COMPLEX MODAL ANALYSIS OF THE SWIMMING MOTION OF A WHITING

B. F. Feeny and A. K. Feeny

Department of Mechanical Engineering
Michigan State University
East Lansing, MI 48824
feeny@egr.msu.edu, feenyalb@msu.edu

ABSTRACT

The kinematics of the transverse motion of a swimming fish are analyzed using a complex modal decomposition. Cinematographic images of a swimming whiting (Gadus merlangus) were obtained from the work of Sir James Gray (Journal of Experimental Biology, 1933). The position of the midline for each image was determined, and used to produce planar positions of virtual markers distributed along the midline of the fish. Transverse deflections of each virtual marker were used for the complex orthogonal decomposition of modes. This method was applied to a normal whiting and an amputated whiting, both of Gray’s paper. The fish motions were well represented by a single complex mode, which was used as a modal filter. The modal coordinate was also extracted. The mode and modal coordinate were used to estimate the frequency, wavelength, and wave speed. The amputated fish was compared to the non-amputated fish, and the different amount of traveling in the respective waveforms was quantified.

1 INTRODUCTION

We study the kinematics of the swimming motion of a whiting (Gadus merlangus) that was examined by Sir James Gray [1, 2]. The motion of a fish during swimming is of interest for several reasons. Some species of fish have the natural ability to swim very efficiently. A greater understanding of their movement allows for better appreciation of fish themselves, as well as valuable insight to a very efficient form of underwater locomotion. The work in this area can also be applied to the development of modern technology, as today there is a lot of interest in biomimetic robotic fish [3–6].

In this paper, we perform complex modal analysis to further analyze and quantify the motion of the fish. The fish is seen as an oscillating structure, and the many degrees of freedom are the positions of points along the midline of the fish. The method applied is a complex orthogonal decomposition [7], which is a generalization of proper orthogonal decomposition [8]. The purpose is to demonstrate how the analysis method can be applied to fish motion data. In this example we look at a brief swimming moment of an individual fish, rather than assess an average representation of swimming patterns of all whitings.

1.1 Background on Fish Kinematics

Brief historical accounts [9, 10] mention early studies, for example by Aristotle, who thought the fish propelled themselves with their pectoral (side) fins, and by Borelli, a disciple of Galileo, who showed that the fish he studied could swim without the pectoral fins and instead relied on motions of the tail. Pertinent interest in fish motion has emerged since the start of the 20th century. Modern analyses begin with Breder [11], who classified body/caudal-fin swimming motions. The classifications of undulatory swimming still used today range from anguilliform, subcarangiform, carangiform, thunniform, and/or ostraciform swimming [10, 12, 13]. The subjective descriptions of these motions vary progressively from large undulation in the anterior through
posterior for anguilliform, to subtle anterior motion in the sub-carangiform motion, to soley caudal (tail) fin motion in the thunniform motion. Lindsey [10] provided a chart comparing motion characteristics and body types associated with these classes of motion. Not all authors have used the same classifications.

In the 1930s, Sir James Gray studied swimming fish in a set of important papers on fish locomotion [1, 2, 14]. In his work [1, 2], he used a series of photographs, and interpretive sketches, to portray the movements of the bodies of various fish while swimming. In [2], he focused on the role of the caudal fin in the whiting species. The whiting is illustrated in Figure 1. Gray experimented with the caudal fin’s role by amputating the caudal fin of one whiting fish, and comparing the swimming motion with its prior non-amputated (normal, or intact) motion. Gray had estimated that the surface area that the caudal fin provides accounts for 40% of the normal fish’s propulsion. However, he observed that the amputated fish did not swim at a significantly different speed than the non-amputated fish, and suggested that the amputated fish makes up for the lost caudal fin by changing its movement pattern so that energy is applied to movement in the rest of its body.

Videler and Hess [13, 15] sensed the motions of mackerel and saithe cinematographically, and processed the images into time series data, which were then analyzed in a variety of ways. They fit the motion at various locations, head to tail, on the body to Fourier series, including both sine and cosine terms. We’ll come back to this later. Gillis [16] studied moving images of aquatic undulatory locomotion in fish and amphibians, in the spirit of Gray’s work, and placed heavy emphasis on the effects of the tail angles while swimming.

