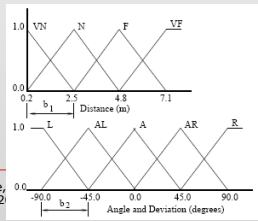


☞ distance: VN, N, F, VF; angle: L, AL, A, AR, R; deviation: L, AL, A, AR, R

If distance is VN & angle is A, then deviation is AL

distance \ angle	L	AL	A	AR	R
VN	A			AL	A
N		A			
F	A		AR		A
VF			A		A





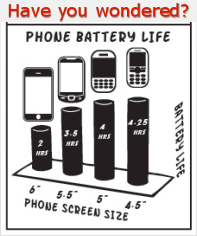
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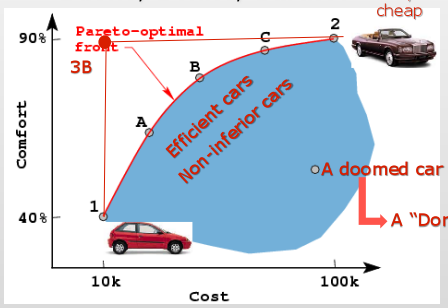
Multi-Objective Optimization:
Handling multiple conflicting objectives

- Bueno, Bonito, Barato?

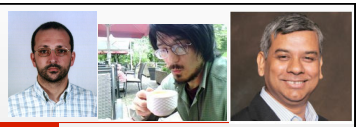
Good, nice, cheap

Have you wondered?





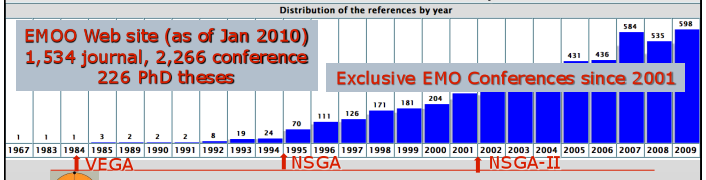
Evolution of EMO



- Early penalty-based approaches
- VEGA (1984)
- Goldberg's (1989) suggestion



- MOGA, NSGA, NPGA (1993-95) used Goldberg's suggestion
- Elitist EMO (SPEA, NSGA-II, PAES, MOMGA etc.) (1998 -- Present)



EMO-2013 in Sheffield, UK



EMO 2013, Sheffield UK, 19-22 March 2013
Submission deadline: 19 August 2012
www.shef.ac.uk/emo2013

Core EMO track
IPC Chair: Carlos Fonseca, University of Coimbra

In the core EMO track, full papers are invited on novel aspects of evolutionary multi-criterion optimization theory and methodology. Topics may include: new aggregation-based, dominance-based, indicator-based algorithms; knowledge mining of the problem landscape; handling problem features such as many objectives, uncertainty, time-variance, and expensive evaluations; interactive preference articulation and exploitation methods; theoretical analyses; new developments in collaborative and parallel approaches; new paradigms for multi-objective search and new developments in EMO-related paradigms; new tools for empirical analyses; hybrid EMO-MCDM methods.

MCDM track
Chair: Salvatore Greco, University of Catania

Full papers are invited for the integrated MCDM track on new developments in the multiple criteria decision making community that might be blended with EMO themes. Original research, tutorial and survey papers are welcome in this track.

Publication Springer Lecture Notes in Computer Science

All accepted works will be published as full papers in Springer-Verlag's Lecture Notes in Computer Science (LNCS) series. Papers must follow the LNCS style and may be **up to 15** pages in length.

