Problem 1 (0.50 pu)

A 400 km, 500 kV, 60 Hz three-phase uncompensated line has a positive-sequence series reactance \( x = 0.3 \Omega/km \) and a positive-sequence shunt admittance \( y = j5 \mu S/km \). Neglecting losses,

1. (0.21 pu) Calculate \( Z_c \), \( \beta l \), and SIL in MW
2. (0.14 pu) Determine the \( \Pi \)-equivalent circuit and its parameters for the line.
3. (0.15 pu) Calculate the theoretical maximum real power, \( P_{\text{max}} \), that the line can deliver with rated voltage on both ends. And explain the meaning of \( P_{\text{max}} \).

Solution

\( l = 400 \) km; \( V_{\text{rated}} = 500 \) kV; \( z = jx = j0.3 \Omega/km \); \( y = j5 \mu S/km \)

1. \( Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j0.3}{j5\mu}} = 245 \) \( \Omega \).

\( (\beta l) = \sqrt{\frac{2y}{l}} = \sqrt{\frac{j0.3 \cdot j5\mu}{400}} = 0.49 \) radian.

\( \text{SIL} = \frac{V_{\text{rated}}^2}{Z_c} = \frac{500^2}{245} = 1020 \) MW.

2. 

\[
\begin{align*}
Z' &= jX' = (jxl) \left( \frac{\sin(\beta l / 2)}{\beta l / 2} \right) = (j0.3 \cdot 400) \left( \frac{\sin(0.49)}{0.49} \right) = j115.3 \Omega, \text{ or } Z' = jZ_c \sin(\beta l) = j245 \cdot \sin(0.49) \\
Y' &= \left( \frac{yl}{2} \right) \left( \tan(\beta l / 2) \right) = \left( \frac{j5\mu \cdot 400}{2} \right) \left( \frac{\tan(0.49/2)}{0.49/2} \right) = j1.02 \mu S, \text{ or } Y' = j \left( \frac{\tan(\beta l / 2)}{Z_c} \right) = j \left( \frac{\tan(0.49/2)}{245} \right)
\end{align*}
\]

3. 

\[
\frac{P_{\text{max}}}{X'} = \frac{V_{\text{rated}}}{X'} = \frac{500^2}{115.3} = 2168 \text{ MW}.
\]

\( P_{\text{max}} \) represents the theoretical steady-state stability limit of the line. If an attempt were made to exceed this steady-state stability limit, the synchronous machines at the sending end would lose synchronism with those at the receiving end. That is, the sending end machines would lead more and more and the receiving end machines lag more and more in phase angle.
Problem 2 (0.50 pu)
The figure shows a one-line diagram of a 3-bus power system. Power flow input data are given in Table 1 and 2.

![Diagram of a 3-bus power system]

Table 1. Bus input data

<table>
<thead>
<tr>
<th>Bus</th>
<th>Type</th>
<th>V</th>
<th>δ</th>
<th>P_G</th>
<th>Q_G</th>
<th>P_L (or P_D)</th>
<th>Q_L (or Q_D)</th>
<th>Q_Gmin</th>
<th>Q_Gmax</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Swing</td>
<td>1.0</td>
<td>0</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>0</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2</td>
<td>Load</td>
<td>---</td>
<td>---</td>
<td>0</td>
<td>0</td>
<td>0.9</td>
<td>0.4</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>Generator</td>
<td>1.0</td>
<td>---</td>
<td>1.0</td>
<td>---</td>
<td>0</td>
<td>0</td>
<td>-5.0</td>
<td>+5.0</td>
</tr>
</tbody>
</table>

Table 2. Line input data

<table>
<thead>
<tr>
<th>Line</th>
<th>Bus-to-Bus</th>
<th>R'</th>
<th>X'</th>
<th>G'</th>
<th>B'</th>
<th>Maximum MVA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>pu</td>
<td>pu</td>
<td>pu</td>
<td>pu</td>
<td>pu</td>
</tr>
<tr>
<td>1</td>
<td>1-2</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>2-3</td>
<td>0</td>
<td>0.4</td>
<td>0</td>
<td>0</td>
<td>3.0</td>
</tr>
<tr>
<td>3</td>
<td>1-3</td>
<td>0</td>
<td>0.3</td>
<td>0</td>
<td>0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

1. (0.25 pu) Determine the 3x3 pu bus admittance matrix \( Y_{bus} \).
2. (0.25 pu) For each bus \( k = 1, 2, \) and 3, determine which of the variables \( V_k, \delta_k, P_k, \) and \( Q_k \) are input data and which are unknowns.

Solution:

1. \[
Y_{bus} = \begin{bmatrix}
\frac{1}{j0.2} + \frac{1}{j0.3} & -\frac{1}{j0.2} & -\frac{1}{j0.3} \\
-\frac{1}{j0.2} & \frac{1}{j0.2} + \frac{1}{j0.4} & -\frac{1}{j0.4} \\
-\frac{1}{j0.3} & -\frac{1}{j0.4} & \frac{1}{j0.3} + \frac{1}{j0.4}
\end{bmatrix} \begin{bmatrix}
-8.33 & j5 & j3.33 \\
-j5 & -7.5 & j2.5 \\
j3.33 & j2.5 & -j5.83
\end{bmatrix} \text{ pu.}
\]

2. | Bus | Bus type | Input Data | Unknowns |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Swing</td>
<td>( V_1=1.0, \ \delta_1=0 )</td>
<td>( P_1, Q_1 )</td>
</tr>
<tr>
<td>2</td>
<td>Load</td>
<td>( P_2= P_{G2} - P_{L2} = -0.9, \ Q_2= Q_{G2} - Q_{L2} = -0.4 )</td>
<td>( V_2, \delta_2 )</td>
</tr>
<tr>
<td>3</td>
<td>Generator</td>
<td>( P_3= P_{G3} - P_{L2} = 1.0, \ V_3= 1.0 )</td>
<td>( Q_3, \delta_3 )</td>
</tr>
</tbody>
</table>