Studying the body motions during undulatory swimming is important because the body kinematics are often used as input to computational models of the fluid flow around the fish. Wolfgang et al. [17] used experimental flow-visualization techniques to study the flow around a swimming giant danio, and compared the experiments to a three-dimensional numerical model of the flow velocity field based on prescribed motions of the mid-

1.2 Complex Modal Decomposition

The method we use to analyze the motion of the fish is the complex orthogonal decomposition (COD), developed for structures [7] and since applied to the movements of worms [23] and waves in beams [24]. COD is a generalization of the well known proper orthogonal decomposition (POD). POD, similar to singular value decomposition (SVD), and principal components analysis (PCA), is a tool for extracting modes that optimize the signal energy distribution in a set of measured time series. It has been used to characterize spatial coherence in turbulence and structures [8, 25, 26], the dimension of the dynamics [25, 27, 28], empirical modes for reduced order models [29, 30], and in system identification [31, 32]. POD, SVD, and PCA have been compared for structural applications [33]. In specific circumstances, the POD produces the normal modes of a structure [34–37]. POD is particularly useful if extracting standing wave components, but is less suited for decomposing nonstanding wave components. The COD leads to complex modes that can be used to describe non-standing and traveling waves.

SVD has been used to study the fluid wake of a fish [38]. In contrast, our work focuses on the body. Both COD and SVD are able to dissect the motion into modes, and indicate a measure of energy associated with the modes. While SVD is a powerful and widely used tool, there are some advantages to using COD, which will be discussed shortly.

The application of COD involves solving the eigenvalue problem $\mathbf{Rw} = \lambda \mathbf{w}$, where $\mathbf{R}$ is a complex “correlation matrix” built from time history measurements of a structure, in this application, the transverse displacement measurements along the body of the fish. The eigenvectors $\mathbf{w}$ of $\mathbf{R}$ are called “complex
orthogonal modes” (COMs), and indicate mode shapes that represent the characteristic movement of the fish. The eigenvalues, $\lambda$, which are the “complex orthogonal values” (COVs), indicate the mean squared amplitude of modal coordinates. The largest COV corresponds to the dominant wave form of the swimming fish. Using the results of COD, we will then be able to calculate other important information of the swimming mechanics of the fish, such as frequency and wave number (or wavelength). Other geometric properties of the mode could also be quantified, such as the amplitude profile, and the tail angle of the fish as it swims through time, a quantity that Gillis [16] focused on.

In this paper, following Gray [2], we analyze the movement of a healthy and an amputated fish. Image processing is first applied to images from Gray’s 1933 paper [2]. Transverse deflections of the midline of the fish are determined and become the subject of this analysis. Indeed, Gray’s results of more than 75 years ago still contain information that becomes accessible as new analysis techniques are developed.

2 METHODS

The treatment of the data involved image processing to obtain transverse deflection data, and then modal decomposition.

2.1 Image Processing

Gray’s papers [1, 2] incorporated photos of multiple species of fish as they swim through time. For this study, we focused on the photos of the movement of the whiting, before and after amputation [2]. The analysis in this work involved image processing to convert Gray’s photographs of swimming fish into displacement data of the midline of the fish. The COD was then applied to the displacement data. In this section, we will summarize the method Gray used to gather the data, and then we will describe our treatment of the data.

Based on information in the sister paper [1], the whiting images were taken every 0.05 seconds. The images were placed over grids so that the various positions of the fish could be easily compared. Each square in the grid is 3 inches by 3 inches (7.62 cm). Inspection of the photos shows that the normal fish was about 12 inches (30.48 cm) in length, while the amputated fish was about 10.5 (26.7 cm) inches long.