EMO Books (Since 2001)

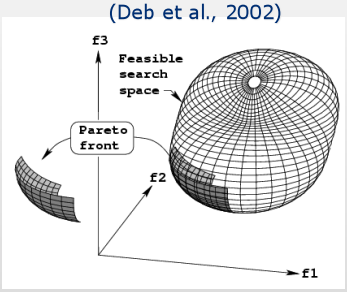


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Elitist Non-dominated Sorting Genetic Algorithm (NSGA-II)

(Deb et al., 2002)

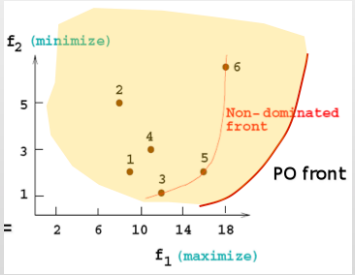
- NSGA-II can extract Pareto-optimal frontier
- Also find a well-distributed set of solutions
- iSIGHT and modeFrontier adopted NSGA-II

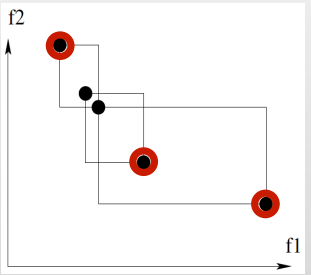


Thomson Citation Laureate Award 2006, Current Classic and Most Highly Cited Paper (~3,300 citations), India's Most Cited Paper among 22 disciplines (2011, Thomson Reuters)

NSGA-II Procedure

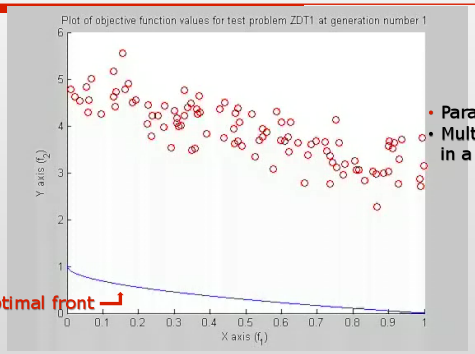
1. Emphasize for non-dominated points
2. Emphasize for less crowded points





Simulation on a Test Problem:

Front-wise Movement Towards PO Front

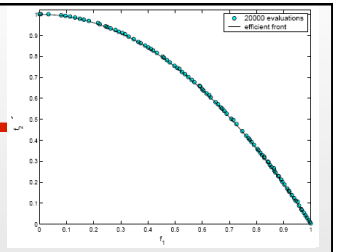


Pareto-optimal front

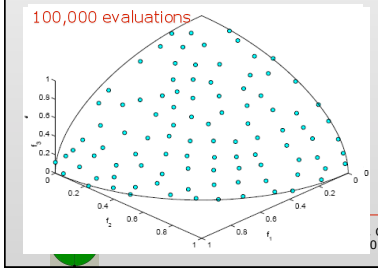
- Parallel search
- Multiple solutions in a single run

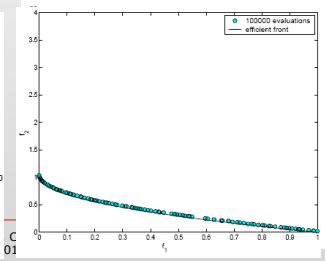
NSGA-II Versus Classical Method

- Population size=100
- Standard parameter setting
- NSGA-II is better



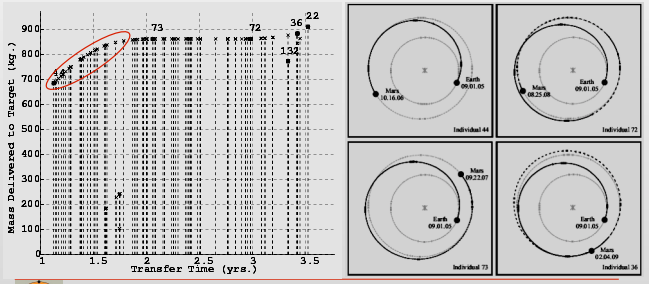
100,000 evaluations





A Case Study:
Inter-planetary Trajectory Optimization at NASA

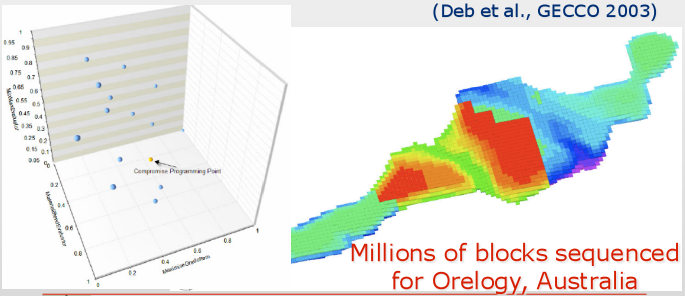
- Find a set of trade-off solutions, then choose one
- More confident decision-making