The photos of the fish were scanned from Gray’s paper [2] at a resolution of 300 pixels per inch (118 pixels per cm). Individual fish images were created with a common background, based on the grid in the original images, to provide a fixed coordinate system. Approximate midlines of the bodies of all individual fish images were created manually with Adobe Illustrator (Figure 2). This manual approach seemed to produce cleaner transverse displacement data than did our attempt to use programmed image processing methods. It is expected that small errors incurred at this step will be modally filtered by the COD [23]. The midlines were then saved as black-and-white images. Representing each fish image, each computer-generated midline was loaded into a MATLAB program. This program assigned each pixel of the midlines a value corresponding to black or white in an array. Each column and row of the image array represented an $\hat{x}$ coordinate and a $\hat{y}$ coordinate. The program recognized the $\hat{x}$ and $\hat{y}$ coordinates of the midline pixels. The length of the midline was determined, and then the $\hat{x}$ and $\hat{y}$ coordinates of $m$ equally spaced virtual markers were established along the midline. This use of virtual markers on images of an organism follows work on the motion studies of nematodes [39, 40]. The normal whiting was tagged with $m = 49$ virtual markers, while the amputated whiting had $m = 43$ virtual markers, such that the virtual markers were spaced by approximately 0.25 inches (6.35 mm).

The first virtual marker was located the tip of the head, while the last virtual marker was located the end of the tail. A set of virtual marker locations was assembled for each fish image, thereby producing $\hat{x}$ and $\hat{y}$ time histories for each virtual marker. In the computations, the fish images were essentially oriented with a horizontal fish axis (a 90-degree clockwise rotation of the images shown in the figures), and so the $\hat{x}$ coordinate information is referred to as ‘axial,’ and $\hat{y}$ coordinate information is referred to as ‘transverse.’ The positions of the virtual markers were then used as points to monitor the fish’s movements as it swam.

Gray’s set of photographs of the normal whiting represented slightly more than a full cycle. Close inspection of these images suggested that the fish spent about the first half cycle swimming nearly straight forward. During the second half cycle, the right pectoral fin flipped forward and pulled back while the heading veered slightly to the left. Our aim is to quantify its straight forward locomotion pattern through one cycle. Henceforth we incorporated the first half cycle, assumed symmetry, and regenerated the second half cycle, thereby obtaining a single cycle of dominantly straight forward locomotion. From further inspection and trial decompositions, we determined that six cycles (we used the first six snapshots) best approximated a half cycle, such that $n = 12$ for a full cycle.

Gray’s photographs of the amputated whiting depicted slightly more than a half cycle of motion. Assuming symmetry between two half cycles of motion, we reconstructed the second half cycle to create a full cycle of motion. We used the data from images two through eleven, such that the half cycle included ten data, and thereby $n = 20$ for a full cycle of amputated fish motion.

We completed both motion cycles by taking the images of the first half cycle of motion, and duplicating them by flipping them about the neutral axis of swimming. As such, the second half cycle was a mirror image of the first half cycle. If the transverse deflection data were flipped about an arbitrary axis, the oscillatory motion of the virtual markers would undergo jumps at the half cycle instants. To prevent this distortion, the neutral axis of swimming was determined by finding the mean of the
marker positions at the first and seventh time samples (for the intact swimmer). If the sampling is approximated as commensurate, the seventh sample would represent the start of the second half cycle, and would serve as the cyclic opposite of the first sample, regardless of the phase of the oscillation. The neutral axis values of each virtual marker was subtracted from its half-cycle time series, so that the transverse data neutral axis was translated approximately to \( y = 0 \), and then the data was reflected about the new neutral \( x \) axis to complete the cycle. (Another option for determining the neutral axis would be to fit a line to the means of the half cycle endpoints. In this case, the virtual marker neutral axis points had a very small variation with a trend, instead of a random distribution, and so the marker means themselves were used.) The neutral axis was a constant axis, and therefore the resulting full cycle of motion remained in a fixed Newtonian \((x, y)\) frame.

The set of \( y \) time histories for all virtual markers were stored in an \( m \times n \) ensemble matrix \( Y \), for which the \( i \)th row included the \( y \) time history of the \( i \)th virtual marker.

A superposition of one cycle of samples of the fish markers, after the treatments above, is shown in Figure 3 (a) for the normal fish, and Figure 4 (a) for the amputated fish.

2.2 Complex Modal Decomposition

Subsequently, a one-dimensional COD was applied solely to the transverse data of the fish. To apply COD, we first needed to convert the sampled real oscillatory \( y \) values into complex analytic oscillatory \( z \) values. We did this by the half-spectrum inversion method, in which the fast Fourier transform (FFT) is computed and multiplied by two, the negative frequencies are nullified, and the inverse FFT is applied [7, 41]. The complex analytic signals can also be obtained by using the Hilbert transform [7, 41]. The \( z \) vector, whose elements represent values for each virtual marker, when sampled through time, comprises the complex ensemble matrix \( Z \) for transverse motion. Similar to \( X \) and \( Y \), the \( i \)th row of \( Z \) included the time history of the \( i \)th virtual marker. The \( m \times m \) complex correlation matrix was then constructed as \( R = ZZ^T/n \). The eigenvectors \( w_i \) and eigenvectors \( \lambda_i, i = 1, \ldots, m \) of \( R \) are obtained as the COMs and COVs. Special properties of this eigensolution include orthogonality, that is \( \bar{w}_i^T \bar{w}_j = 1 \) for two of the same normalized eigenvectors, and \( \bar{w}_i^T \bar{w}_j = 0 \) for two different eigenvectors [7]. The rank of \( R \) is bounded by the minimum of \( n \) and \( m \), which in our case is \( n \). As such, at most, \( n \) meaningful modes can be expected. Indeed, fewer than \( n \) meaningful modes are expected, as many modes tend to have insignificant participation and are noise dominated. Since our fish undergoes a single cycle of motion, we expect a mode corresponding to the fundamental frequency of undulation, with higher modes corresponding to harmonics, most of which will be small noisy contributions.

Once the COMs, \( w_i \), are obtained, we can look at the motion associated with these modes. The premise is that the total motion is a sum of modal motions (e.g., see [42]), such that, in ensemble form, \( Z = WQ \), where \( W \) is the complex “modal matrix” whose columns are the dimensionless normalized modal vectors \( w_j \), and the rows of \( Q \) are samplings of the complex “modal coordinates.” Then, this complex modal coordinate ensemble is \( Q = W^{-1}Z = W^T Z \). Ensemble matrices \( Z \) and \( Q \) both have units of length. Reduced modal motion can be constructed as \( Z_r = W_r Q_r \), where \( W_r = [w_1, \ldots, w_k] \), \( k < m \), and the rows of \( Q_r \) are samplings of the associated modal coordinates \( q_j(t) \),
is the modally reduced motion [42]. If a small number of modes are “active”, then $Z_r \approx Z$. This process can be used as a filter to “purify” the response based on the deterministic modal activity. The modal filter is not perfect, because noise will also infiltrate the dominant modal coordinates. But the modal filter can be effective.

It may be useful to sketch an interpretation of a complex modal motion. The harmonic motion in a complex mode $z(t) = e^{\omega t} u$, where $z$ is a vector of complex analytic particle positions, $t$ is time, $\alpha = \gamma + \omega i$, and $u = c + di$ is a complex mode, with $\gamma$, $\omega$, $c$ and $d$ being real scalars and vectors, can be expressed in real form (for example by combining with its complex conjugate, $\bar{z}(t)$) as

$$y(t) = e^{\omega t} \left[ \cos(\omega t) c - \sin(\omega t) d \right].$$  \hfill (2)

Thus a complex mode induces an oscillation with a continual cyclic transition between the shapes $c$ and $d$. The relative sizes and degree of independence of $c$ and $d$ dictate the “amounts” of standing and traveling in the wave.