Another Case Study:
Mine Scheduling in Australia

- 3D scheduling for three objectives

(Deb et al., GECCO 2003)

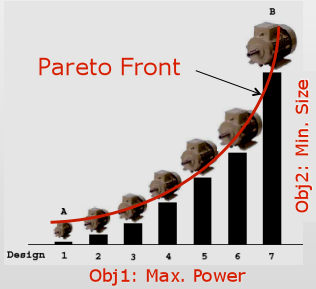


Millions of blocks sequenced for Orelogy, Australia

EMO for Knowledge Discovery
Innovization: Innovation through Optimization

Unveil important design principles in a routine design scenario

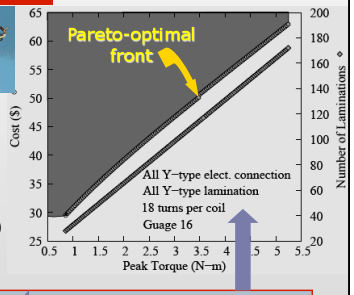
- Example: Electric motor design with varying ratings, say 1 to 10 kW
  - Each will vary in size and power
  - Armature size, number of turns etc.
- How do solutions vary?
  - Any common principles!



Brushless DC Permanent Magnet Motor Design for Cost and Peak Torque

- Five variables (all discrete), three constraints
- Non-convex, disconnected P-O fronts
- *Innovization*:
  - Connection: Y (betn. Y & Δ)
  - Lamination Type: Y (X, Y, Z)
  - 1 out of 16 wire guages
  - 18 turns per coil (10,80)
  - *More peak torque by adding linearly more laminations*



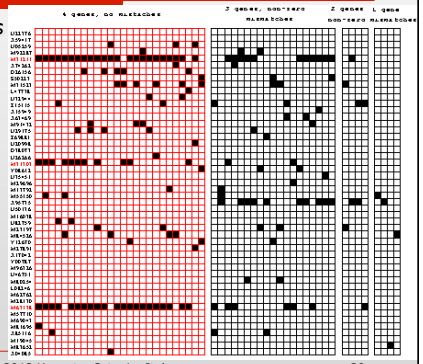


Design Innovation

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Multiple Gene Subsets for Leukemia Samples

- Identifying classifiers for two types of Leukemia
- 26 four-gene combinations for 100% classification
- 7,024 genes to 2 genes with confidence
- Deb and Reddy (BioSystems, 2003)

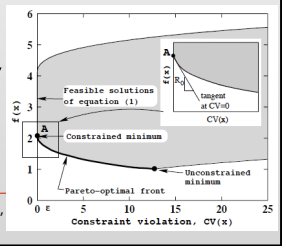


Multi-objectivization: *Solving Problems Using MO*

- Constrained handling
  - Constraint violations as additional objectives
- Multimodal problems
- Bloating in Genetic Programming (Blueler et al, 2001)
- Diversity preservation in EAs (Jensen, 2003, Abbas and Deb, 2003)
- Fuzzy clustering methods
- Goal programming



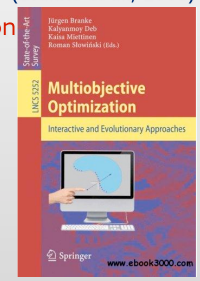
(Knowles, Corne & Deb, 2008)