Indeed, the work of Videler and Hess [13, 15] on the kinematics of saithe and mackerel was founded on essentially the same concept, although the terminology “complex modes” was not used. Videler and Hess expressed the motion in a Fourier series comprising of discretizations of associated functions $c_j(x)$ and $d_j(x)$, which play the role of real and imaginary part vectors $c$ and $d$, as coefficients of time varying harmonic terms $\cos(j \omega t)$ and $\sin(j \omega t)$ for a given point $x$ along the midline. In their work, the elements of vectors $c$ and $d$ were estimated by using Fourier transforms and splines for up to three odd harmonics. The COD presented here is a method of packaging this estimation and quickly estimating these coefficients as complex modal vectors, which result from an eigenvalue problem.

The non-dimensional nonsynchronicity index, or “traveling index” for mixed traveling waves, quantifies the dependency between the real and imaginary parts of a COM as the reciprocal of the relative condition number between the real and imaginary parts of the vector. A traveling index value of zero indicates no independence between the real and imaginary vectors, thus a standing wave, and a traveling index value of 1 represents complete independence between the two vectors, meaning a “fully” traveling wave [7].

Furthermore, we can dissect the COM vector as $w = w_s + w_t$, where $w_s$ is a purely standing addend, and $w_t$ is a purely traveling (or nonsynchronous) addend. Then $w_s = c_s + i d_s$ and $w_t = c_t + i d_t$. For example, if $\|c\| \geq \|d\|$, then the standing addend of $d$ is the vector addend that is parallel to $c$, such that $d_t = d \cdot e_c$, where $e_c = c / \|c\|$ is the unit vector in the direction of $c$. Then the traveling vector addend of $d$ is $d_t = d - d_t$, which is the part that is normal to $c$. The traveling addend of $c$ is the piece of $c$ of the same size as $d_t$, such that $c_t = |d_t| e_c$.

Finally, the standing part of $c_t = c - c_t$. Then the modal motion $Z_r = w Q = w_s Q + w_t Q = Z_s + Z_t$ is now separated into standing and traveling parts.

The eigenvalues, $\lambda$, of $R$ produce mean squared amplitudes, in units of length squared, of the modal coordinates. Examination of the $\lambda_j$ can indicate how many modes are active and significant. Since $|w_j| = 1$, the average of the mean amplitude squared of the markers on the fish is $\lambda_j / m$. Thus $\lambda_j / m$ provides an estimate of the mean modal amplitude squared of deflection along the midline of the fish. The eigenvalues can also be used as indicators of the “signal energy” of the associated modes.

COD can be performed with the goal of isolating a single mode of interest, and then representing the mode by the real and imaginary parts of the extracted complex mode, quantifying the motion parameters based on the complex mode and modal coordinate, enabling visualization and computation of additional quantities of interest, and “purifying” or isolating the motion in terms of the extracted mode. This is the aim of the current study. Another goal of COD can be to extract multiple modes and study properties of the modal spectrum, which can be a topic of future study.

3 RESULTS

We applied the image processing, data analysis, and COD, as discussed above, to both the normal and amputated fish. This section shows results of the COVs, COMs, and further analysis including the use of the modal coordinates, applied to both the normal and amputated fish.
The COVs were used to establish modal dominance. For the intact whiting fish, the primary COV had a value of 7.98 in², and the next highest values were 0.111 in² and 0.0077 in². The rest of the COVs were below 10⁻¹⁴ in². As such, the primary mode captured 98.5% of the signal energy. Scaling out the number of markers, the dominant mode indicates a mean squared amplitude of 0.163 in² on average per marker, or a root mean squared amplitude of about 0.404 in on average per marker. We will study the dominant mode as a representation of the swimming motion, and consider the second mode as mainly composed of the transverse drift and rotation of the fish, as well as the errors of the digitization of the photographic images. If the dominant swimming mode is a nonlinear mode, the second mode may also include higher-order information about the mode. The COVs for the amputated whiting were 13.01, 0.128, 0.0229, 0.00677, and 0.00336 in², and smaller. Once again, the dominant eigenvalues suggest that the amputated-fish motion can be well approximated by one mode, as the dominant mode had 98.8% of the signal energy. The dominant mode suggests a root mean squared amplitude of 0.550 in on average per marker.

The eigenvector (COM) associated with the greatest eigenvalue (COV) corresponds to the shape of the dominant complex mode. The dominant COM, \( \mathbf{w}_1 \), in the complex plane, and also as both the real and imaginary parts of the complex mode, are plotted in Figure 5 for the intact whiting.

The first modal coordinate was extracted as \( \mathbf{Q} = \mathbf{w}_1^T \mathbf{Z} \) and is plotted in Figure 6 in the complex plane and as an oscillation via its real part. The complex modal coordinate goes through nearly a single period, as the next sample would come close to the starting point (and be part of the next cycle). This examination of the modal coordinate for various values of \( n \) helped determine \( n = 12 \) as a nearly complete cycle of oscillation.

Applying the modal coordinate according to equation (1) produces the single mode motion shown in Figure 3 (b). Comparison with Figure 3 (a) shows how noise from the raw motions is removed from the modally filtered motions. The modally filtered motion of the intact whiting bears some visual similarity to the animated snapshots of carangiform motion for a mackerel model shown in [18]. We calculated the traveling (or nonsynchronicity) index of the dominant mode for the normal whiting fish as 0.5205, which indicates significant, but not pure, traveling.

The dominant complex mode of the amputated whiting is shown in Figure 7. Applying the modal coordinate according to equation (1) produced the single mode motion shown in Figure 4 (b). Again, comparison with the raw motions in Figure 4 (a) suggests that the motions are dominantly of a single mode, and that small noise from the raw motions is removed from the modally filtered motions. The traveling index was 0.5209. The index suggests a similar amount of nonsynchronicity, which we take to be
traveling, in the movements of both the intact and amputated fish in the Newtonian frame. We will discuss this later.

The first modal coordinate is plotted in Figure 8 in the complex plane and as an oscillation via its real part. Again, the complex modal coordinate goes through nearly a single period, as the next sample would come close to the starting point (and be part of the next cycle). This examination of the modal coordinate for various values of \( n \) helped determine which images to include in the complete half cycle, and in determining \( n = 20 \) as the number of images in a nearly complete cycle of oscillation. In comparison with the normal fish, the amputated fish modal coordinate shows a more sawtoothed wave form in the plot of the real part versus time (Figure 8 (b)).

We estimated the frequency, wave number, and wavelength, from the primary COM and modal coordinate. The frequency, \( f \), was estimated from the mean whirl rate of the dominant modal coordinate in the complex plane (Figure 6 (a)). The mean frequency through the full cycle for the normal whiting was \( f = 1.6462 \) cycles per second. This compares closely to the frequency of the nearly one-period time record of \( 1/(14\Delta t) = 1.6667 \) Hz.

Wave speed \( c \) can be estimated from estimates of frequency and wavelength \( L \), as \( c = Lf \). The wavelength can be obtained from estimate of the wave number \( k = 2\pi/L \). Calculating the wave number was a delicate issue. The spatial waveforms are not harmonic and may not make a full cycle across the length of the fish. Particularly, that of the amputated fish seems to display only about a half cycle across the length of the fish. Inspection of the spacing of zero crossings of the real and imaginary parts of the normal whiting’s complex mode suggests that the wavelength varies in space. Inspection of the real and imaginary parts of the amputated whiting’s complex mode suggests that the two parts have different wavelengths. These features make it difficult to estimate the wavelength from the peak-to-peak characteristic of an oscillatory waveform, and points us toward estimating the wave number from spatial whirling rates as done in [23]. Thus, in calculating the wave number we examined the whirl of the complex mode in the complex plane. One whirl corresponds to a wavelength. Furthermore, since the waves have mixed standing parts, which do not travel, we also estimated wave numbers from spatial whirl rates of the traveling addends of the modal vectors. This provides a local wave number, and hence a local wavelength, along the fish’s body (Figure 10 (a)).