EMO and Decision-Making

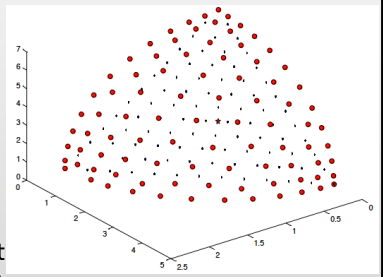
- Finding a P-O set (using EMO) is half the story
- How to choose one preferred solution (MCDM)
- A priori approach:
  - First MCDM, then EMO
- A posteriori approach:
  - First EMO, then MCDM
- Interactive approach:
  - MCDM during EMO

(Branke et al., 2008)



Many-Objective Problems

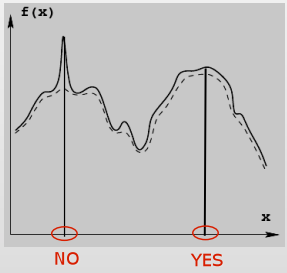
- Finding PO front for 4 or more objectives
- Getting a good distribution is a challenge
  - Large population size
  - Domination principle doesn't work
  - Visualization difficult
- Algorithm development underway (M-NSGA-II)



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Robust Optimization:
Handling uncertainties in variables/parameters

- Not noise in obj/constraints
- DV/Parameters uncertain and sensitive to implementation
  - Tolerances in manufacturing
  - Material properties are uncertain
  - Loading is uncertain
- Who wants a sensitive optimum solution?
  - Real Issue: How to perform EAs sub-optimally?

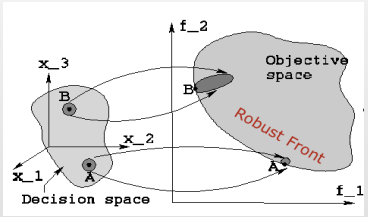


Robustness in Evolutionary Optimization

- Relate to solution quality or performance
  - Every $\boldsymbol{x}$ represents a set of neighboring solutions $\boldsymbol{y}^j \in B(\boldsymbol{x})$, $j=1,\ldots,H$
  - Good expected performance
    - Minimize μ_f instead of f
  - Good worst-case performance
    - Minimize (max $\boldsymbol{y}^j$)
  - Low variability in performance
    - Minimize σ_f <- *not a good idea alone*
    - Minimize $\mu_f + k\,\sigma_f$ is a better idea

Robust EAs for Multi-Objective Optimization

- Not all Pareto-optimal points may be robust
- A is robust, but B is not
- Decision-makers will be interested in knowing robust part of the front
- Generic decision-making idea



Multi-Objective Robust Solutions of Type I and II *(Deb and Gupta, ECJ 2004)*

- Similar to single-objective robust solution of type I

$$\begin{aligned} &\textit{Minimize} && (f_1^{\text{eff}}(\mathbf{x}), f_2^{\text{eff}}(\mathbf{x}), \ldots, f_M^{\text{eff}}(\mathbf{x})), \\ &\textit{subject to} && \mathbf{x} \in \mathcal{S}, \end{aligned}$$

- Type II

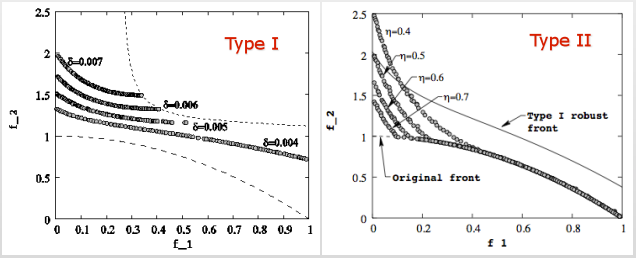
$$\begin{aligned} &\textit{Minimize} && \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})), \\ &\textit{subject to} && \frac{\|\mathbf{f}^p(\mathbf{x}) - \mathbf{f}(\mathbf{x})\|}{\|\mathbf{f}(\mathbf{x})\|} \leq \eta, \\ & && \mathbf{x} \in \mathcal{S}. \end{aligned}$$