Wavelength makes more sense for the traveling part of the wave. The traveling and standing addends were separated and re-animated as \( Z_t = w_t Q \) and \( Z_s = w_s Q \) (in ensemble form), and are shown in Figure 9 for the normal whiting. To the casual eye, the “traveling” addend in Figure 9 (a) does not seem to have a traveling advantage over the full modal oscillation shown in Figure 3 (b). However, the traveling addend indeed has a traveling index of one, such that it is fully nonsynchronous, and about double that of the full modal oscillation (0.5205). The standing addend in Figure 9 (b) is clearly standing. The sum of these addends produces the motion in Figure 3 (b). The distribution of the local wavelength is shown in Figure 10 for the traveling addend.

We sought wave number estimations of the normal swimming motions, and found that the estimated wave number was not uniform. Presuming the thrust bot be dominated in the posterior of the fish, we take interest in the wavelength of the tail area, which fluctuates about 10 in/cyc (25 cm/cyc). The wave speed is then about 16 in/sec, well above mean observed speed of the fish (5.2 in/sec) for this interval of its swimming.

The mean frequency from the whirl rate of the amputated fish’s complex modal coordinate was 0.9809 cyc/sec. For reference, the 20 images used to approximate a cycle corresponds to a frequency of 1.0 Hz. The separated traveling and standing aspects of the motion are shown in Figure 11. The relative amplitudes of each aspect of motion is visually slightly more comparable than that of the normal fish (Figure 9), although the traveling index of the amputated motion is not strikingly different.

![Figure 7](image1.png) ![Figure 8](image2.png)
Wavelengths obtained from the full complex mode and from the traveling addend (Figure 10), calculated in the same way as for the intact fish, show large variability across the length of the fish. The mean full complex wavelength, which due to its coarseness may not be highly relevant for relating to the propulsion of the fish, was 30 in/cycle, while that of the traveling addend was 35 in/cycle, corresponding to mean wavespeeds of about 30 and 35 in/sec.

4 DISCUSSION

Interpretations will be made first for the normal whiting, and then the amputated whiting. In both cases, limited data was used; we analyzed a single cycle built from a measured half cycle, assumed to be a symmetric part of a full cycle and reflected about the neutral axis. In working with this extrapolated single cycle, we were not able to quantify long-term average swimming behavior, rather behavior specific to a single cycle, in a single fish.

We would expect the modal characteristics to vary during transient swimming, such as burst swimming or start up, turning, and breaking. Even with steady swimming, it would still be beneficial to examine large data sets. The computational tool is easily feasible for much larger time records. For example, in studying nematode crawling, 1440 time samples for 13 virtual markers were used with computational ease [23].

The single cycle of data from Gray [2], as it was noted by Videler [13], were obtained when the pectoral fins were extended, indicating that the fish was not swimming in its most efficient manner. Nonetheless, the usefulness and feasibility of the approach can be seen. In this cycle of this swimmer, a single mode was sufficient to represent the dominant characteristics of the transverse deflection of the midline. Higher modes may include small deterministic components, but in this case are noise dominated.

The frequency and wave speed can be estimated from the output of the COD process. Wave speed, however, depends on wavelength, whose accuracy is uncertain as it was computed from a mode that was not simply sinusoidal. The wave number was estimated from the geometry of the mode in the complex plane. This calculation suggested that the wavelength varies across the axis of the fish. Indeed, from Euler-Bernoulli beam theory, the wavelength is expected to be a function of frequency, dependent on beam parameters such as the Young’s modulus, $E$, area moment of inertia, $I$, and mass density [43]. The fish is clearly inhomogeneous, such that the values of $I$ and $E$ (should it be defined) would not be uniform. Thus, for a fish forced at a given frequency, it would not be unreasonable to produce an inhomogeneous wavelength. The roughly estimated wave speeds well exceeded the swimming speed.

Previous works on hydrodynamical modeling have used fish motion profiles as input. The motion model, $y = h(x - ct) = a(x)\sin(kx - \omega t)$, with an experimentally observed amplitude.
a(x) and uniform wave speed, has been used as a good first approximation, but the option of using complex modal oscillations may help in refinement of the hydrodynamical modeling. For this purpose, the motion can be reanimated using a complex exponential with the estimated frequency, modulating the complex mode shape, as shown in Figure 12 for both the intact and amputated fish.

Interesting observations arose from Gray’s photos of the amputated swimmer. The absence of the caudal fin, according to Gray, does not inhibit the fish from swimming effectively, even though the motion was observed to be quite different. The normal fish swam at about 5.2 inches per second (about one half length per second), while the amputated fish swam at about 4.6 inches per second. Gray made the subjective comment that he saw “very little evidence of a transmitted wave along the muscular tail.” Gray’s sketch of the animated motion of the amputated fish supports this comment, as the sketch shows the oscillation of the tail, relative to the anterior half of the body drawn fixed, visually indicates a standing wave. While the fish did not produce the transmitted wave along the muscular tail, it produced a transmitted wavy motion relative to a fixed frame, in which the traveling index is significant, and the motion has modulating traveling and standing addends. Indeed, Gray believed that the amputated animal generated thrust from the interaction of its body with the water as a result of its movement relative to a fixed frame. The whiting fish has a full set of dorsal and ventral fins (Figure 1), and any action of these fins during the swimming motion has not been isolated from Gray’s images, nor referenced in Gray’s paper.

Not shown are decompositions under variations in how the data was processed. When the transverse component of the fish centroid was removed from each frame, the modal motion resembled that of Borazjani and Sotiropoulos [18] more closely. Decompositions from an anterior-body-fixed frame yielded traveling indices of 0.48 for the intact fish versus 0.046 for the amputated fish, consistent with Gray’s observations about wave transmissions along the body.

Gray concluded that mechanical action of the water and caudal fin helps realize the traveling wave in the intact fish. We might speculate that while the caudal fin in the intact fish contributes thrust in the swimming direction of the non-amputated fish, it may act as a nonproportional damping element for the transverse deflection of the fish, and thereby mechanically contribute to the complex nature of the mode. It may be worth considering that a strong swimming fish such as a carangiform swimmer may resonate the mode shape of its body in interaction with the water when swimming. Exploiting resonance may allow the fish to achieve significant oscillation amplitudes with minimal effort, and thereby play a role in how the fish can swim efficiently.

Much of the interpretation of our results is therefore dependent on the accuracy and conditions of Gray’s work. For example, we were able to quantify modal characteristics over a single cycle of motion, for a fish swimming with extended pectoral fins, and for a swimming amputated fish. Our results also hinge on the accuracy of the transverse deflections we obtained from the images. We are following up with transverse motion estimations with an improved datum.

5 CONCLUSION

This work demonstrated the application of a modern vibration analysis tool to the photographic images of a swimming fish by Gray, thereby expanding on his studies of fish movement. We processed Gray’s images to obtain transverse deflections of the midline of the fish, and then conducted a complex modal decomposition to quantify the motion pattern.

The decomposition produced a complex modal vector whose real and imaginary vector parts contain information on the mixed standing and traveling wave characteristics. The motion was dominated by a single mode. Measurement noise and minor fluctuations were “modally filtered” and the single-mode motion was examined. The frequency of oscillation was cross checked using the complex modal coordinate. The wavelength was approximated, and the results suggest a wavelength that varies with the location along the axis of the fish, which is not surprising if the fish is considered with reference to a non-uniform slender beam.

Quantifying the mechanics of the swimming motion serves multiple purposes. It gives us a better understanding of the motion of a fish as it swims, which is of value for the understanding of efficient water borne motion, application to biomimetic robotic locomotion, and for the knowledge of fish themselves. The kinematics are important as input to fluid mechanics analyses. Perhaps the methods here can be applied as input in future
refinements of hydrodynamical models.

In this work, new information was extracted from old images collected by Gray in the 1930s. The work demonstrates a method that can be added to the toolbox for quantifying fish kinematic parameters. It would be interesting, and feasible, to apply the method to longer time series data of transverse midline deflections during steady or unsteady swimming. Quantification of average steady behavior can be applied to classification of the types of swimming motion, variation among species, or even individuals.

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6 References

REFERENCES