- Other ideas can be used as well

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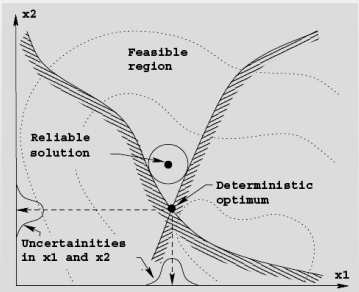
Robust Frontier for Two Objectives

- Identify robust region
- Allows a control on desired robustness



Reliability-Based Optimization:
Making designs safe against failures

- Deterministic optimum is not usually reliable
- Reliable solution is an interior point
- *Chance* constraints with a given reliability



Minimize $\mu_f + k\sigma_f$
Subject to $Pr(g_j(x) \geq 0) \geq \beta_j$
β_j is user-supplied

RBDO: Basics

- x: uncertain variables
- d: fixed variables
- p: uncertain parameters

Original Problem

$$\begin{aligned} \underset{(\mathbf{x},\mathbf{d})}{\text{Minimize}} \quad & f(\mathbf{x},\mathbf{d},\mathbf{p}), \\ \text{Subject to} \quad & g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0, \quad j = 1,2,\ldots,J, \\ & h_k(\mathbf{d}) \geq 0, \quad k = 1,2,\ldots,K, \\ & \mathbf{x}^{(L)} \leq \mathbf{x} \leq \mathbf{x}^{(U)}, \\ & \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$

- x and p are assumed to vary with (μ, σ)
- Functions f and g_j are uncertain and have distributions of variation
- Makes it difficult to use an optimization method
  - Dealing with function distributions

RBDO Basics (cont.)

- Reliability R
- Revised problem
- Joint prob. P difficult to compute

$$\begin{aligned} \underset{(\mu_{\mathbf{X}},\mathbf{d})}{\text{Minimize}} \quad & f(\mu_{\mathbf{X}},\mathbf{d},\mu_{\mathbf{p}}), \\ \text{Subject to} \quad & P(\textstyle\bigwedge_{j=1}^{J}(g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0)) \geq R \\ & h_k(\mathbf{d}) \geq 0, \quad k = 1,2,\ldots,K, \\ & \mathbf{x}^{(L)} \leq \mu_{\mathbf{X}} \leq \mathbf{x}^{(U)}, \\ & \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$

- Most studies use $P(g_j(\mathbf{x},\mathbf{d},\mathbf{p}) \geq 0) \geq R_j, \quad j = 1,2,\ldots,J.$ R_j=R
- System reliability versus component reliability
- Thus, the worst constraint govern the reliability constraint
- First, we discuss single-constraint strategies

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Determining a Solution's Reliability

- Probability of success against constraint satisfaction

$$P_j(\mu_{\mathbf{x}}, \mathbf{d}, \mu_{\mathbf{p}}) = \int_{g_j(\mathbf{x},\mathbf{d},\mathbf{p})<0} \varphi_{\mu_{\mathbf{x}},\mu_{\mathbf{p}}}(\mathbf{x},\mathbf{p}) d\mathbf{x} d\mathbf{p},$$

- Difficult to compute exactly
- Monte-Carlo simulation: r_j success of N scenarios: $1 - \frac{r_j}{N} \geq R_j$ (Loughlin and Ranjithan, 1999)
- If R=0.9999, N>10,000 (Expensive!)
- Importance sampling: May still be large

Optimization Based Procedure: *Classical Approaches*

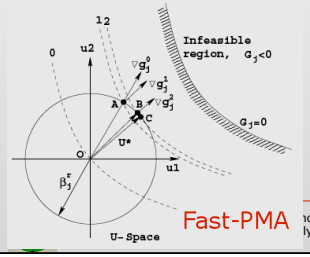
- Given a solution, determine the closest point on the constraint boundary
  - Most probable point (MPP)
- To generalize, computation is performed on std. normal coordinate system U for every x
- $g_j(x,d,p)=0 \Rightarrow G_j(U)=0$
- Two optimization approaches:
  - Performance Measure Approach (PMA)
  - Reliability Index Approach (RIA)

Reliability Based Optimization Procedures

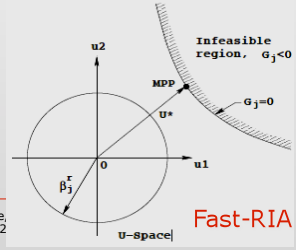
- Double-loop methods (Agarwal, 2004)
  - Nested optimization with PMA or RIA
    Computationally expensive, but accurate
- Single-loop methods (Liang et al., 2004)
  - Approximate methods of PMA/RIA
  - Computationally fast, but not accurate
- Decoupled methods (Du and Chen, 2004)
  - Sequential, repetitive application of both optimization tasks
  - A good compromise

Reliability Based EAs

- For every EA solution, reliability constraints are checked using faster methods of PMA/RIA
  - Fast-PMA: Gradient of G is computed at rel. circle but projected from origin
  - Fast-RIA: First Fast-PMA, then Newton's search



Fast-PMA



Fast-RIA

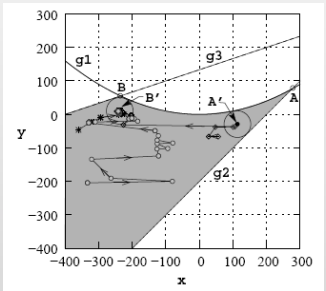
Three Problem Niches for EAs

1. Finding reliable optimum in multi-modal problems
2. Enhancing knowledge of reliable solutions with increased reliability
3. Locating reliable frontier in multi-objective optimization problems

1. Finding Global Reliable Solution: *Single-objective reliable solution*

- Multiple deterministic optima
- Reliable solution for local is better
- Classical methods starts from deterministic global optimum
- EAs solve the overall problem and are better

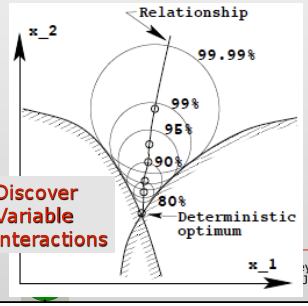
PMA approach is used

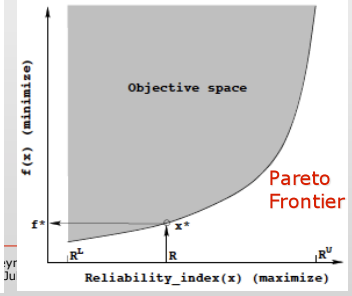


2. Solutions with Increased Reliability

- Bi-objective problem
- Use an EMO

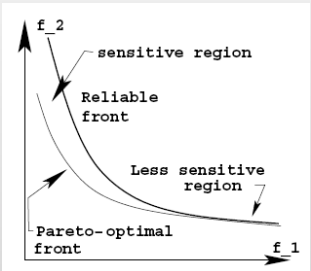
$$\begin{aligned} &\underset{(\mu_{\mathbf{x}},\mathbf{d})}{\text{Minimize}} && f(\mu_{\mathbf{x}},\mathbf{d},\mu_{\mathbf{p}}),\\ &\underset{(\mu_{\mathbf{x}},\mathbf{d})}{\text{Maximize}} && R(\mu_{\mathbf{x}},\mathbf{d},\mu_{\mathbf{p}})\\ &\text{Subject to} && h_k(\mathbf{d}) \geq 0,\\ & && \mathbf{x}^{(L)} \leq \mu_{\mathbf{x}} \leq \mathbf{x}^{(U)},\\ & && \mathbf{d}^{(L)} \leq \mathbf{d} \leq \mathbf{d}^{(U)}. \end{aligned}$$





3. Multi-Objective Reliable Frontier

- Instead of finding deterministic Pareto-optimal front, find the reliable front
- Chance constraints
- Objectives as they are
- PMA approach is used



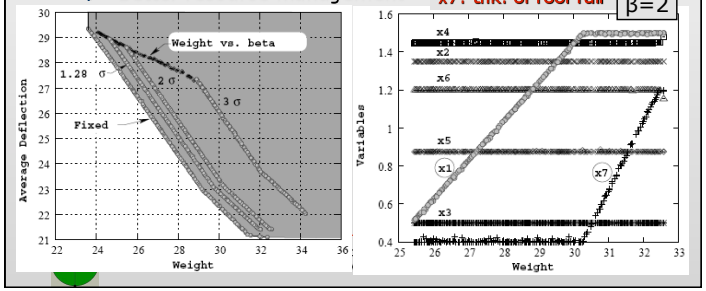
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Innovization: Innovative design principles through optimization

A Car Side Impact Design

- Reliable fronts show rate of movement
- What remains unchanged and what gets changed!
  - Better resource management

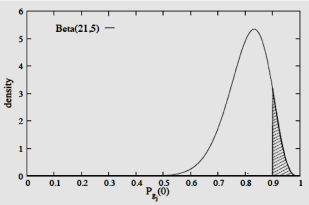
x1: thk. of B-Pillar inner
x7: thk. of roof rail
β=2



Bayesian Design Optimization (BDO)

- Epistemic uncertainty: A few samples are known
- Equivalent to a Bernoulli sequence
- Expected # of realizations:

$$E_j(r) = \sum_{k=1}^{N} Pr\left[g_j\left(\mathbf{X_t}, \mathbf{P_t}\right) \leq 0 \,|\, (\mathbf{X_s}, \mathbf{P_s})_k\right]$$



A Beta distribution

- Confidence level

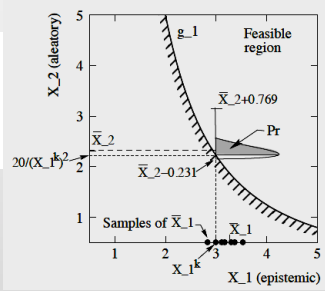
$$\zeta_j = Pr\left[P_{g_j}(0)|_{\mu_\mathbf{X}} \geq R_j\right]; \quad j = 1, \ldots, J.$$

BDO Algorithm

- Optimize obj. and confidence level:

$$\begin{aligned} &\underset{\mu_\mathbf{X}}{\text{minimize}} \;\; f\left(\mu_\mathbf{X}, \mu_\mathbf{P}\right), \\ &\underset{\mu_\mathbf{X}}{\text{maximize}} \;\; \zeta_s\left(\mu_\mathbf{X}\right), \\ &\text{subject to:} \;\; 0 \leq \zeta_s\left(\mu_\mathbf{X}\right) \leq 1. \end{aligned}$$

- A mixed scenario:

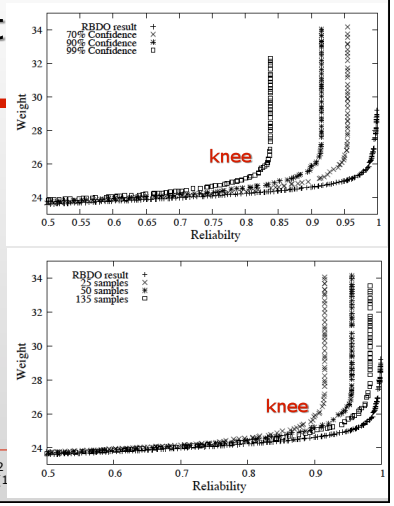


Car Side Impact Problem

- Reliability:

$$\zeta_s^{max} = 1 - R^{N+1}.$$

- 3 epsitemic variables
- Worse than RBDO results
- More samples, approach RBDO
- Presence of ***knee*** interesting



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Evidence-Based Optimization (EBDO)

- Possibility of intervals are known:
- Belief and possibility measures:

$$Bel(A_1 \cup A_2) \geq Bel(A_1) + Bel(A_2) - Bel(A_1 \cap A_2).$$

$$Pl(A_1 \cap A_2) \leq Pl(A_1) + Pl(A_2) - Pl(A_1 \cup A_2).$$

x_1 Interval	BPA
$[x_{1,N}$ - 1.0, $x_{1,N}$ - 0.5]	4.78 %
$[x_{1,N}$ - 0.5, $x_{1,N}]$	45.22 %
$[x_{1,N}, x_{1,N} + 0.5]$	45.22 %
$[x_{1,N} + 0.5, x_{1,N} + 1.0]$	4.78 %

- Fuzzy measure: m(A)
- $Pl(A) \geq Bel(A).$

$$Bel(A) = \sum_{B \subseteq A} m(B).$$

$$Pl(A) = \sum_{B \cap A \neq 0} m(B).$$

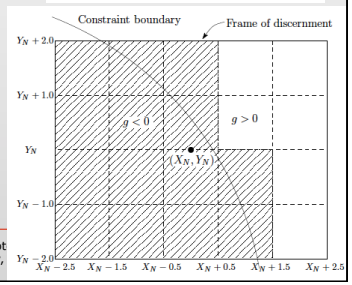
EBDO Algorithm

- Optimize obj. and Pl_{max}:

$$Pl_{\max} = \max_{j=1}^{J}(Pl\,[g_j\,(\mathbf{X},\mathbf{P}) \leq 0])$$

$$\begin{aligned} \underset{\mathbf{X},\mathbf{P}}{\text{minimize}} \quad & f\,(\mathbf{X},\mathbf{P})\,, \\ \underset{\mathbf{X},\mathbf{P}}{\text{minimize}} \quad & Pl_{\max}, \\ \text{subject to:} \quad & 0 \leq Pl_{\max} \leq P_{lim} \end{aligned}$$

- Grid the search space and compute Pl_{max}
- g>0: Belief measure
- g<0: Plausibility mea.
- Plaus. values are added
- Constraint having smallest Pl. chosen

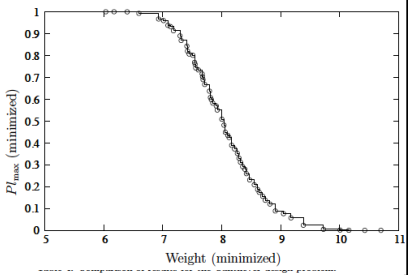


Cantilever Beam Design

Epsitemic Variables:

Z Interval	BPA	y ($\times 10^3$) Interval	BPA
[200, 300]	2.2 %	[35, 37]	6.1 %
[300, 400]	13.6 %	[37, 38]	9.2 %
[400, 450]	15 %	[38, 39]	15 %
[450, 500]	19.2 %	[39, 40]	19.2 %
[500, 550]	19.2 %	[40, 41]	19.2 %
[550, 600]	15 %	[41, 42]	15 %
[600, 700]	13.6 %	[42, 43]	9.2 %
[700, 800]	2.2 %	[43, 45]	7.1 %

Y Interval	BPA	E ($\times 10^6$) Interval	BPA
[700, 800]	2.2 %	[26.5, 27.5]	10 %
[800, 900]	13.6 %	[27.5, 28.5]	21 %
[900, 1000]	34.1 %	[28.5, 29]	13.5 %
[1000, 1100]	34.1 %	[29, 29.5]	13.5 %
[1100, 1200]	13.6 %	[29.5, 30.5]	21 %
[1200, 1300]	2.4 %	[30.5, 31.3]	21 %



	Evidence theory based results Algorithm	p_{lim} = 0.2	p_{lim} = 0.1	p_{lim} = 0.0013	Reliable Optimum R = 0.9987	Deterministic optimum
Weight	DIRECT [7]	8.6448	10.217	10.556	9.5212	7.6679
	NSGA-II	8.5751	8.8832	9.9580		

Conclusions

- EAs have an edge in applied optimization
  - Flexibility, population approach, modularity
- EMO provide promise to collaborate with MCDM, Math. Opt and other Soft Computing approaches
- Uncertainty in variables and parameters
  - Highly pragmatic and desired
- Further uncertainties need to be considered

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